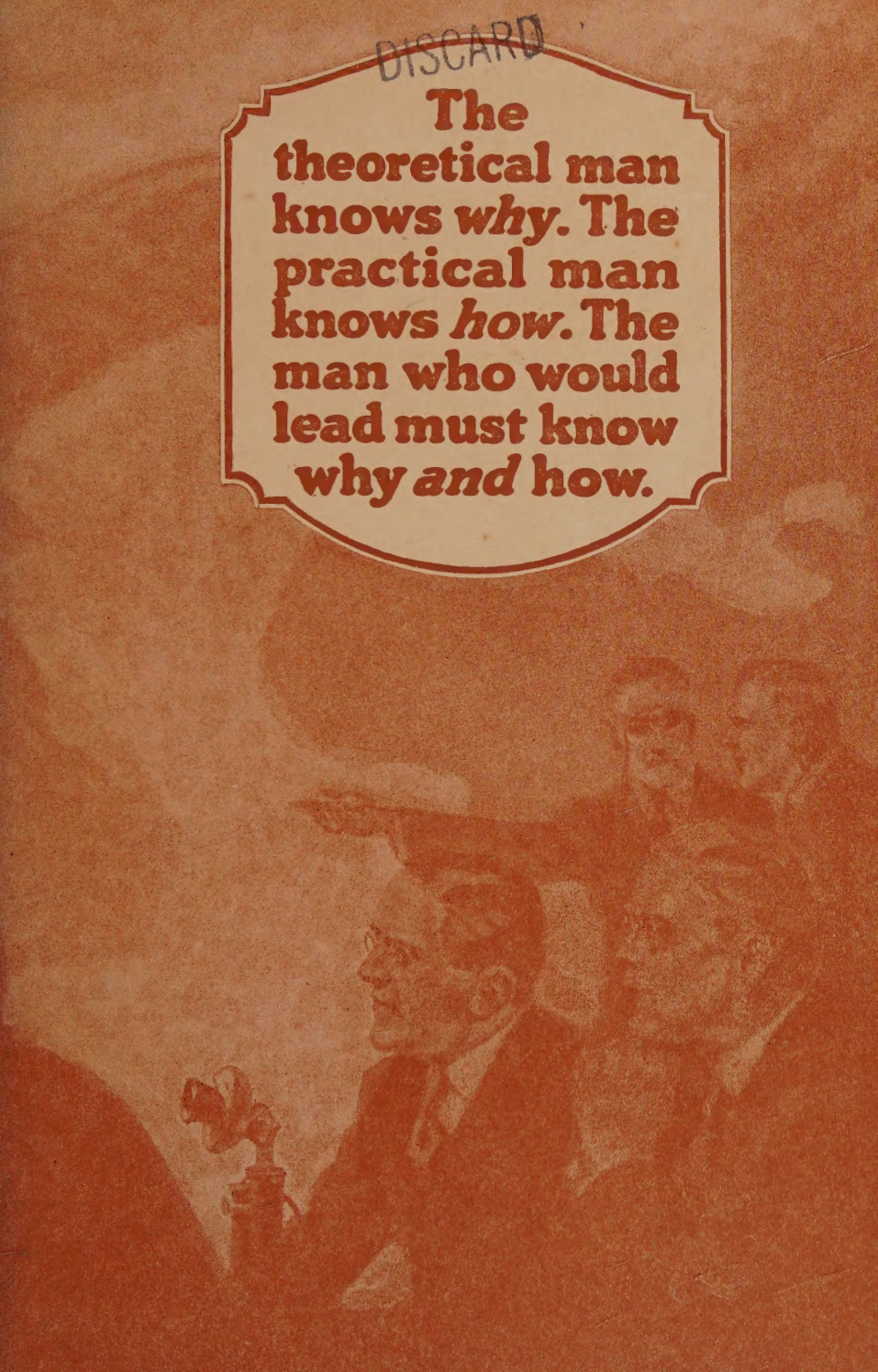
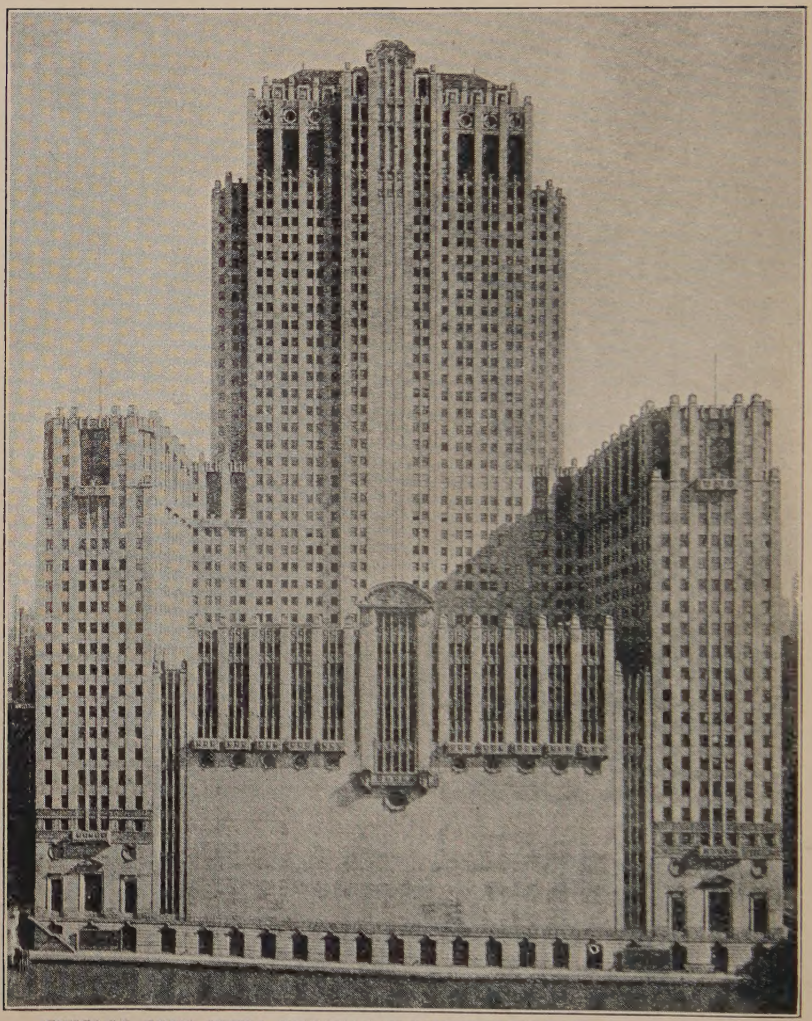
CYCLOPEDIA OF DRAWING

DISCARD

The theoretical man knows *why*. The practical man knows *how*. The man who would lead must know why *and* how.





CHICAGO CIVIC OPERA BUILDING IS EXCEPTIONALLY WELL DESIGNED
This view is the west elevation overlooking the Chicago River.
Courtesy of the International Newsreel

Cyclopedia *of*
Drawing

A Reading Course and General Reference Work on

Mechanical and Architectural Drawing, including Lettering and Design, Blueprint Reading, Machine Drawing and Design, Structural Drafting, Perspective and Freehand Drawing, Rendering in Wash and in Pen and Ink, Sheet-Metal Pattern Work, Shades and Shadows, Etc.

Prepared by a Staff of
Architects, Engineers and Instructors

Illustrated With Over Seventeen Hundred Engravings
Five Volumes

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Grateful acknowledgment is here made also for the valuable cooperation and advice of many of the foremost architects, engineers, and designers in making these volumes thoroughly representative of the latest and best in drafting practice.

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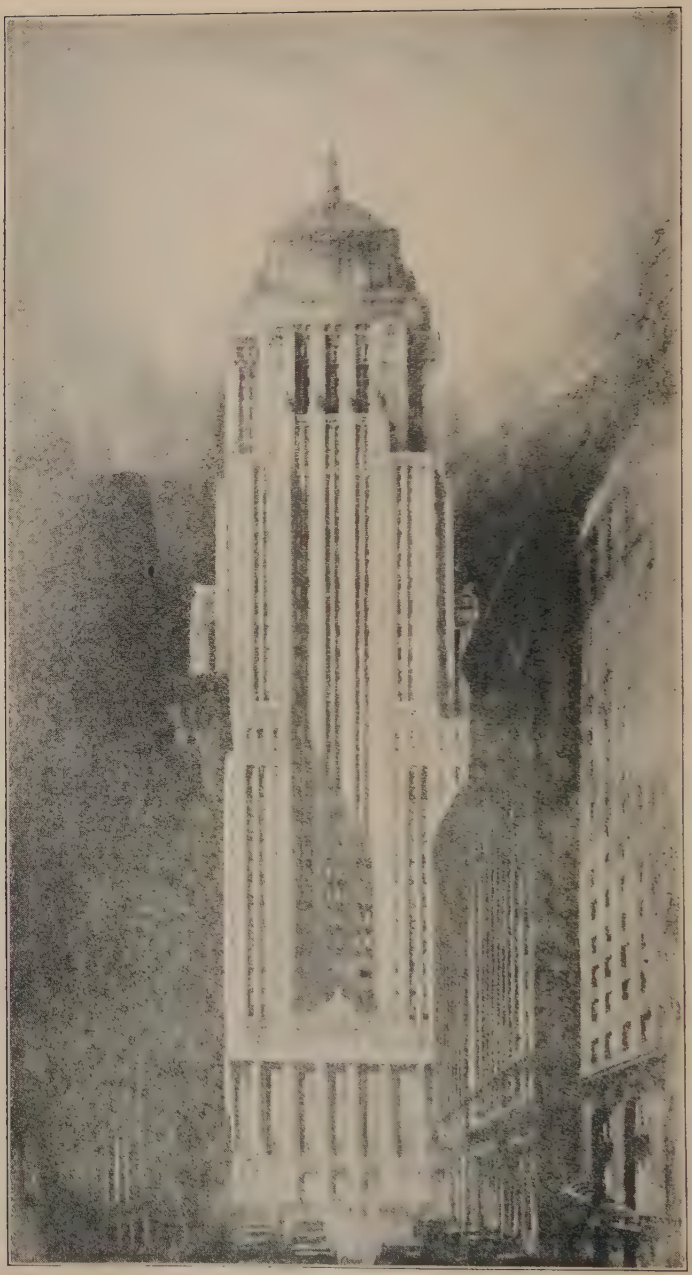

ARCHITECT'S DRAWING OF THE NEW BOARD OF TRADE BUILDING, CHICAGO

Courtesy of Underwood & Underwood

Foreword

THE position of draftsman in architectural or engineering work is a very important one. The trained architectural draftsman will execute a set of plans for a house or flat building with a dispatch and precision which is a delight to anyone sufficiently acquainted with these details to appreciate the ability shown. The knowledge of the conventions used, the accuracy of scale, the methods of representing this or that detail of construction show a wonderful facility of execution and broad acquaintance with such work. Likewise, the structural draftsman must know the many conventions used in designing the beams and girders in a sixteen-story steel building; he must know the methods of designing; he must, in a way, check the calculations of the designer; and he must give just the specifications necessary for the steel fabricator and for the contractor on the work. Again, the machine draftsman and designer must be able to translate in perfectly understood terms the details of a complicated machine; he must understand mechanics so that he will visualize the operation of the machine; he must draw just the details necessary to make clear to the pattern maker, the die cutter, or the shop man how the different parts are to be fashioned.

¶ All of these details represent advanced processes in architectural and engineering drafting. And yet, these same men who show such skill had to acquire their training through careful and painstaking study and practice of the rudiments of mechanical drawing. They had first to lay the foundation for the more advanced work by learning the kind of equipment necessary for drawings of various kinds, the use of the T-square, the triangle, the ruling pen, and all the other instruments which must be used from time to time. They had to train their eyes to visual-

ize objects and measure distances and their hands to draw, with precision, lines of uniform width and accurate direction. They had to learn the rules of geometrical construction; the methods of representing plans and elevations of objects; and the principles of orthographic and isometric projection and profile work.

¶ These volumes have been compiled with the idea of unfolding to the reader the whole subject of drawing and design, of giving draftsmen trained in one line accurate information along other lines. The material will also be found interesting and instructive for those who are not directly engaged in drafting work. An especially attractive feature is the section on "Blueprint Reading," which has been written with the idea of giving any workman the required amount of practical knowledge to properly interpret the blueprints which are such a necessary part of every job to be performed.

¶ Each article has been prepared with the utmost care by an acknowledged expert, and the wealth of material in the form of plates, art studies, and practical problems should make the work an invaluable source of information and reference.

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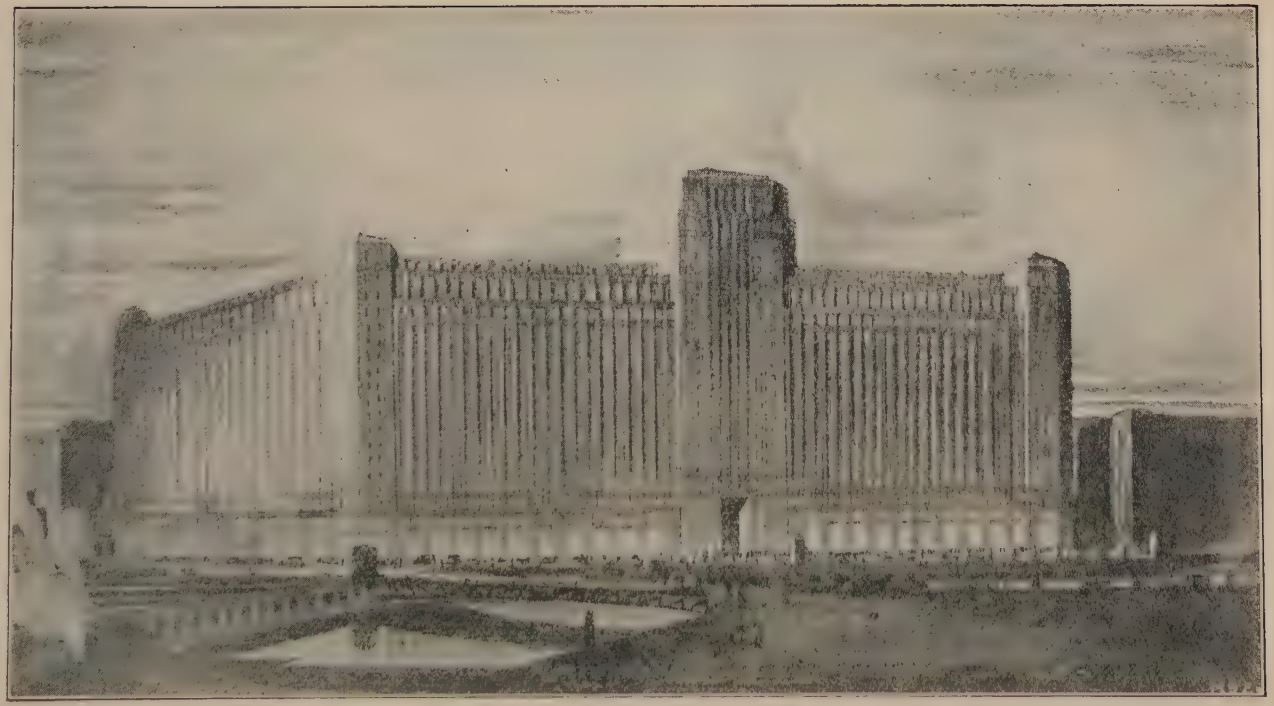
†For professional standing of authors, see list of Authors and Collaborators at front of volume.



ARCHITECT'S DRAWING OF THE MERCHANDISE MART OF CHICAGO

The World's Largest Building

Courtesy of Underwood & Underwood

MECHANICAL DRAWING

The subject of mechanical drawing is of great interest and importance to all mechanics and engineers. Drawing is a method of showing graphically the minute details of machinery; it is the language by which the designer speaks to the workman; it is the most graphical way of placing ideas and calculation on record. A brief inspection of an accurate, well-executed working drawing gives a better idea of a machine than a lengthy written or verbal description. The better and more clearly a drawing is made, the more intelligently the workman can comprehend the ideas of the designer. Thorough training in this important subject is necessary to the success of everyone engaged in drafting or construction work.

INSTRUMENTS AND MATERIALS

The draftsman is dependent for his success, to a certain extent, upon the quality of the instruments and materials which he uses. While he will find a cheap grade of instrument sufficient for his needs at the start, as he becomes more expert it will be necessary to procure those of better construction and finish in order to do accurate work. It is better to purchase the well-made instruments at the start, if possible.

Drawing Paper. In selecting drawing paper, the first thing to be considered is the kind of paper most suitable for the proposed work. For all the plates in Mechanical Drawing a good quality of cream drawing paper will be found satisfactory. However, if drawings are to be kept for a number of years, the best white drawing paper should be used. A good drawing paper should be strong; should have uniform thickness and surface; should stretch evenly and lie smoothly when stretched or when ink or colors are used; should neither repel nor absorb liquids; and should allow considerable erasing without spoiling the surface. It is, of course, impossible to find all of these qualities in any one paper, as great strength cannot be combined with fine surface. However, a kind should be chosen which combines the greatest number of these qualities for the given work. Of the higher grades of papers, What-

man's are considered by far the best. This paper, either side of which may be used, is made in three grades: the *hot pressed*, which has a smooth surface and is especially adapted for pencil and very fine line drawing; the *cold pressed*, which is rougher than the hot pressed, has a finely grained surface, and is more suitable for water color drawing; and the *rough*, which is used for tinting. For general work, the cold pressed is the best as erasures do not show as plainly on it, but it does not take ink as well as the hot pressed.

Whatman's paper comes in sheets of standard sizes as follows:

Cap	13×17 inches	Imperial	22×30 inches
Demy	15×20 "	Atlas	26×34 "
Medium	17×11 "	Double Elephant	27×40 "
Royal	19×24 "	Antiquarian	31×53 "
Super-Royal	19×27 "		

The usual method of fastening paper to a drawing board is by means of thumb tacks or small one-ounce copper or iron tacks. First fasten the upper left-hand corner and then the lower right, pulling the paper taut. The other two corners are then fastened, and a sufficient number of tacks placed along the edges to make the paper lie smoothly. For very fine work, however, it is better to stretch the paper and glue it to the board. Turn up the edges of the paper all the way round—the margin being at least one inch—then moisten the surface of the paper by means of a sponge or soft cloth, and spread paste or glue on the turned-up edges. After removing all the surplus water on the paper, press the edges down on the board, commencing at one corner and stretching the paper *slightly*—if stretched too much it is liable to split in drying. Place the drawing board in a horizontal position until the paper is dry, when it will be found to be as smooth and tight as a drum head.

Drawing Board. The drawing board, Fig. 1, is usually made of well-seasoned and straight-grained soft pine, the grain running lengthwise of the board. Each end of the board is protected by a side strip—$1\frac{3}{4}$ to 2 inches in width—the edge of which is made perfectly straight for accuracy in using the T-square. Frequently the end pieces are fastened by a glued matched joint, nails, or screws. Two cleats on the bottom, extending the whole width of the board, will reduce the tendency to warp. Drawing boards are made in sizes to accommodate the sizes of paper in general use.

Thumb Tacks. Thumb tacks are used to fasten the paper to the drawing board. They are usually made of steel, pressed into shape—as in the cheaper grades—or with heads of German silver, the points being screwed and riveted to them. For most work, draftsmen use small one-ounce copper or iron tacks, as they are cheap and can be forced flush with the drawing-paper, thus offering no obstruction to the T-square.

Pencils. Lead pencils are graded according to their hardness, the degree of which is indicated by the letter H—as HH, 4H, 6H,

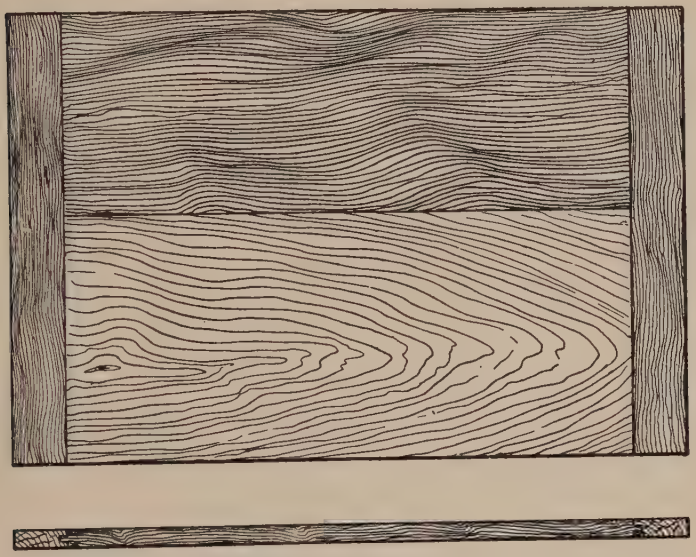


Fig. 1. Drawing Board

etc. For general use a lead pencil of 5H or 6H should be used, although a softer 4H pencil is better for making letters, figures, and points. The hard lead pencil should be sharpened as shown in Fig. 2, so that in penciling a drawing the lines may be made very fine and light. The wood is cut away so that about $\frac{1}{4}$ or $\frac{1}{2}$ inch of lead projects. The lead can then be sharpened to a chisel edge by rubbing it against a bit of sand paper or a fine file, and the corners slightly rounded. In drawing the lines the draftsman should place the chisel edge against the T-square or triangle, which will enable him to draw a fine line exactly through a given point. If the

drawing is not to be inked but is made for tracing or for rough usage in the shop, a softer pencil, 3H or 4H, may be used, so as to make the lines somewhat thicker and heavier. The lead for compasses may also be sharpened to a point although some draftsmen prefer to use a chisel edge for the compasses as well as for the pencil.

Fig. 2. Pencils Sharpened to a Chisel Point

In using a very hard lead pencil a light pressure should be used as otherwise the chisel edge will make a deep impression in the paper which cannot be erased.

Erasers. What little erasing is necessary in making drawings should be done with a soft rubber. To avoid erasing the surrounding work some draftsmen use a card in which a slit is cut about 3 inches long and $\frac{1}{8}$ to $\frac{1}{4}$ inch wide, Fig. 3. An erasing shield of thin metal, Fig. 4, is also very convenient, especially in erasing letters. For cleaning drawings when they are completed, a sponge rubber or a preparation called "art gum" may be used, but in either case care should be taken not to make the lines dull by too hard rubbing.

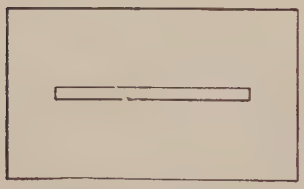


Fig. 3. Erasing Shield

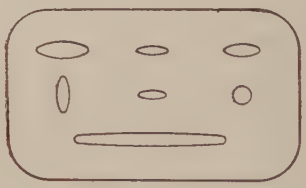


Fig. 4. Metal Erasing Shield

T-Square. The T-square, which gets its name from its general shape, consists of a thin straight-edge, called the *blade*, and a short piece, called the *head*, fastened at right angles to it, Fig. 5. T-squares are usually made of wood, the pear and maple woods being used in the cheaper grades, and the harder woods, like mahogany, with protecting edges of ebony or celluloid, Fig. 6, in the more expensive instruments. The head is designed to fit against the edge of the drawing board, allowing the blade to extend across the surface of the

board. It is desirable to have the blade of the T-square make a right angle with the head, but this is not absolutely necessary if the head is always placed against the left-hand edge of the board, for the lines drawn with the T-square will then be referred to one edge of the board only, and if this edge is straight, the lines will be parallel to each other.

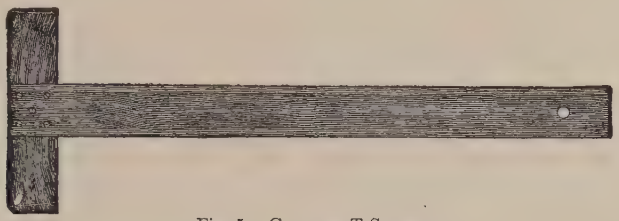


Fig. 5. Common T-Square

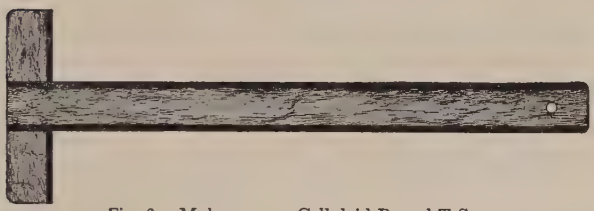


Fig. 6. Mahogany or Celluloid-Bound T-Square

T-squares are sometimes provided with swiveled heads as it is frequently very convenient to draw lines parallel to each other which are not at right angles to the left-hand edge of the board.

To use the T-square when drawing parallel lines, place the head of the T-square against the edge of the board and hold it down firmly with the left hand, as shown in Fig. 7. The lines should be drawn with the pencil against the upper edge of the T-square.

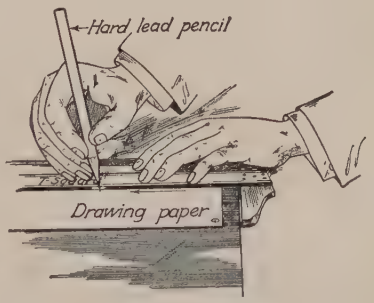


Fig. 7. Holding T-Square
Note position of hands.

It is customary to use the left-hand edge of the board, but in any event the position of the head of the T-square should never be changed from one edge of the board to another during the making of a drawing, because if lines

are drawn with the head of the T-square against a different edge than the one used when the drawing was started, they will not be parallel.

Move the T-square up or down the board when drawing the horizontal parallel lines to the different points, Fig. 8, and keep the point of the pencil squarely against the edge of the T-square. Take care not to slope the top of the pencil towards you when getting near the end of the line, because this throws the point of the pencil away from the end of the T-square, and the lines will not be parallel their full length.

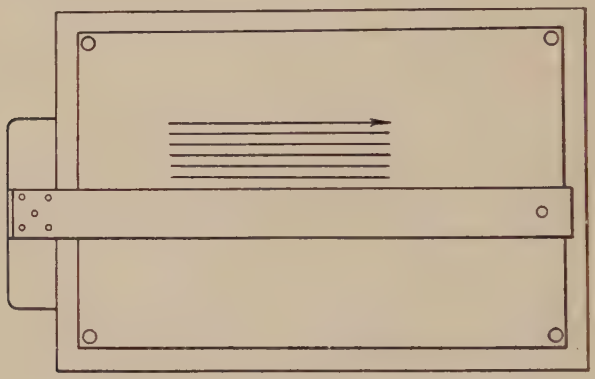


Fig. 8. Direction of Drawing Horizontal Lines

If the paper is to be cut to a certain size, it is best to use a pair of shears, but if a knife is used with the edge of a T-square as a guide, use the lower edge of the T-square.

Triangles. Triangles are made of various substances, such as wood, rubber, celluloid, and steel. Wooden triangles are cheap but are likely to warp out of shape; rubber triangles are frequently used and are, in general, satisfactory; celluloid triangles are extensively used on account of their transparency, which enables the draftsman to see the work already done even when covered with the triangle.

In using a rubber or celluloid triangle take care that it lies perfectly flat and is hung up when not in use; when allowed to lie on the drawing board with a pencil or an eraser under one corner, it will become warped in a short time, especially if the room is hot or the sun happens to strike the triangle. Triangles that are warped

can often be straightened by putting them in boiling water until they are thoroughly heated through and then laying them on a flat surface under a heavy weight and allowing them to cool in that position.

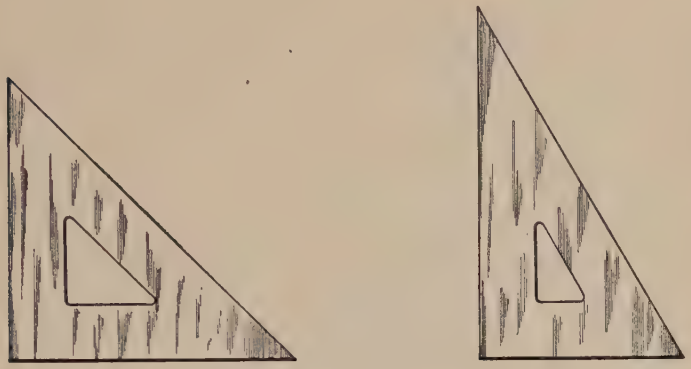


Fig. 9. 45° Triangle and 30°—60° Triangle

Triangles from 6 to 8 inches on a side will be found convenient for most work, although there are many cases where a small triangle measuring about 4 inches on a side or a large 14-inch triangle for

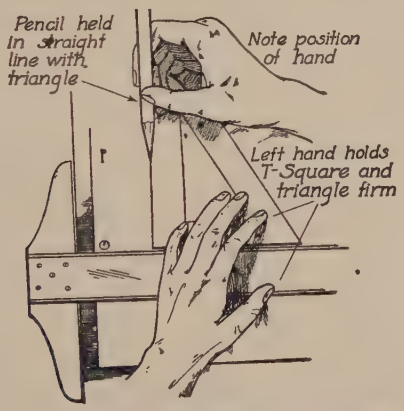


Fig. 10. Holding T-Square and Triangle
Note position of hands.

long vertical lines will be found useful. Every draftsman should have at least two triangles, one having two angles of 45 degrees and one right angle; and the other having angles of 30, 60, and 90 degrees, respectively, Fig. 9.

Triangles are used in drawing lines at right angles to the T-square, Fig. 10, and at an angle with the horizontal, Fig. 11. Notice the position of the hand when holding the T-square and triangle. When drawing vertical lines, the correct position is to have the drawing edge of the triangle to the left, and the position of the triangle should not be reversed, otherwise the lines drawn will not be parallel.

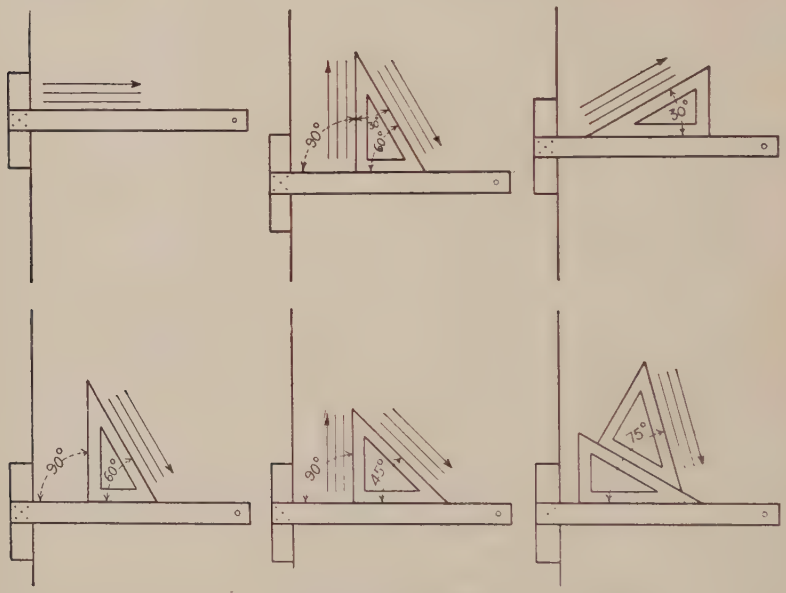


Fig. 11. Drawing Lines with T-Square and Triangles

The arrows shown in Figs. 8 and 11 show the direction in which the lines are drawn. The pencil is always pulled along, and never pushed.

If it is desired to draw a line through the point P, Fig. 12, parallel to a given line EF, two triangles should be used. First, place triangle A with one edge coinciding with the given line. Now take triangle B and place one of its edges in contact with the bottom edge of triangle A. Holding triangle B firmly with the left hand, slide triangle A to the right or to the left until its edge reaches the point P. The line MN may then be drawn passing through the point P. In place of the triangle B any straightedge, such as a T-square, may be used.

A line may be drawn through a point, perpendicular to a given line, by means of triangles as follows: Let *EF*, Fig. 13, be the given line and let *D* be the point. Place the longest side of triangle *A* so that it coincides with the line *EF*. Place the other triangle (or any straightedge) in the position of the triangle *B;* then holding *B* with

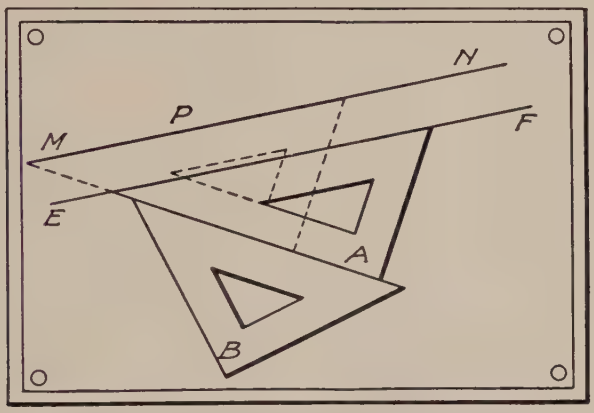


Fig. 12. Drawing Parallel Lines with Two Triangles

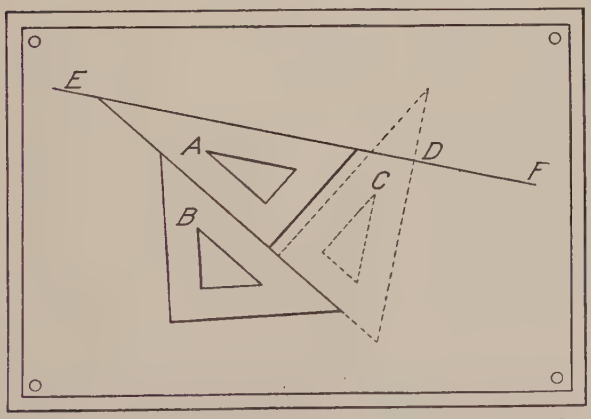


Fig. 13. Drawing a Line Perpendicular to a Given Line through a Given Point with Two Triangles

the left hand, place the triangle *A* in the position *C* so that the longest side passes through the point *D*. A line may then be drawn through the point *D* perpendicular to *EF*.

It has been shown how lines may be drawn making angles of 30, 45, 60, and 90 degrees with the horizontal. It is also possible

to draw lines forming angles of 15 and 75 degrees by placing the triangles as shown in Fig. 14.

By the use of the triangles and T-square almost any line may be drawn. Suppose it is desired to draw a rectangle having one side horizontal. First draw by means of the T-square the sides *AB*

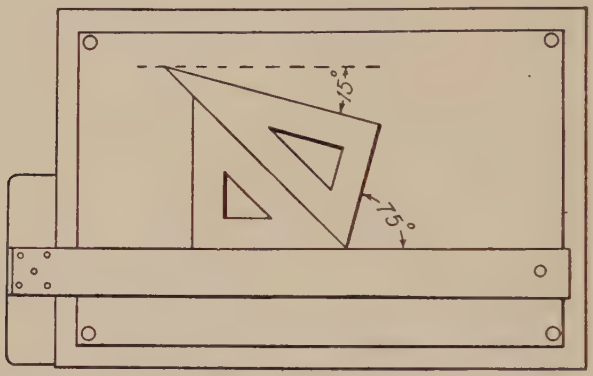


Fig. 14. Drawing a Line at 15° to the Horizontal and 75° to the Vertical with Aid of Two Triangles and T-Square

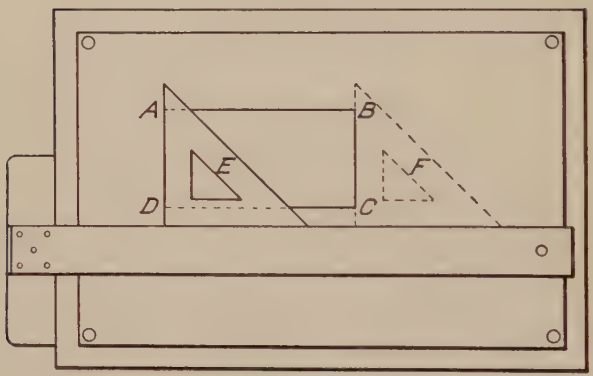


Fig. 15. Drawing Parallel Vertical Lines with Aid of T-Square and Triangle

and *DC* horizontal and parallel, Fig. 15. Now place one of the triangles on the T-square and in positions *E* and *F* draw the vertical lines *DA* and *BC*.

If the rectangle is to be drawn in some other position on the board, as shown in Fig. 16, place the 45-degree triangle *F* so that

the longest edge is in the required direction of the side *DC*. Now, hold the triangle *F* in position and place another triangle in position

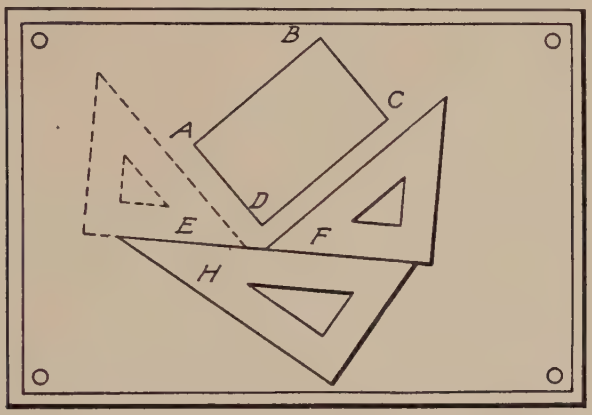


Fig. 16. Drawing Rectangle with Aid of Two Triangles

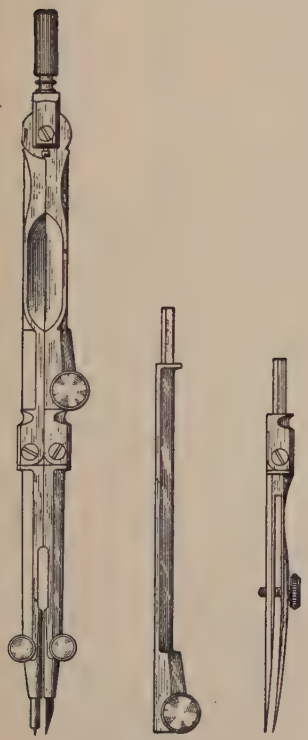


Fig. 17. Compasses and Attachments

H. By holding *H* in position and sliding triangle *F*, the sides *AB* and *DC* may be drawn. To draw the sides *AD* and *BC*, change triangle *F* to position *E* and repeat the process.

Compasses. Compasses are used for drawing circles and arcs of circles. They are sometimes made in the form shown in Fig. 17 and are provided with pencil and pen points. One leg has a socket into which the shank of the pencil or pen mounting may be inserted. The other leg is fitted with a needle point which is placed at the center of the circle.

When drawing large circles in ink, it is necessary that the needle point and the pen be at right angles or perpendicular to the paper, as shown in Fig. 18. To enable the draftsman to do this the compass legs are made with joints in them as shown in Figs. 19, 20, and 21

Some compasses are so made that the leg in which the pen and pencil attachment is to be placed is jointed in the middle, allowing the pen to be placed perpendicular to the paper, while the needle point is movable in the other leg, Fig. 22, and can be so placed that the needle point is perpendicular to the paper. The advantage of having the needle point movable is that a much larger radius can be drawn with this type of pencil than with a compass that has both legs jointed in the middle.

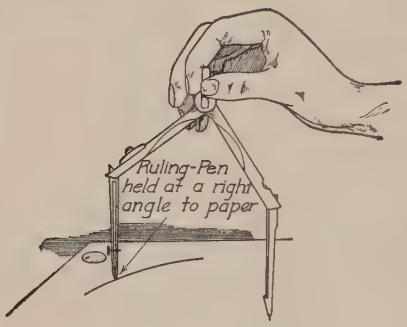


Fig. 18. Drawing Circles in Ink
Note position of compass legs.

In Fig. 23 are shown the methods of adjusting, setting, holding, and revolving the compass for drawing circles. Notice how the top of the compass is held between the thumb and the forefinger when revolving the compass.

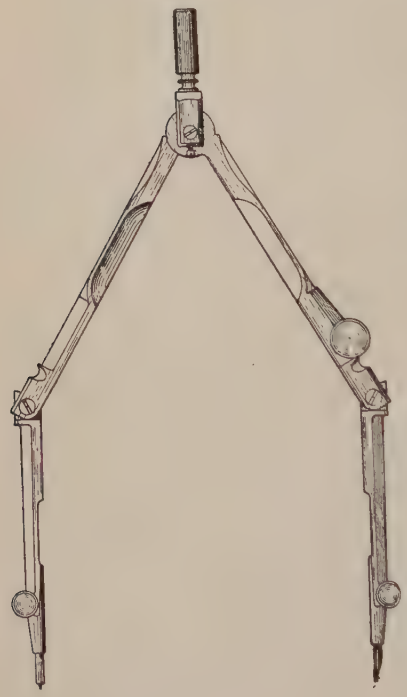


Fig. 19. Compasses Set for Drawing Circles

Bow Pen and Bow Pencil. Ordinary large compasses are too heavy and the leverage of the long leg is too great to allow small circles to be drawn accurately. For this reason the bow compasses, Figs. 24 and 25, should be used on all arcs and circles having a radius of less than $\frac{3}{4}$ inch, such as those which represent boiler tubes and bolt holes. The drop compass, Fig. 25, is made especially for drawing rivet holes. When small circles are drawn, the needle point must be adjusted to the same length as the pen or pencil point. If a considerable change in radius is made,

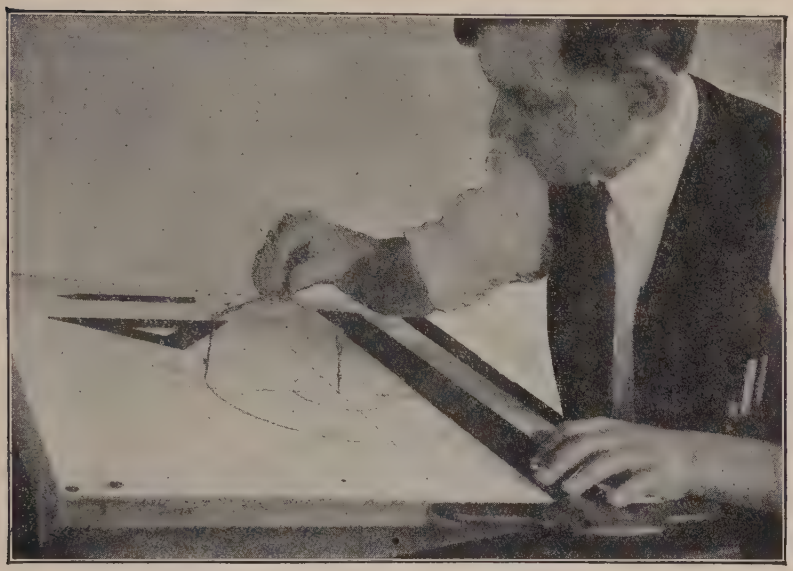


Fig. 20. Drawing Circles
Note position of hands.

Fig. 21. Drawing Large Circles
Note position of hands.

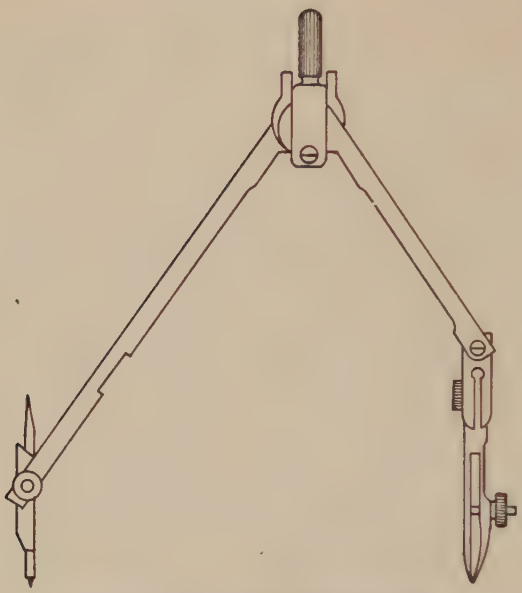


Fig. 22. Movable Needle Point Compass
Compare this instrument with Fig. 19.

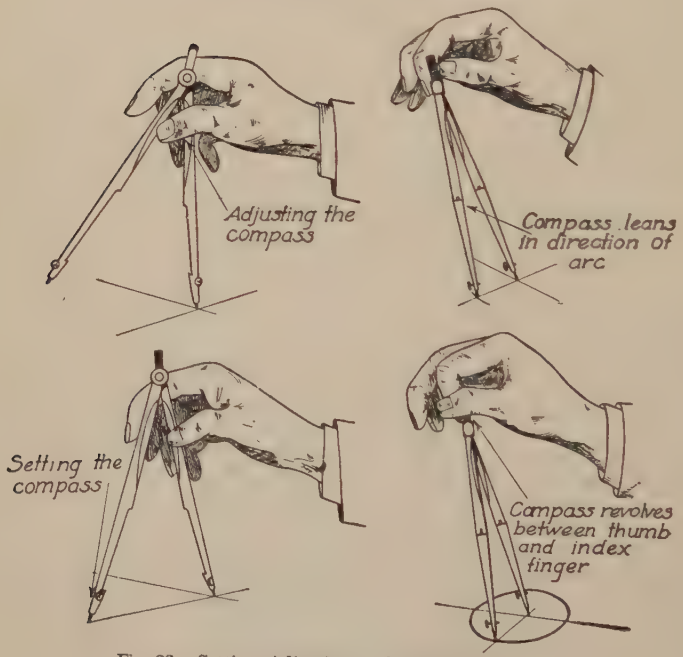

Adjusting the compass
Compass leans in direction of arc
Setting the compass
Compass revolves between thumb and index finger

Fig. 23. Setting, Adjusting, and Revolving Compass
Note position of fingers.

press the points together before turning the nut so as to prevent wear in the screw threads, Fig. 26.

When making close adjustments, the bow compass should be held as shown in Fig. 27. Notice the first finger is placed on the top of the compass, and the thumb and second finger make the

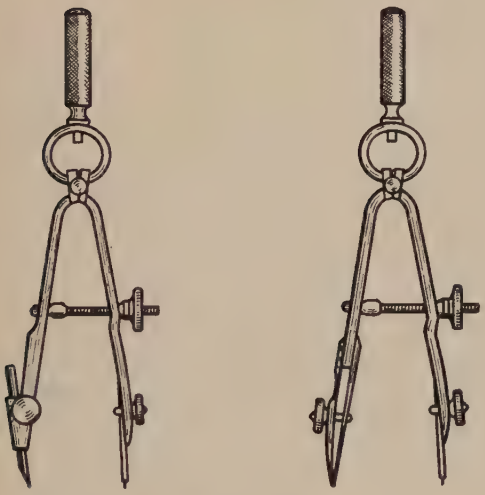


Fig. 24. Bow Pencil and Bow Pen

Fig. 25. Drop Compass

adjustment. In using the bow compass, care should be taken not to make a hole in the paper, and the pressure should be light on the top of the compass, as shown in Fig. 28.

Dividers. Dividers, Fig. 29, are similar to compasses. They are used to lay off distances on the drawing, either from a scale or from other parts of the drawing. They are also used for dividing a line into equal parts, Fig. 30. To do this turn the dividers in the opposite direction each time, or, in other words, move the point alternately to the right and to the left as shown in Fig. 31.

If the setting of the dividers is too short, as shown in Fig. 31, the distance between the legs of the dividers should be increased a little and another trial made. If the setting of the dividers is too

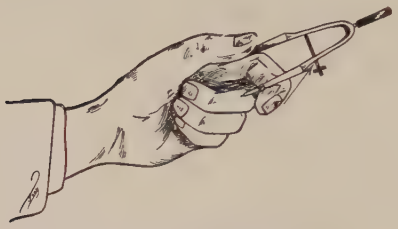


Fig. 26. Making Quick Adjustment on Small Compass
Note position of fingers.

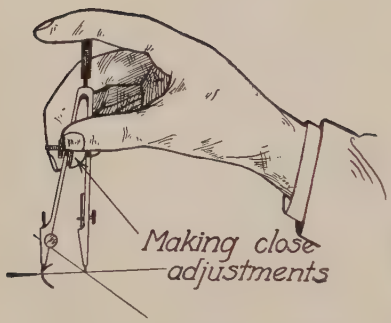


Fig. 27. Making Close Adjustments on Small Compass
Note position of fingers.

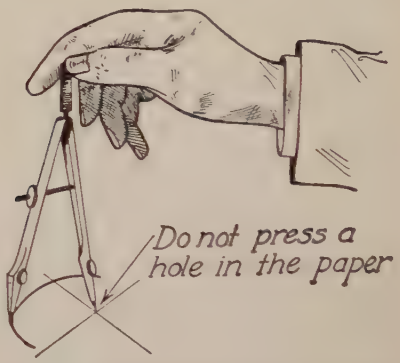
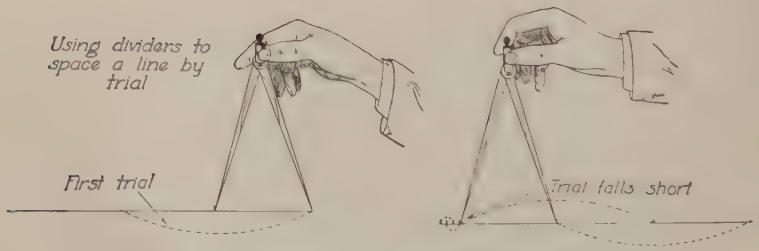


Fig. 28. Drawing Circle with Small Compass
Note position of fingers.

Fig. 29. Dividers



Figs. 30 and 31. Using Dividers for Stepping off Distances by Trial

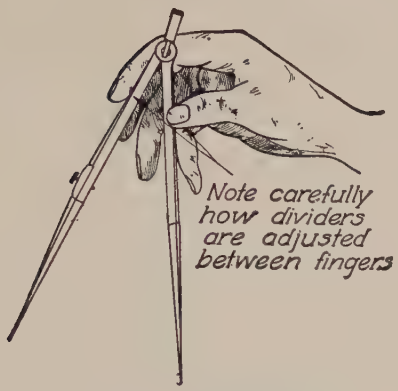


Fig. 32. Adjusting Dividers
Note position of fingers.

Fig. 33. Setting Dividers to a Drawing
Note position of fingers.

great, the distance between the legs should be shortened a little and another trial made. This method should be continued until a setting of the dividers is found that will enable the draftsman to divide the line up into the proper number of equal spaces.

Notice how the fingers are used to move the legs of the dividers when changing their setting, Figs. 32 and 33.

The bow dividers, Fig. 34, replace the ordinary dividers in small work and have the advantage of a fixed adjustment.

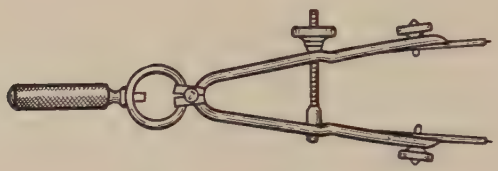


Fig. 34. Bow Dividers

The points of the dividers should be very sharp so that the holes made in the paper will be small, thus assuring accurate spacing. Compasses may be used as dividers by substituting for the pencil or pen point an extra steel point, usually furnished with the instrument. In place of dividers many draftsmen use a needle point. The needle, with the eye-end broken off, is forced into a handle of soft pine. This makes a convenient instrument for marking line intersections and distances.

Ruling Pen. For drawing straight lines and curves that are not arcs or circles, the ruling pen—sometimes called the line pen—is used, Fig. 35.

Fig. 35. Ruling or Lining Pen

The distance between the pen points, which regulates the width of line to be drawn, is adjusted by the thumb screw, and the blades are given a slight curvature so that there will be a cavity for ink when the points are close together.

When using the ruling pen, it should be held lightly against the edge of the T-square, and pulled along with the top of the pen leaning in the direction in which the pen is being moved, as shown in Fig. 36.

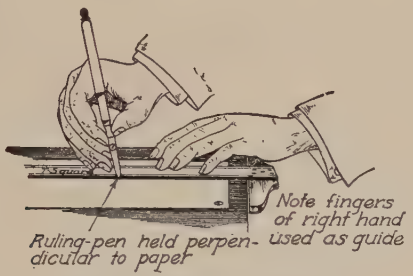


Fig. 36. Using the Ruling Pen
Note position of hand, fingers, and pen.

Scales. The scales used for obtaining measurements on drawings are made in several forms, the most convenient being the *flat*, with beveled edges, and the *triangular*. The scale is usually graduated for a distance of 12 inches. The triangular scale, Fig. 37, has six surfaces for different graduations, and the scales are arranged so that the drawings may be made in any proportion to the actual size. For mechanical work, the common divisions are multiples of two; thus drawings are made full size, $\frac{1}{2}$ size, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, etc. If

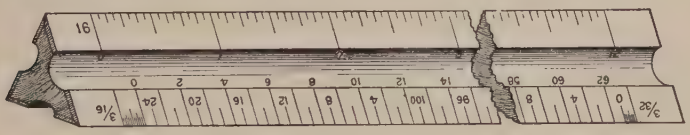


Fig. 37. Rule or Scale

a drawing is $\frac{1}{4}$ size, 3 inches equals 1 foot, hence 3 inches is divided into 12 equal parts and each division represents one inch. If the smallest division on a scale represents $\frac{1}{16}$ inch, the scale is said to read to $\frac{1}{16}$ inch.

Scales are often divided into $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, etc., for civil engineers, and for measuring indicator cards.

It is very necessary to be as accurate as possible in making measurements and in laying off dimensions. There are two methods

of laying off dimensions. One method is to lay the scale or rule on the paper and take the measurements directly from it; the other method is to set the dividers to the distance on the scale and then transfer it to the drawing by lightly pricking the paper with the point of the dividers.

The use of the dividers is the more accurate method of the two, but the use of the scale is the quicker.

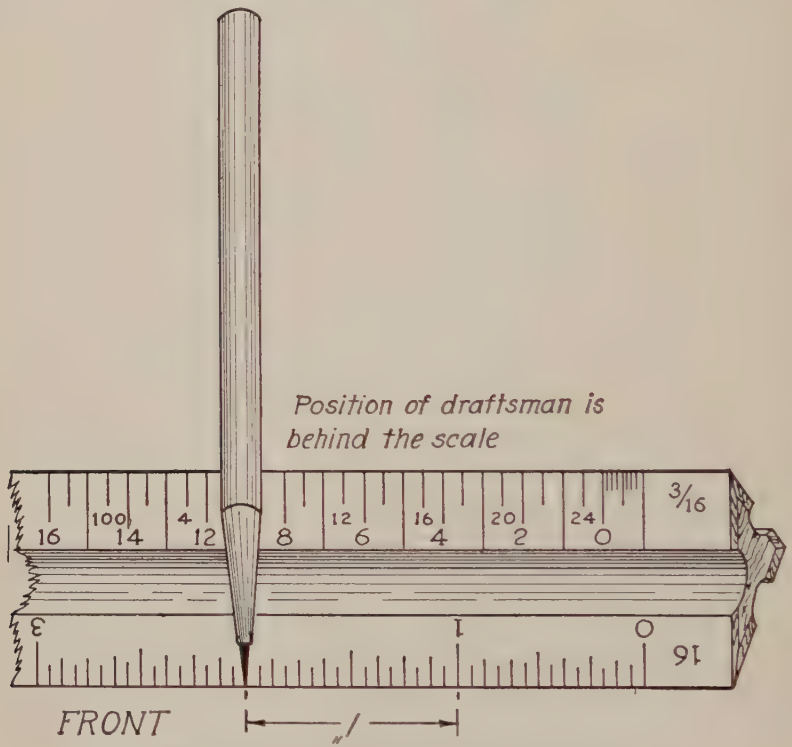


Fig. 38. Laying off Distances with Aid of Scale or Rule

When using the scale, the eye should be directly above and behind the division or line being used on the scale, and then pencil marks made on the paper in front of the scale with a sharp pencil or slight prick marks made in the paper opposite the lines between which the dimension is taken, as shown in Fig. 38.

In laying off a number of dimensions from a point or from a line, in one continuous direction, if the measurements are made progressively, as shown in Fig. 39, and there is a slight mistake

made in the first measurement, it will throw out all the others; but if the measurements are made from one definite point, as in Fig. 40, the total amount of error will be much smaller.

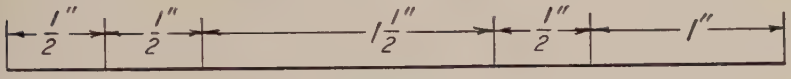


Fig. 39. Measuring Progressively
Not correct

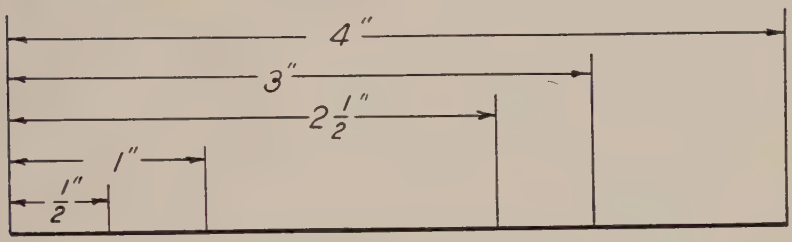


Fig. 40. Measuring from a Definite Point
Compare this with Fig. 39.

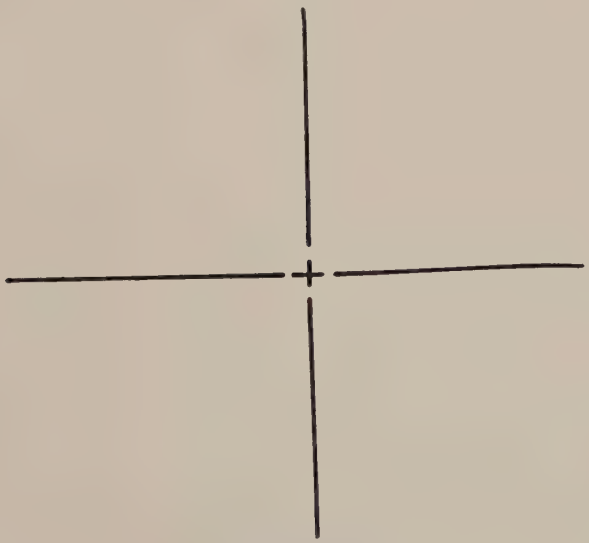


Fig. 41. Center Lines

Where accurate measurements are to be laid off, such as equal distances each side of a center line or the division of a circle into equal parts, the dividers are the best instrument to use. When

stepping distances off along a line or around a circle, the points of the dividers should be placed as near the center of the line as possible, because the slightest mistake will be multiplied by the number of spaces or distances being laid off.

Let us suppose that a dimension of $2\frac{1}{4}$ inches is to be laid off horizontally each side of the vertical center line shown in Fig. 41.

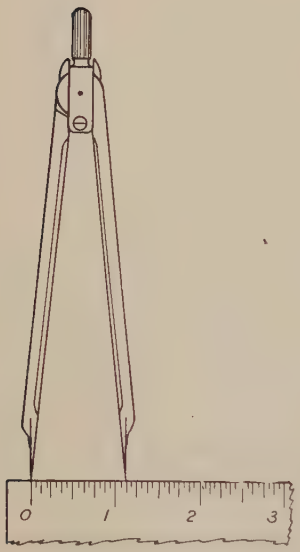


Fig. 42. Setting Dividers to Distance on Scale or Rule

Fig. 43. Laying Off Distance Each Side of Center Line

Take the scale and set the dividers to half the distance, or $1\frac{1}{8}$ inches, as shown in Fig. 42. Next place one point of the dividers on the intersection of the horizontal and vertical lines, and lightly press the other point into the paper, on one side of the line, and then repeat the operation on the other side of the line, Fig. 43.

The scale should never be used as a substitute for the triangle or T-square in drawing lines.

Protractor. The protractor is an instrument used for laying off and measuring angles. It is made of steel, brass, horn, or paper. When made of metal, the central portion is cut out, Fig. 44, so that the draftsman may see the drawing. The outer edge is divided into degrees and tenths of degrees. To lay off the required angle—

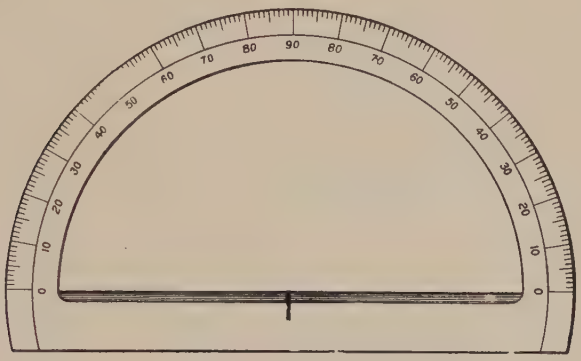


Fig. 44. Protractor

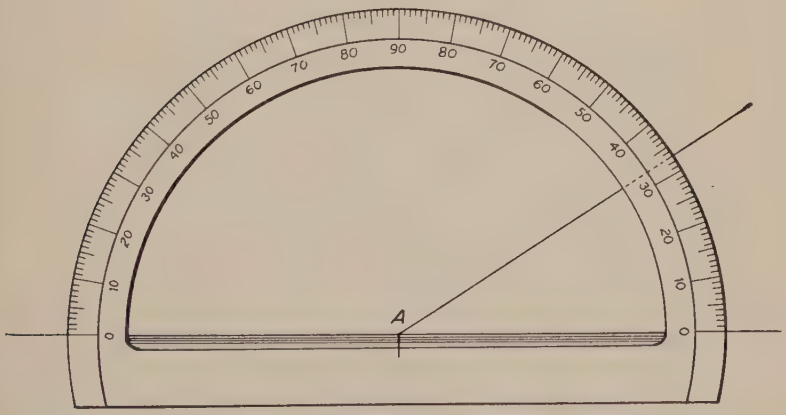


Fig. 45. Laying out a Line at an Angle to the Horizontal with Aid of Protractor

use a very sharp, hard pencil in order that the measurements may be accurate—place the protractor so that the two zero marks are on the given line, produced, if necessary, and the center mark on the horizontal part of the protractor is on the point through which the line is to be drawn. In Fig. 45 the protractor is being used to

draw a line through the point A and at 33 degrees to the horizontal line on which the two zero marks are located; while in Fig. 46 the protractor is being used to draw a line through point A and at 33 degrees to the vertical line.

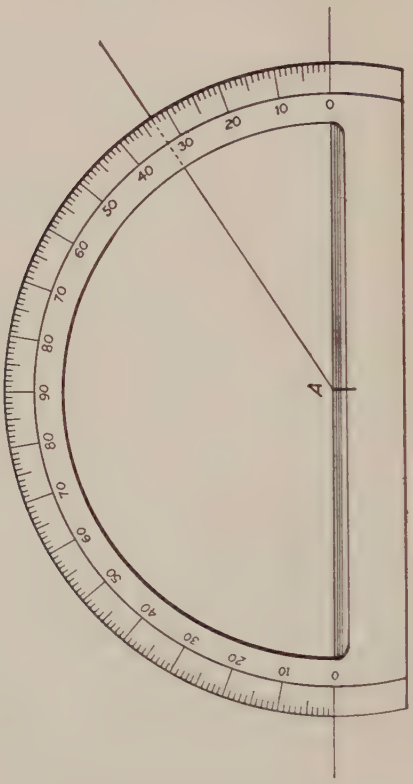


Fig. 46. Laying Out a line at an Angle to Vertical with aid of Protractor

Irregular Curve. One of the conveniences of a draftsman's outfit is the French or irregular curve, which is used for drawing curves other than arcs of circles, with either pencil or line pen. This instrument, which is made of wood, hard rubber, or celluloid—celluloid being the best—is made in various shapes, one of the most common being shown in Fig. 47.

To draw a curve through a series of located points, find that position of the irregular curve that passes through three points, say, and draw the line through them, Fig. 48. Now shift the curve

so as to include a part of the curve already drawn and two or three more points. Draw the curve through these points, continuing this process until the curve is completed. If, at each new setting, the line is not carried quite as far as the coincidence of the irregular curve would permit, a smoother curve will result. It frequently facilitates the work and improves its appearance to draw a pencil

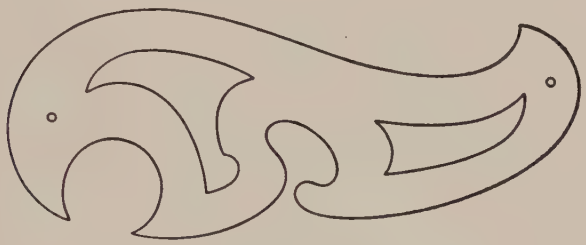


Fig. 47. French or Irregular Curve

Fig. 48. Using French or Irregular Curve

curve free-hand through the points and then use the irregular curve, taking care that it always fits at least three points. In inking the curve, the blades of the pen must be kept tangent to the curve. For certain kinds of work, irregular curves of plastic metal are sometimes used to fit exceptionally erratic curves.

Beam Compasses. The ordinary compasses are suitable for drawing circles up to 8 or 10 inches diameter. For larger circles beam compasses are provided. The two parts, called *channels*, carry the pen or pencil and the needle point and are clamped to

a wooden beam at a distance equal to the radius of the circle. The thumb nut underneath one of the channel pieces makes accurate adjustment possible.

Essential "Dont's"

Don't fold a drawing.
Don't stick the dividers into the drawing board.
Don't use the dividers as picks.
Don't use the scale to rule lines.
Don't fail to clean the table, board, and instruments when beginning work.
Don't draw on the lower edge of the T-square.
Don't cut the sheets of drawing paper with the upper edge of the T-square and a knife; use the lower edge.
Don't put the end of a pencil in the mouth.
Don't oil the compass joints.
Don't put away the instruments without cleaning, especially pens.
Don't use the cheapest materials.
Don't use the T-square as a hammer.

MECHANICAL DRAWING

LETTERING

No mechanical drawing is finished unless all headings, titles, and dimensions are drawn in plain, neat type. Many drawings are accurate, well planned, and finely executed, but do not present a good appearance because the draftsman did not think it worth while to letter carefully. Lettering requires time and practice, and skill can only be obtained by incessant practice.

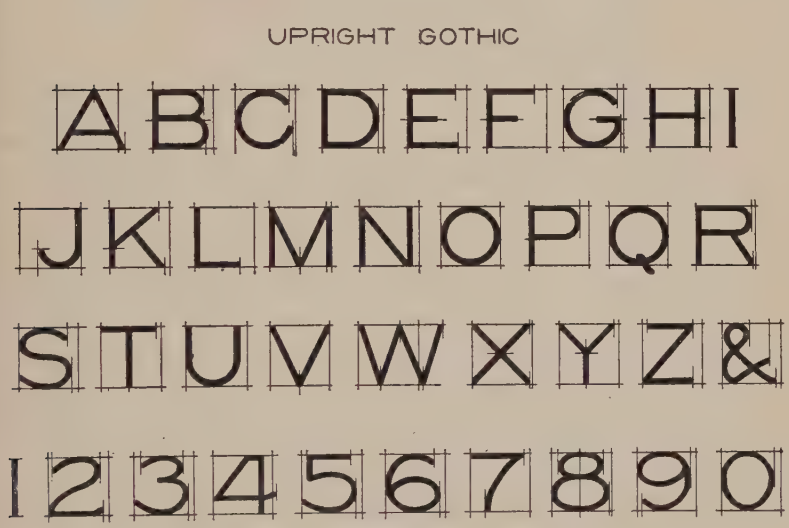


Fig. 1. Upright Gothic Capitals

Style. There are many styles of letters used by draftsmen, but almost any neat letter free from ornamentation is acceptable in regular practice. For titles, large Roman capitals are preferred, although Gothic and black letters also look well and are much easier to make. The vertical and inclined or italicized Gothic capitals shown in Figs. 1 and 2 are neat, plain, and easily made. The inclined letter possesses the advantage over the vertical type in that a slight difference in inclination is not apparent.

The curves of the inclined Gothic letters such as those in *B*, *C*, *G*, *J*, etc., are somewhat difficult to make free-hand, especially if the letters are about one-half inch high. In the alphabet, Fig. 3, the letters are made almost wholly of straight lines, the corners only being curved.

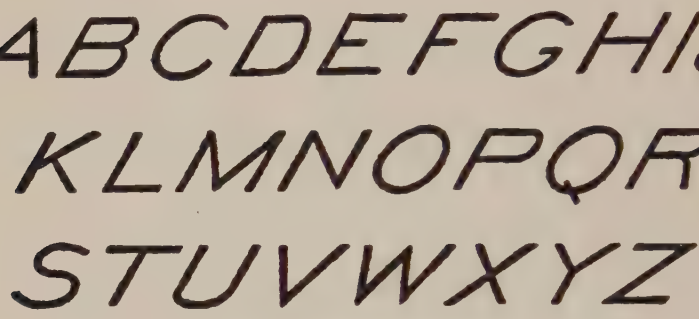


Fig. 2. Inclined Gothic Capitals

The style of the Arabic numerals is also given in Figs. 1 and 3. Roman numerals are made of straight lines.

To make the inclined Gothic letters, first draw two parallel lines $\frac{5}{32}$ inch apart to mark the height for the letters of the date, name, and plate number. This is the height to be used on all plates throughout this work, unless other directions are given. When two sizes of letters are used, the smaller should be about two-thirds as

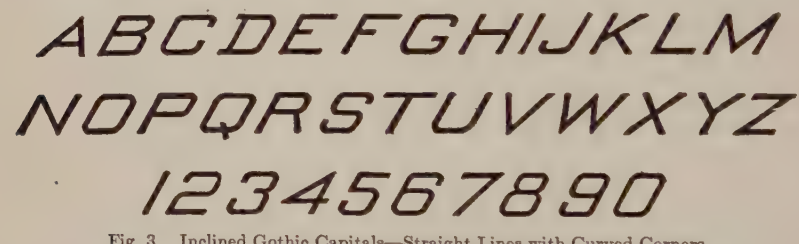


Fig. 3. Inclined Gothic Capitals—Straight Lines with Curved Corners

high as the larger. The inclination of the letters should be the same for all, and as an aid to the beginner, light pencil lines may be drawn about $\frac{1}{4}$ inch apart, forming the proper angle with the parallel lines already drawn; this angle is usually about 70°, but if a 60° triangle is at hand, it may be used in connection with the T-square.

Capital letters such as *D*, *E*, *F*, *L*, *Z*, etc., should have their top and bottom lines coincide with the horizontal guide lines or the work will look uneven. Letters, of which *C*, *G*, *O*, and *Q* are types, may be formed of curved or straight lines. If made of curved lines, their height should be a little greater than the guide lines to prevent their appearing smaller than the other letters. In this work they may be made of straight lines with rounded corners as such letters are easily constructed and may be made of standard height.

To construct the letter *A*, use one of the 60° lines as a center line. Then from its intersection with the upper horizontal line drop a perpendicular to the lower guide line. Draw another line from the vertex meeting the lower guide line at the same distance on the other side of the center line. The cross line of the *A* should be a

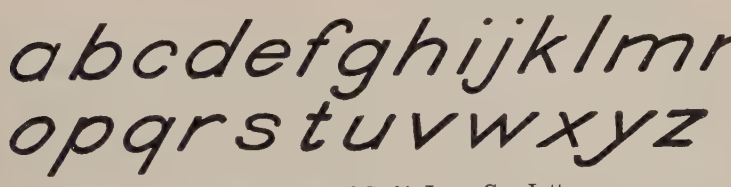


Fig. 4. Inclined Gothic Lower-Case Letters

little below the center. The *V* is an inverted *A* without the cross line. For the letter *M*, the side lines should be parallel and about the same distance apart as the guide lines. The side lines of the *W* are not parallel but are farther apart at the top. The *J* is not quite as wide as such letters as *H*, *E*, *N*, *R*, etc. To make a *Y*, use the same spread as in making a *V* but let the diverging lines meet the center line a little below the middle.

The lower-case letters are shown in Fig. 4. In such letters as *m*, *n*, *r*, etc., make the corners slightly rounding. The letters *a*, *b*, *c*, *e*, *g*, *o*, *p*, *q* should be full and rounding.

The other forms of lower-case letters in general use among draftsmen are shown in Figs. 5, 6, and 7. The height of the lower-case letters is about two-thirds that of capital letters. This proportion, however, varies in special cases.

A strictly mechanical letter, like the block letter, Fig. 8, is made on the same general plan as the Gothic, but much heavier. Small squares are taken as the unit of measurement, as shown.

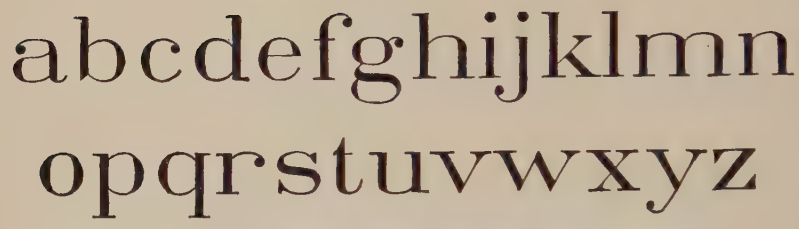


Fig. 5. Vertical Roman Lower-Case Letters

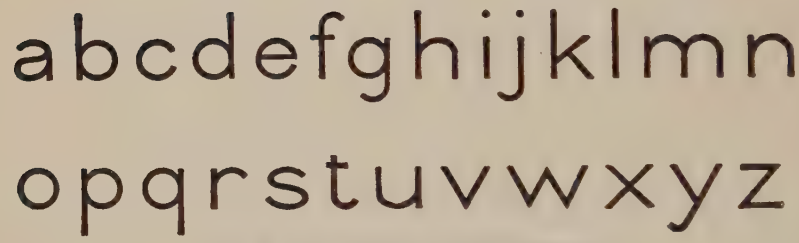


Fig. 6. Vertical Gothic Lower-Case Letters

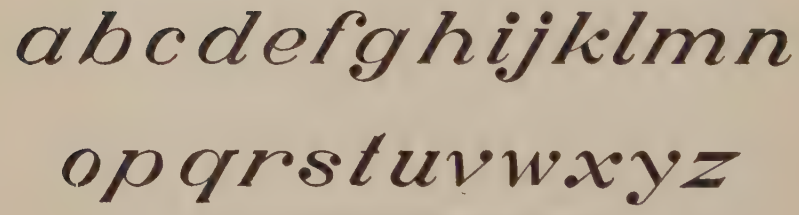


Fig. 7. Inclined Roman Lower-Case Letters

The use of this letter is not advocated for general work, although if made merely in outline the effect is pleasing.

The styles of numbers correspond with the styles of letters. When a fraction, such as $2\frac{5}{8}$, is to be made, the proportion should be about as shown in Fig. 9.

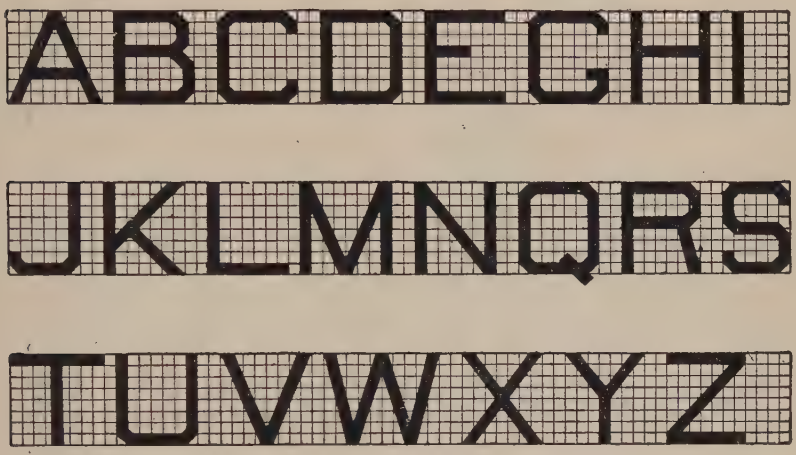


Fig. 8. Block Letters

Size and Inclination of Letters. There is no absolute size or proportion of letters, as the dimensions are regulated by the amount of space in which the letters are to be placed, the size of the drawing, the effect desired, etc. In some cases letters are made so that the height is greater than the width, and sometimes the reverse; sometimes the height and width are the same. This last proportion is the most common. Certain relations of width, however, should be observed. Thus, in whatever style of alphabet used, the W should be the widest letter; J the narrowest, M and T the next widest to W, then A and B. The other letters are of about the same width.

Vertical Gothic. In the vertical Gothic alphabet, Fig. 9, the average height is that of B, D, E, F, etc., and the additional height of the curved letters and of the A and V is very slight. The horizontal cross lines of such letters as E, F, H, etc., are slightly above the center; those of A, G, and P slightly below.

Inclined Gothic. For the inclined letters, Fig. 10, 60 degrees is a convenient angle, although they may be at any other angle

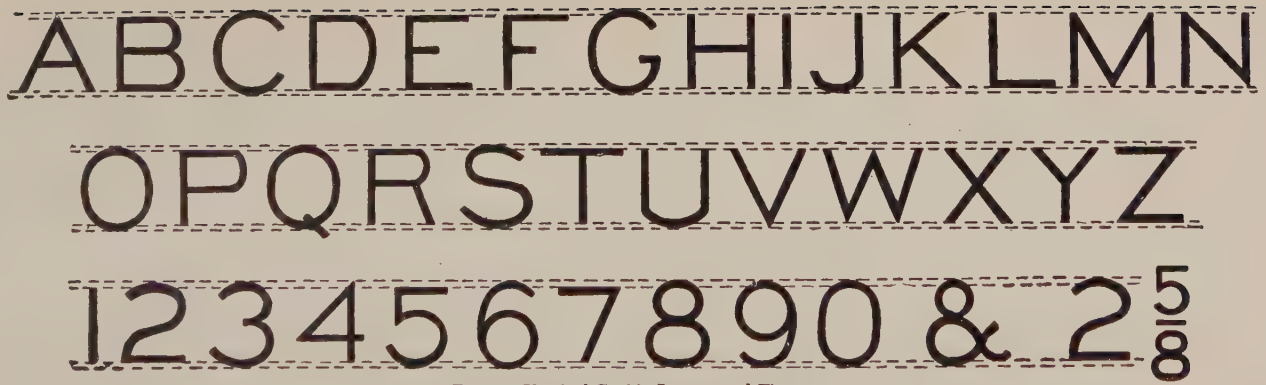


Fig. 9. Vertical Gothic Letters and Figures

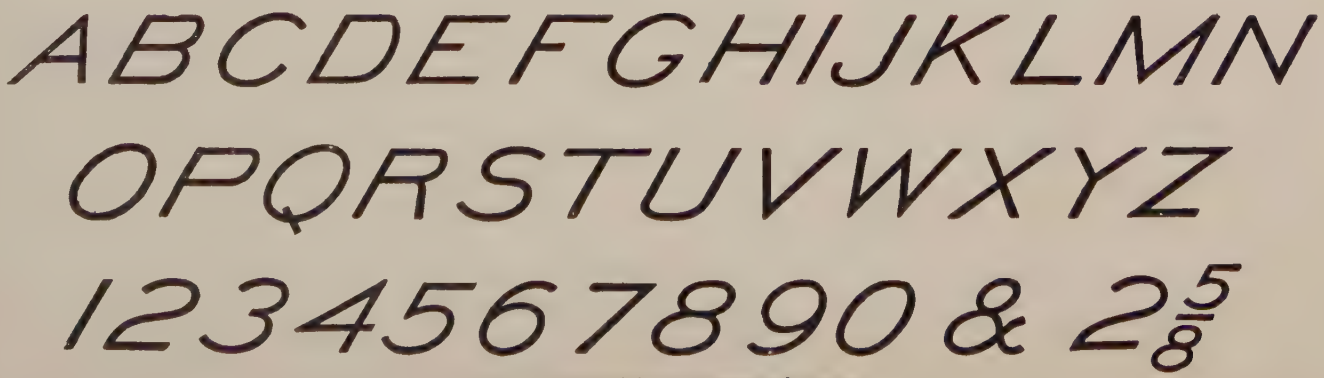


Fig. 10. Inclined Gothic Letters and Figures

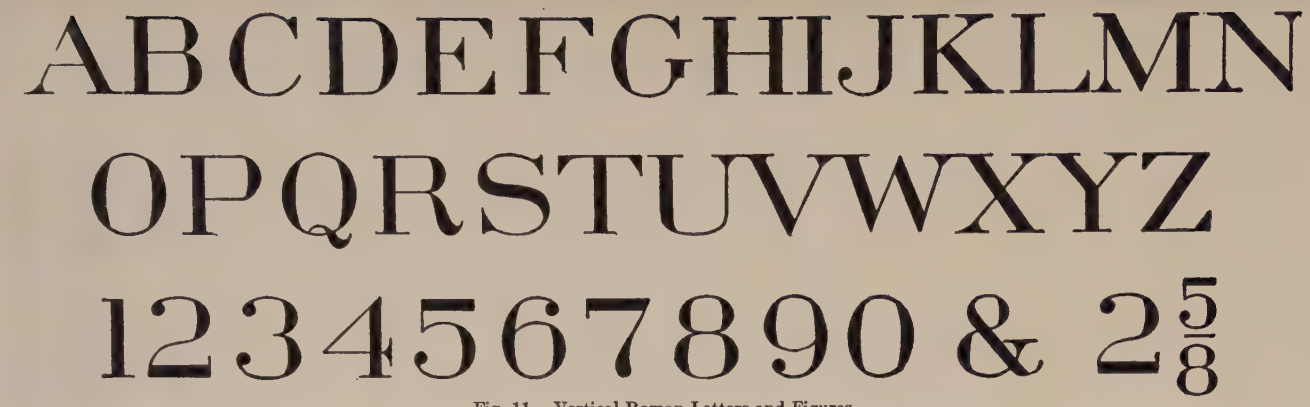


Fig. 11. Vertical Roman Letters and Figures

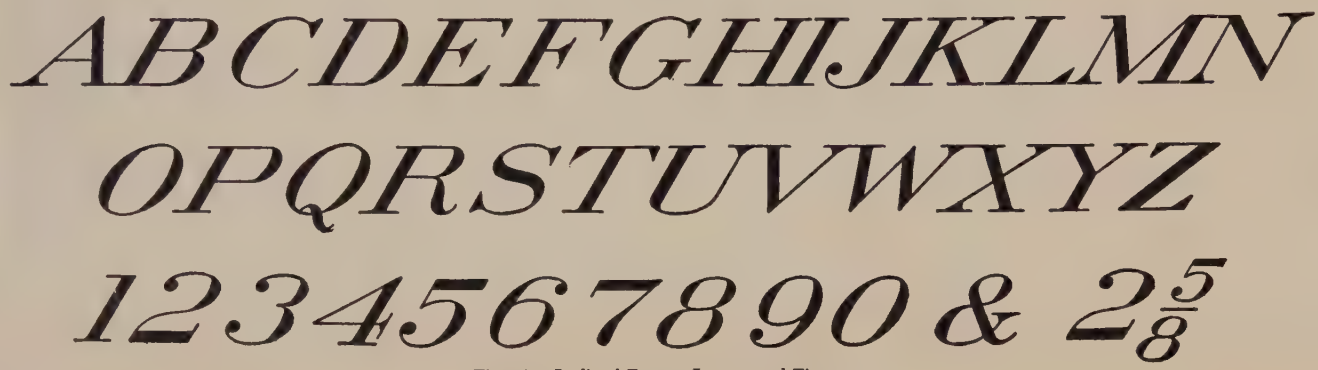


Fig. 12. Inclined Roman Letters and Figures

suited to the convenience or fancy of the draftsman. Many draftsmen use an angle of about 70 degrees.

Roman. The letters of the Roman alphabet, whether vertical, Fig. 11, or inclined, Fig. 12, are quite ornamental in effect if well made, the inclined Roman being a particularly attractive letter.

Letter Forms. The student is apt to think that lettering is a form of mechanical drawing; that the use of the straight-edge is the principal operation; and that letter forms and the spaces between are to be figured out by measurement. On the contrary, lettering is design, and the draftsman so distributes the letters in the spaces arranged for them as to make a combination that will be pleasing to the eye. The requirements for a good design are simplicity and uniformity. These are acquired by accuracy in detail and by good judgment and taste, as no practical rules can be followed which will

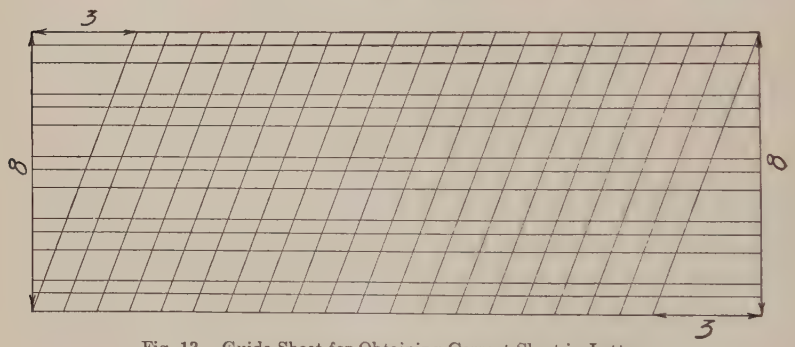


Fig. 13. Guide Sheet for Obtaining Correct Slant in Letters

invariably produce the same result. Letter forms are, to a certain extent, standard. The lettering for a title is usually done very carefully and accurately, while practically all of the other lettering on a drawing is done rapidly and in a simple style. To develop a letter, use the same method of procedure as in drawing a straight line between two points. First, draw the guide lines rather carefully and then block out the general form of the letter by a series of short strokes of the pencil. Continue this method, straightening the lines and rounding the curves of the latter until its form is satisfactory.

Letters and figures should always be made by beginners by first preparing guide lines drawn with a pencil. Even in case the guide lines have been drawn upon the detailed paper, it is also

advisable to draw them upon the tracing cloth, or place under the cloth a sheet similar to Fig. 13, with the lines drawn in ink, to act as a guide. This practice should be continued until enough skill has been acquired to make the letters uniform without the assistance of more than a line or two.

The only way in which a person can become proficient in lettering is through practice. A piece of paper ruled up and having the slant of the letter placed upon it, as shown in Fig. 13, will be found an excellent thing on which to practice lettering. Letters cannot be made nicely and quickly as one would suppose. Care and time are required until the draftsman becomes proficient in this respect.

Spacing. The spacing of the letters is very important and is best obtained by the unaided eye just as are the proportions of the letters. Care must be taken to allow a clear distance between letters, the space varying according to the combination. For instance, such letters as *A*, *V*, and *W* spread more at one part than at another and therefore do not fill the space completely. Of course, when the distance between letters is large, any such irregularities will not be noticeable. The best method for obtaining good space values is by

TECHNICALITY

Fig. 14. Sample of Letter Spacing

sketching in the letters roughly and then bringing them to a good appearance by correction and adjustment. The first results will be unsatisfactory, but after the eye and hand have become trained, great improvement will be noticed. A simple aid to this development will be found in the use of a piece of cardboard with the widths of the enclosing rectangles or parallelograms of the different letters marked on its edge, by which the spacing made by the eye may be checked.

Rules are of little value on account of the many combinations in which letters are found. A few directions, however, may be found helpful. For instance, take the word TECHNICALITY, Fig. 14. If all the spaces were made equal, the space between the L and the I would appear to be too great, and the same would apply to the space between the I and the T. The space between the H

and the N and that between the N and the I would be insufficient. Usually, when the vertical side of one letter is followed by the vertical side of another, as in H, E, H, B, I, R, etc., the maximum space should be allowed. Where T and A come together the least space is given, for in this case the top of the T frequently extends over the bottom of the A. In general, the spacing should be such that a uniform appearance is obtained. For the distances between words in a sentence, a space of about $1\frac{1}{2}$ the width of the average letter may be used. The space, however, depends largely upon the desired effect.

Titles for Working Drawings. The titles for working drawings are generally placed in the lower right-hand corner. Usually a

Fig. 15. Sample Title

draftsman has his choice of letters, mainly because after he has become used to making one style he can do it rapidly and accurately. However, in some drafting rooms the head draftsman decides what lettering should be used. In making these titles, the different alphabets are selected to give the best results without spending too much time. In most work the letters are made in straight lines, although frequently a portion of the title is found lettered on an arc of a circle.

In Fig. 15 is shown a title having the words CONNECTING ROD lettered on an arc of a circle. To do this work requires considerable patience and practice. First, draw the vertical center line as shown at *C* in Fig. 15; then, draw horizontal lines for the horizontal letters. The radii of the arcs depend upon the general arrangement of the entire title, and this is a matter of taste. The

difference between the arcs should equal the height of the letters. After the arc is drawn, the letters should be sketched in pencil to find their approximate positions. After this is done, draw radial lines from the center of the letters to the center of the arcs. These lines will be the centers of the letters, as shown at *A*, *B*, *D*, and *E*. The vertical lines of the letters should not radiate from the center of the arc, but should be parallel to the center lines already drawn; otherwise the letters will appear distorted. Thus, in the letter *N* the two verticals are parallel to the line *A*. The same applies to the other letters in the alphabet. In making the curved letters such as O and C, the centers of the arcs will fall upon these center lines; and if the compasses are used, the lettering is a comparatively simple matter. In Fig. 16 is shown another title in which all the letters are in horizontal lines.

Fig. 16. Sample Title

Penciling before Inking. For large titles, such as those placed on charts, maps, and some large working drawings, the letters should be penciled before inking. If the height is made equal to the width, considerable time and labor will be saved in laying out the work. This is especially true with such Gothic letters as O, Q, C, etc., as these letters may then be made with compasses. If the letters are of sufficient size, the outlines may be drawn with the ruling pen or compasses, and the spaces between filled in with a fine brush.

Keep the pencil point sharp. This will help to keep the lines fine and light. Use a pencil with a round point for lettering. Make the pencil drawings accurate, and the finished drawings will be accurate.

Inking. In practical work, most of the lettering is penciled in and then finished in ink. As faults in letters which may not be noticed in the pencil work stand out clearly after inking, it is not advisable to ink in the penciled letter accurately, but rather to improve upon it.

The standard form of lettering used on general working drawings is the single stroke "Gothic." Both the upright and slant letters are

used. The slant letter is used more than the upright letter. The term "single stroke" means that the width of the letter is made by a single stroke of the pen. A very satisfactory pen is the Hyland's No. 2. The Hunt pen is used for titles and large letters.

Ink all of the Practice Sheets for Lettering and Numerals and be sure to make the strokes for the various letters in the order shown on these sheets. In Fig. 17 suggestions are given for holding

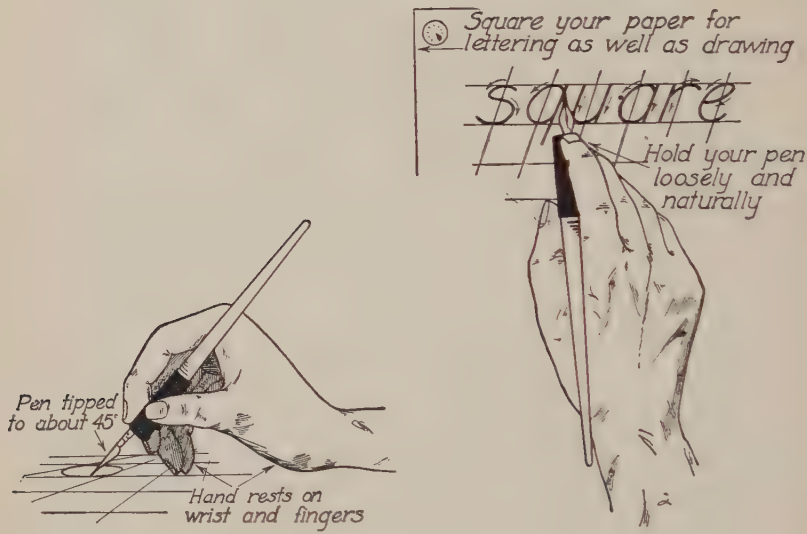

Square your paper for lettering as well as drawing
Square
Hold your pen loosely and naturally
Pen tipped to about 45°
Hand rests on wrist and fingers

Fig. 17. Method of Holding Lettering Pen

the pen. To get good results in inking, the pen must be kept absolutely clean. It pays to wash the pen thoroughly, even in the middle of a page of lettering, as the ink is apt to dry between the nibs and thus interfere with the free flow of the ink. Dip the pen in the ink frequently; this keeps the ink from drying. A coarse pen must be used on rough paper. To remove a faulty line or a blot, let the ink dry thoroughly and then, with a sand rubber, erase the spot carefully, rubbing around it as well. Clean the sand out of the surface with a pencil eraser, and finally polish down with a piece of ivory or smooth wood.

MECHANICAL DRAWING

The beginning problems are designed to teach the elementary operations of mechanical drawing—how to use the instruments, how to draw a straight and true line, and how to make up simple figures. In the more complicated problems of mechanical drawing, however, it is absolutely essential to have a knowledge of geometrical figures and their properties.

GEOMETRICAL DEFINITIONS

A *point* is used for marking position; it has neither length, breadth, nor thickness.

LINES

A *line* has length only; it is produced by the motion of a point.

A *straight line* or *right line* is one that has the same direction throughout. It is the shortest distance between two points.

A *curved line* is one that is constantly changing in direction. It is sometimes called a curve.

A *broken line* is one made up of several straight lines.

Parallel lines are lines which lie in the same plane and are equally distant from each other at all points.

A *horizontal line* is one having the direction of a line drawn upon the surface of water that is at rest. It is a line parallel to the horizon.

A *vertical line* is one that lies in the direction of a thread suspended from its upper end and having a weight at the lower end. It is a line that is perpendicular to a horizontal plane.

An *oblique line* is one that is neither vertical nor horizontal.

In Mechanical Drawing, lines drawn along the edge of the T-square, when the head of the T-square is resting against the left-

hand edge of the board, are called *horizontal* lines. Those drawn at right angles or perpendicular to the edge of the T-square are called *vertical* lines.

If two lines cut each other, they are called *intersecting lines*, and the point at which they cross is called the *point of intersection*.

ANGLES

An *angle* is the measure of the difference in direction of two lines. The lines are called *sides*, and the point of meeting, the *vertex*. The size of an angle is independent of the length of the lines.

If one straight line meets another (extended if necessary), Fig. 1, so that the two angles thus formed are equal, the lines are

Fig. 1. Right Angle

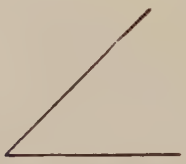


Fig. 2. Acute Angle

Fig. 3. Obtuse Angle

said to be *perpendicular* to each other and the angles formed are called *right angles*.

An *acute angle* is less than a right angle, Fig. 2.

An *obtuse angle* is greater than a right angle, Fig. 3.

SURFACES

A *surface* is produced by the motion of a line; it has two dimensions—length and breadth.

A *plane figure* is a plane bounded on all sides by lines; the space included within these lines (if they are straight lines) is called a *polygon* or a *rectilinear figure*.

POLYGONS

A *polygon* is a plane figure bounded by straight lines. The boundary lines are called the *sides* and the sum of the sides is called the *perimeter*. Polygons are classified according to the number of sides.

A *triangle* is a polygon of *three* sides.

A *quadrilateral* is a polygon of *four* sides.

A *pentagon* is a polygon of *five* sides, Fig. 4.
A *hexagon* is a polygon of *six* sides, Fig. 5.
A *heptagon* is a polygon of *seven* sides.

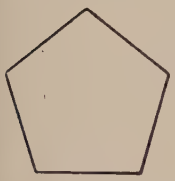


Fig. 4. Pentagon

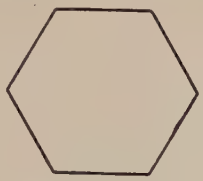


Fig. 5. Hexagon

Fig. 6. Octagon

An *octagon* is a polygon of *eight* sides, Fig. 6.
A *decagon* is a polygon of *ten* sides.
A *dodecagon* is a polygon of *twelve* sides.
An *equilateral* polygon is one whose sides are all equal.
An *equiangular* polygon is one whose angles are all equal.
A *regular* polygon is one whose angles are all equal and whose sides are all equal.

Triangles. A triangle is a polygon enclosed by three straight lines called *sides*. The *angles* of a triangle are the angles formed by the sides.

A *right-angle* triangle, often called a *right* triangle, Fig. 7, is one that has a right angle. The longest side (the one opposite

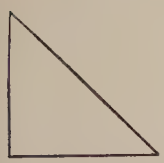


Fig. 7. Right-Angled Triangle

Fig. 8. Acute-Angled Triangle

Fig. 9. Obtuse-Angled Triangle

the right angle) is called the *hypotenuse*, and the other sides are sometimes called *legs*.

An *acute-angle* triangle is one that has all of its angles acute, Fig. 8.

An *obtuse-angle* triangle is one that has an obtuse angle, Fig. 9.
An *equilateral* triangle is one having all of its sides equal, Fig. 10.
An *equiangular* triangle is one having all of its angles equal.

An *isosceles* triangle, Fig. 11, is one, two of whose sides are equal.

A *scalene* triangle, Fig. 12, is one, no two of whose sides are equal.

The base of a triangle is the lowest side; it is the side upon which the triangle is supposed to stand. Any side may, however, be taken

Fig. 10. Equilateral Triangle

Fig. 11. Isosceles Triangle

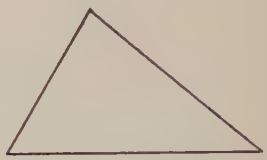


Fig. 12. Scalene Triangle

as the base. In an isosceles triangle, the side which is not one of the equal sides is usually considered as the base.

The *altitude* of a triangle is the perpendicular drawn from the vertex to the base.

Quadrilaterals. A quadrilateral is a polygon bounded by four straight lines, as Fig. 13.

The *diagonal* of a quadrilateral is a straight line joining two opposite vertices.

Trapezium. A trapezium is a quadrilateral, no two of whose sides are parallel.

Trapezoid. A trapezoid is a quadrilateral having two sides parallel, Fig. 14. The parallel sides are called the *bases* and the perpendicular distance between the bases is called the *altitude*.

Fig. 13. Quadrilateral

Fig. 14. Trapezoid

Fig. 15. Parallelogram

Parallelogram. A parallelogram is a quadrilateral whose opposite sides are parallel, Fig. 15.

There are four kinds of parallelograms: rectangle, square, rhombus, and rhomboid.

The *rectangle,* Fig. 16, is a parallelogram whose angles are right angles.

The *square*, Fig. 17, is a parallelogram all of whose sides are equal and whose angles are right angles.

The *rhombus*, Fig. 18, is a parallelogram whose sides are equal but whose angles are not right angles.

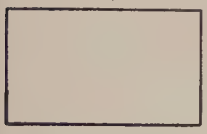


Fig. 16. Rectangle

Fig. 17. Square

Fig. 18. Rhombus

The *rhomboid* is a parallelogram whose adjacent sides are unequal, and whose angles are not right angles.

CIRCLES

A *circle* is a plane figure bounded by a curved line called the *circumference*, every point of which is equally distant from a point within called the *center*, Fig. 19.

A *diameter* of a circle is a straight line drawn through the center, terminating at both ends in the circumference, Fig. 20.

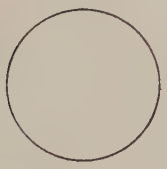


Fig. 19. Circle

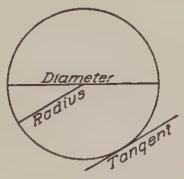


Fig. 20. Diameter, Radius, Tangent

Fig. 21. Quadrant

A *radius* of a circle is a straight line joining the center with the circumference. All radii of the same circle are equal and their length is always one-half that of the diameter.

A *tangent* is a straight line which touches the circumference at only one point, called the *point of tangency or contact*, Fig. 20.

An *arc* is any part of the circumferences of a circle. An arc equal to one-half the circumference is called a *semi-circumference*, and an arc equal to one-quarter of the circumference is called a *quadrant*, Fig. 21. A quadrant may mean the arc or angle.

A *chord,* Fig. 22, is a straight line which joins the extremities of an arc but does not pass through the center of the circle.

A *secant* is a straight line which intersects the circumference in two points, Fig. 22.

Fig. 22. Chord and Secant

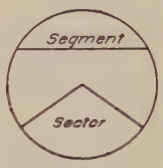


Fig. 23. Segment and Sector

Fig. 24. Concentric Circles

A *segment* of a circle, Fig. 23, is the area included between an arc and a chord.

A *sector* is the area included between an arc and two radii drawn to the extremities of the arc, Fig. 23.

Concentric circles are circles having the same center, Fig. 24.

An *inscribed angle* is an angle whose vertex lies in the circumference and whose sides are chords. It is measured by one-half the intercepted arc, Fig. 25.

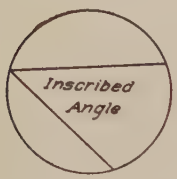


Fig. 25. Inscribed Angle

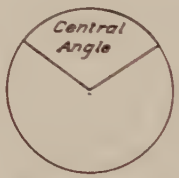


Fig. 26. Central Angle

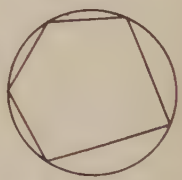


Fig. 27. Inscribed Polygon

A *central angle* is an angle whose vertex is at the center of the circle and whose sides are radii, Fig. 26.

An *inscribed polygon* is one whose vertices lie in the circumference and whose sides are chords, Fig. 27.

MEASUREMENT OF ANGLES

To measure an angle, take any convenient radius and describe an arc with the center at the vertex of the angle. The portion of the arc included between the sides of the angle is the *measure of*

the angle. If the arc has a constant radius, the greater the divergence of the sides, the longer will be the arc. If there are several arcs drawn with the same center, the intercepted arcs will have different lengths but they will all be the *same fraction* of the entire circumference.

In order that the size of an angle or arc may be stated without saying that it is a certain fraction of a circumference, the circumference is divided into 360 equal parts, called *degrees*, Fig. 28.

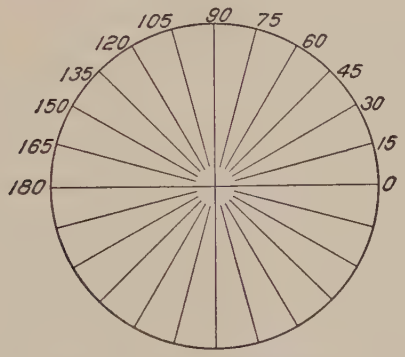


Fig. 28. Angular Measurement

Thus, it may be said that a certain angle contains 45 degrees, *i.e.*, it is $\frac{45}{360}=\frac{1}{8}$ of a circumference. In order to obtain accurate measurements each degree is divided into 60 equal parts, called *minutes;* and each minute is divided into 60 equal parts, called *seconds.*

SOLIDS

A *solid* has three dimensions—length, breadth, and thickness. The most common forms of solids are *polyhedrons, cylinders, cones,* and *spheres.*

POLYHEDRONS

A *polyhedron* is a solid bounded by planes. The bounding planes are called *faces* and their intersections are called *edges.* The intersections of the edges are called *vertices.*

A polyhedron having four faces is called a *tetrahedron,* one having six faces, a *hexahedron,* one having eight faces, an *octahedron,* Fig. 29; one having twelve faces, a *dodecahedron,* etc.

Prisms. A prism is a polyhedron having two opposite faces, called *bases,* which are equal and parallel, and other faces, called *lateral faces,* which are parallelograms, Fig. 30. The *altitude* of a prism is the perpendicular distances between the bases. The area of the lateral faces is called the *lateral area.*

Prisms are called *triangular, rectangular, hexagonal,* etc., according to the shape of the bases. Further classifications are as follows:

A *right* prism is one whose lateral faces are perpendicular to the bases, Fig. 31.

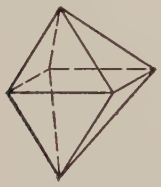


Fig. 29. Octahedron

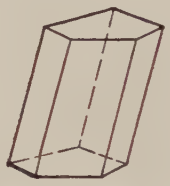


Fig. 30. Prism

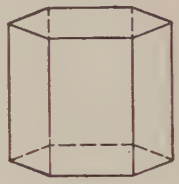


Fig. 31. Right Prism

A *regular* prism is a right prism having regular polygons for bases.

Parallelopiped. A parallelopiped is a prism whose bases are parallelograms, Fig. 32. If all the edges are perpendicular to the bases, it is called a *right parallelopiped.*

A *rectangular parallelopiped* is a right parallelopiped whose bases and lateral faces are rectangles, Fig. 33.

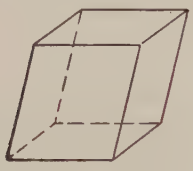


Fig. 32. Parallelopiped

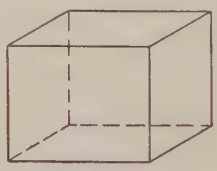


Fig. 33. Rectangular Parallelopiped

Fig. 34. Truncated Prism

A *cube* is a rectangular parallelopiped all of whose faces are squares.

A *truncated prism* is the portion of a prism included between the base and a plane not parallel to the base, Fig. 34.

Pyramids. A pyramid is a polyhedron whose base is a polygon and whose lateral faces are triangles having a common vertex, called the *vertex* of the pyramid.

The *altitude* of the pyramid is the perpendicular distance from the vertex to the base.

Pyramids are named according to the kind of polygon forming the base, viz., *triangular, quadriangular,* Fig. 35, *pentagonal,* Fig. 36, *hexagonal.*

A *regular pyramid* is one whose base is a regular polygon and whose vertex lies in a perpendicular erected at the center of the base, Fig. 36.

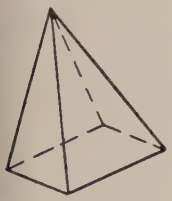


Fig. 35. Pyramid

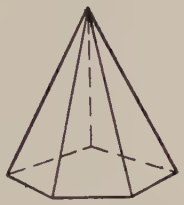


Fig. 36. Regular Pyramid

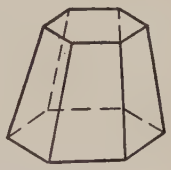


Fig. 37. Frustum of Pyramid

A *truncated pyramid* is the portion of a pyramid included between the base and a plane not parallel to the base.

A *frustum* of a pyramid is the solid included between the base and a plane parallel to the base, Fig. 37; its *altitude* is the perpendicular distance between the bases.

CYLINDERS

A *cylinder* is a solid having as bases two equal parallel surfaces bounded by curved lines, and as its lateral face the continuous

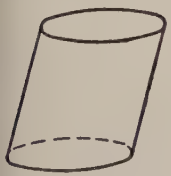


Fig. 38. Cylinder

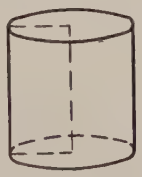


Fig. 39. Right Cylinder

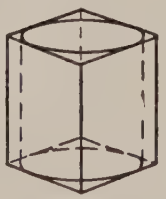


Fig. 40. Inscribed Cylinder

surface generated by a straight line connecting the bases and moving along their circumferences. The bases are usually circles and such a cylinder is called a *circular cylinder,* Fig. 38.

A *right cylinder,* Fig. 39, is one whose side is perpendicular to the bases.

The *altitude* of a cylinder is the perpendicular distance between the bases.

A prism whose base is a regular polygon may be inscribed in or circumscribed about a circular cylinder, Fig. 40.

CONES

A *cone* is a solid bounded by a conical surface and a plane which cuts the conical surface. It may be considered as a pyramid with an infinite number of sides, Fig. 41.

The conical surface is called the *lateral area* and it tapers to a point called the *vertex;* the plane is called the *base.*

The *altitude* of a cone is the perpendicular distance from the vertex to the base.

Fig. 41. Cone

Fig. 42. Right Circular Cone

Fig. 43. Frustum of Cone

An *element of a cone* is any straight line from the vertex to the circumference of the base.

A *circular cone* is a cone whose base is a circle.

A *right circular cone,* or *cone of revolution,* Fig. 42, is a cone whose axis is perpendicular to the base. It may be generated by the revolution of a right triangle about one of the legs as an axis.

A *frustum* of a cone, Fig. 43, is the portion of the cone included between the base and a plane parallel to the base; its *altitude* is the perpendicular distance between the bases.

SPHERES

A *sphere* is a solid bounded by a curved surface, every point of which is equally distant from a point within called the *center.*

The *diameter* is a straight line drawn through the center and having its extremities in the curved surface. The *radius*—$\frac{1}{2}$ diameter—is the straight line from the center to a point on the surface.

A *plane is tangent to a sphere* when it touches the sphere in only one point. A plane perpendicular to a radius at its outer extremity is tangent to the sphere, Fig. 44.

An *inscribed polyhedron* is a polyhedron whose vertices lie in the surface of the sphere.

A *circumscribed polyhedron* is a polyhedron whose faces are tangent to a sphere.

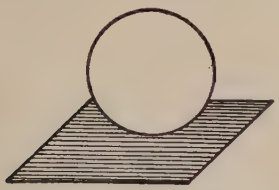


Fig. 44. Plane Tangent to Sphere

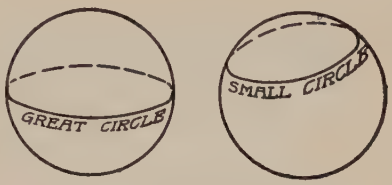


Fig. 45. Great and Small Circle

A *great circle* is the intersection of the spherical surface and a plane passing through the center of the sphere, Fig. 45.

A *small circle* is the intersection of the spherical surface and a plane which does not pass through the center, Fig. 45.

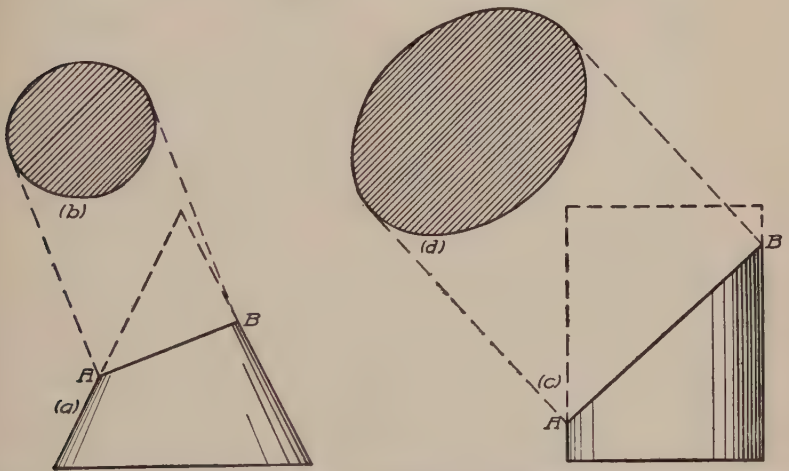


Fig. 46. Intersections of Plane with Cone and Cylinder Giving Ellipses as Shown in (*b*) and (*d*)

CONIC SECTIONS

If a plane intersects a cone at various angles with the base, the geometrical figures thus formed are called *conic sections*. A plane perpendicular to the base passing through the vertex of a right

circular cone forms an isosceles triangle. If the plane is parallel to the base, the intersection of the plane and the conical surfaces will be the circumference of a circle.

Ellipse. If a plane AB, Fig. 46 (a), cuts a cone oblique to the axis of the cone, but not cutting the base, the curve formed is called an ellipse, as shown in Fig. 46 (b), this view being taken perpendicular to the plane AB. If the plane cuts a cylinder as shown in Fig. 46 (c), the ellipse shown in Fig. 46 (d) is the result, this view being also taken perpendicular to the plane AB. An ellipse may be defined as *a curve generated by a point moving in a plane in such a manner that the sum of the distances from the point to two fixed points shall always be constant.*

The two fixed points are called *foci*, Fig. 47, and shall lie on the longest line that can be drawn in the ellipse, which is called the

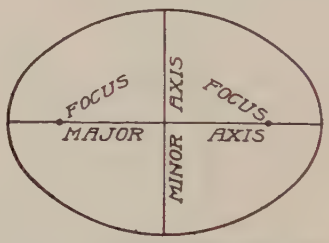


Fig. 47. Diagram Showing Constants of Ellipse

major axis; the shortest line is called the *minor axis*; it is perpendicular to the major axis at its middle point, called the *center*.

An ellipse may be constructed if the major and minor axes are given or if the foci and one axis are known.

Parabola. If a plane AB, Fig. 48 (a), cuts a cone parallel to an element of the cone, the curve resulting from this intersection is called a parabola, as shown in Fig. 48 (b), the view being taken perpendicular to the plane AB. This curve is not a closed curve for the branches approach parallelism, Fig. 49.

Hyperbola. If a plane AB, Fig. 50 (a), cuts a cone parallel to its axis, the resulting curve is called a hyperbola, Fig. 50 (b), the view being taken perpendicular to the plane AB. Like the parabola, the curve is not closed, the branches constantly diverging, Fig. 51.

Rectangular Hyperbola. The form of hyperbola most used in mechanical engineering is called the rectangular hyperbola because

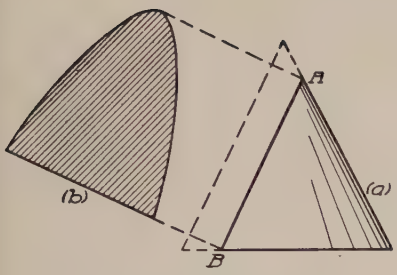


Fig. 48. Intersection of Plane with Cone, Parallel to Element of Cone and Parabolic Section Produced

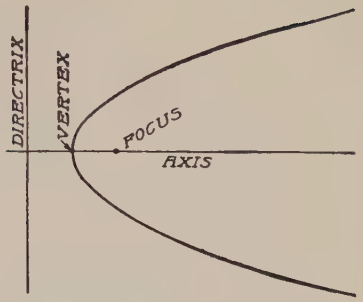


Fig. 49. Diagram Showing Constants of Parabola

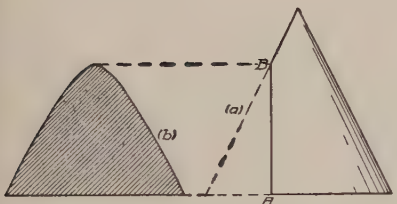


Fig. 50. Intersection of Plane with Cone, Parallel to Axis and Hyperbolic Section Produced

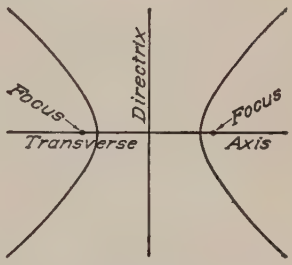


Fig. 51. Diagram Showing Constants of Hyperbola

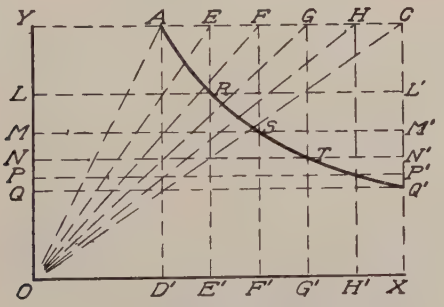


Fig. 52. Construction of Rectangular Hyperbola

it is drawn with reference to rectangular coördinates. This curve is constructed as follows: In Fig. 52, OX and OY are the two coördinate axes drawn at right angles to each other. These lines are also called *asymptotes*. Assume A to be a known point on the curve. Draw AC parallel to OX and AD' perpendicular to OX. Mark off any convenient points on AC, such as E, F, G, and H, and through these points draw EE', FF', GG', and HH', perpendicular to OX. Connect E, F, G, H, and C with O. Through the points of intersection of the oblique lines and the vertical line AD' draw the horizontal lines LL', MM', NN', PP', QQ'. The first point on the curve is the assumed point A, the second point is R, the intersection of LL' and EE', the third the intersection S, and so on.

In this curve the products of the coördinates of all points are equal. Thus $LRXRE' = MSXSF' = NTXTG'$.

ODONTOIDAL CURVES

Cycloidal Curves. *Cycloid.* The cycloid is a curve generated by a point on the circumference of a circle which rolls on a straight line tangent to the circle, as shown at the left, Fig. 53.

Fig. 53. Geometrical Construction for Cycloid and Hypocycloid

The rolling circle is called the *describing* or *generating circle*, the point on the circle, the *describing* or *generating point*, and the tangent along which the circle rolls, the *director*. In order that the curve described by the point may be a true cycloid the circle must roll without any slipping.

Hypocycloid. In case the generating circle rolls upon the inside of an arc or circle, the curve thus generated is a hypocycloid, Fig. 53. If the generating circle has a diameter equal to the radius of the director circle, the hypocycloid becomes a straight line.

Epicycloid. If the generating circle rolls upon the *outside* of the *director circle*, the curve generated is an epicycloid, Fig. 54.

Involute Curves. If a thread of fine wire is wound around a cylinder or circle and then unwound, the end will describe an involute curve. The involute may be defined as *a curve generated by a point in a tangent rolling on a circle, known as the base circle,* Fig. 55.

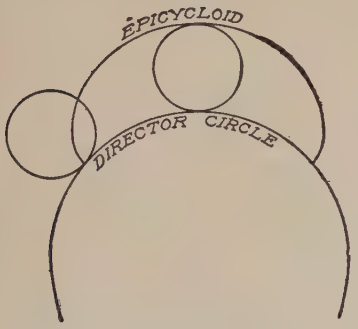


Fig. 54. Geometrical Construction for an Epicycloid

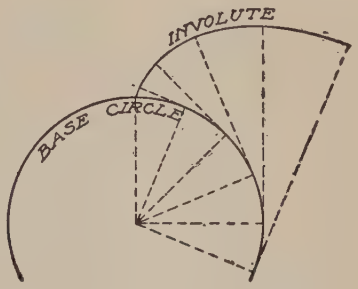
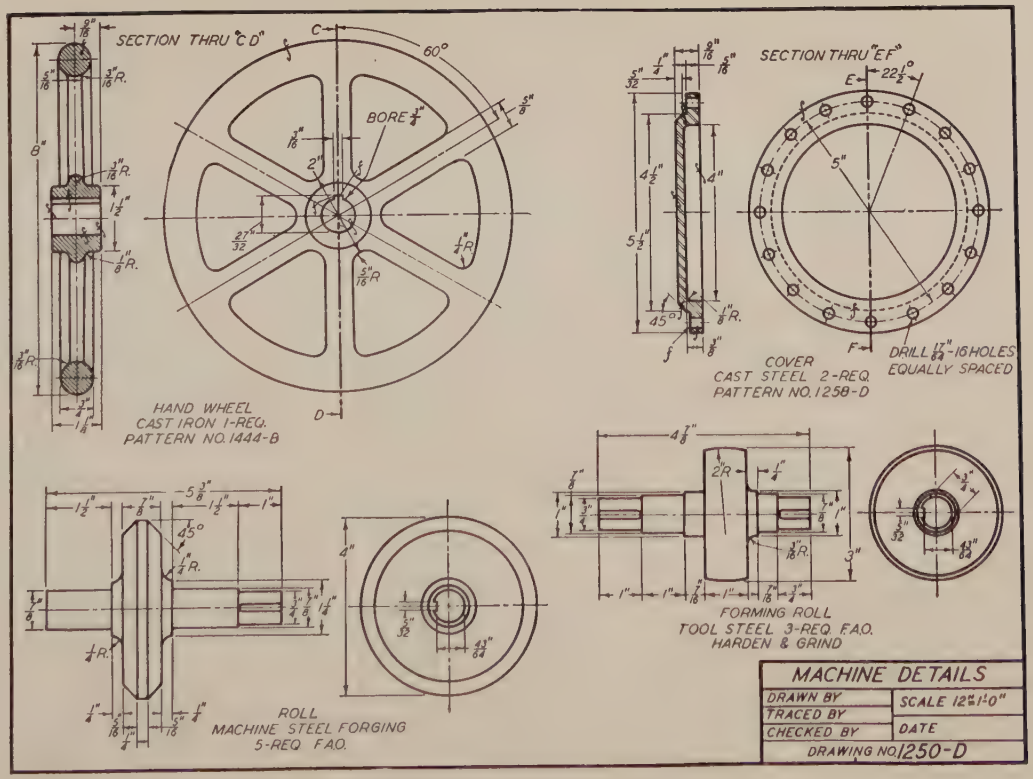


Fig. 55. Geometrical Construction for an Involute

The most important application of the cycloidal and involute curves is in the cutting of all forms of gear teeth. It has been found that the teeth of gears when cut accurately to either of these curves will mesh with the least friction and run with exceptional smoothness. The development of these gears and of the machines for cutting them has reached a high state of perfection.



Detail Drawing of Machine Parts

MECHANICAL DRAWING

GEOMETRICAL PROBLEMS

These geometrical problems are given because of their particular bearing on the work of the draftsman.

The problems should be solved with great care. The application of the principles of these problems will be used in later work. Pencil and compass points should be kept sharp so that line intersections will be distinct.

All of these problems should be drawn in order to become familiar with their construction. Draw the construction lines light and make the outlines of the figure in a heavier line. In penciling, all intersections must be formed with great care as the accuracy of the results depends upon it. Keep the pencil points in good order at all times and draw lines *exactly* through intersections.

Problem 1. *To bisect a given straight line.*

Draw the horizontal straight line AC about 3 inches long. With the extremity A as a center and any convenient radius—about 2 inches—describe arcs above and below the line AC. With the other extremity C as a center and with the same radius draw similar arcs intersecting the first arcs at D and E. The radius of these arcs must be greater than one-half the length of the line in order that they may intersect. Now draw the straight line DE passing through the intersections D and E. This line will cut AC at its middle point F. Therefore AF equals FC.

Proof. Since the points D and E are equally distant from A and C, a straight line drawn through them is perpendicular to AC at its middle point F.

Problem 2. *To construct an angle equal to a given angle.*

Draw the line OC about 2 inches long and the line OA about the same length. The angle formed by these lines may be any convenient size—about 45 degrees is suitable. This angle AOC is the given angle.

Now draw FG, a horizontal line about $2\frac{1}{4}$ inches long, and let F, the left-hand extremity, be the vertex of the angle to be constructed.

With O as a center and any convenient radius—about $1\frac{1}{2}$ inches —describe the arc LM cutting both OA and OC. With F as a center and the same radius draw the indefinite arc OQ. Now set the compass so that the distance between the pencil and the needle point is equal to the chord LM. With Q as a center and a radius equal to LM draw an arc cutting the arc OQ at P. Through F and P draw the straight line FE. The angle EFG is the required angle since it is equal to AOC.

Proof. Since the chords of the arcs LM and PQ are equal, the arcs are equal. The angles are equal because with equal radii equal arcs are intercepted by equal angles.

Problems 3 and 4. *To draw through a given point a line parallel to a given line.*

First Method. Draw the straight line AC about $3\frac{1}{2}$ inches long and assume the point P about $1\frac{1}{2}$ inches above AC. Through the point P draw an oblique line FE forming any convenient angle —about 60 degrees—with AC. Now construct an angle equal to PFC and having its vertex at P and the line EP as one side. (See Problem 2.) The straight line PO forming the other side of the angle EPO will be parallel to AC.

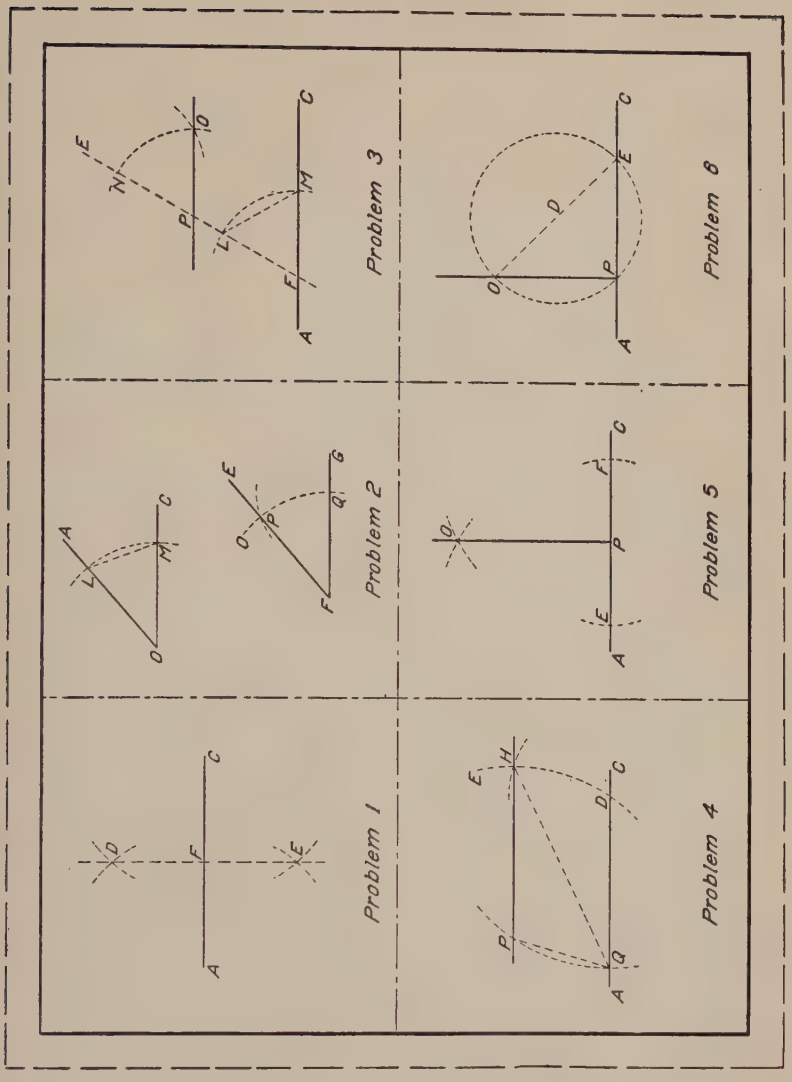
Proof. If two straight lines are cut by a third making the corresponding angles equal, the lines are parallel.

Second Method. Draw the straight line AC about $3\frac{1}{4}$ inches long and assume the point P about $1\frac{1}{2}$ inches above AC. With P as a center and any convenient radius—about $2\frac{1}{2}$ inches—draw the indefinite arc ED cutting the line AC. Now with the same radius and with D as a center draw an arc PQ. Set the compass so that the distance between the needle point and the pencil is equal to the chord PQ. With D as a center and a radius equal to PQ describe an arc cutting the arc ED at H. A line drawn through P and H will be parallel to AC.

Proof. Draw the line QH. Since the arcs PQ and HD are equal and have the same radii, the angles PHQ and HQD are equal. Two lines are parallel if the alternate interior angles are equal.

Problem 5. *To draw a perpendicular to a line from a point in the line when the point is near the middle of the line.*

Draw the line AC about $3\frac{1}{2}$ inches long and assume the point


Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6

P near the middle of the line. With P as a center and any convenient radius—about $1\frac{1}{4}$ inches—draw two arcs cutting the line AC at E and F. Now with E and F as centers and any convenient radius—about $2\frac{1}{2}$ inches—describe arcs intersecting at O. The line OP will be perpendicular to AC at P.

Proof. The points P and O are both equally distant from E and F. Hence a line drawn through them is perpendicular to EF at P.

Problem 6. *To draw a perpendicular to a line from a point in the line when the point is near the end of the line.*

Draw the line AC about $3\frac{1}{2}$ inches long. Assume the given point P to be about $\frac{3}{4}$ inch from the end A. With any point D as a center and a radius equal to DP describe an arc cutting AC at E. Through E and D draw the diameter EO. A line from O to P is perpendicular to AC at P.

Proof. The angle OPE is inscribed in a semicircle; hence it is a right angle, and the sides OP and PE are perpendicular to each other.

Problem 7. *To draw a perpendicular to a line from a point without the line.*

Draw the straight line AC about $3\frac{1}{4}$ inches long, and assume the point P about $1\frac{1}{2}$ inches above the line. With P as a center and any convenient radius—about 2 inches—describe an arc cutting AC at E and F. The radius of this arc must always be such that it will cut AC in two points; the nearer the points E and F are to A and C, the greater will be the accuracy of the work.

Now with E and F as centers and any convenient radius—about $2\frac{1}{4}$ inches—draw the arcs intersecting below AC at T. A line through the points P and T will be perpendicular to AC. In case there is not room below AC to draw the arcs, they may be drawn intersecting above the line as shown at N. Whenever convenient draw the arcs below AC for greater accuracy.

Proof. Since P and T are both equally distant from E and F, the line PT is perpendicular to AC.

Problem 8. *To bisect a given angle when the sides intersect.*

Draw the lines OC and OA—about 3 inches long—forming any angle of 45 to 60 degrees. With O as a center and any convenient radius—about 2 inches—draw an arc intersecting the sides

of the angle at E and F. With E and F as centers and a radius of $1\frac{1}{2}$ or $1\frac{3}{4}$ inches describe short arcs intersecting at I. A line OD drawn through the points O and I bisects the angle.

In solving this problem the arc EF should not be too near the vertex if accuracy is desired.

Proof. The central angles AOD and DOC are equal because the arc EF is bisected by the line OD. The point I is equally distant from E and F.

Problem 9. *To bisect a given angle when the lines do not intersect.*

Draw the lines AC and EF about 4 inches long, making an angle approximately as shown. Draw $A'C'$ and $E'F'$ parallel to AC and EF and at such equal distances from them that they will intersect at O. Now bisect the angle $C'O'F'$ by the method given in Problem 8. The line OR bisects the given angle.

Proof. Since $A'C'$ is parallel to AC and $E'F'$ is parallel to EF, the angle $C'O'F'$ is equal to the angle formed by the lines AC and EF. Hence as OR bisects angle $C'O'F'$, it also bisects the angle formed by the lines AC and EF.

Problem 10. *To divide a line into any number of equal parts.*

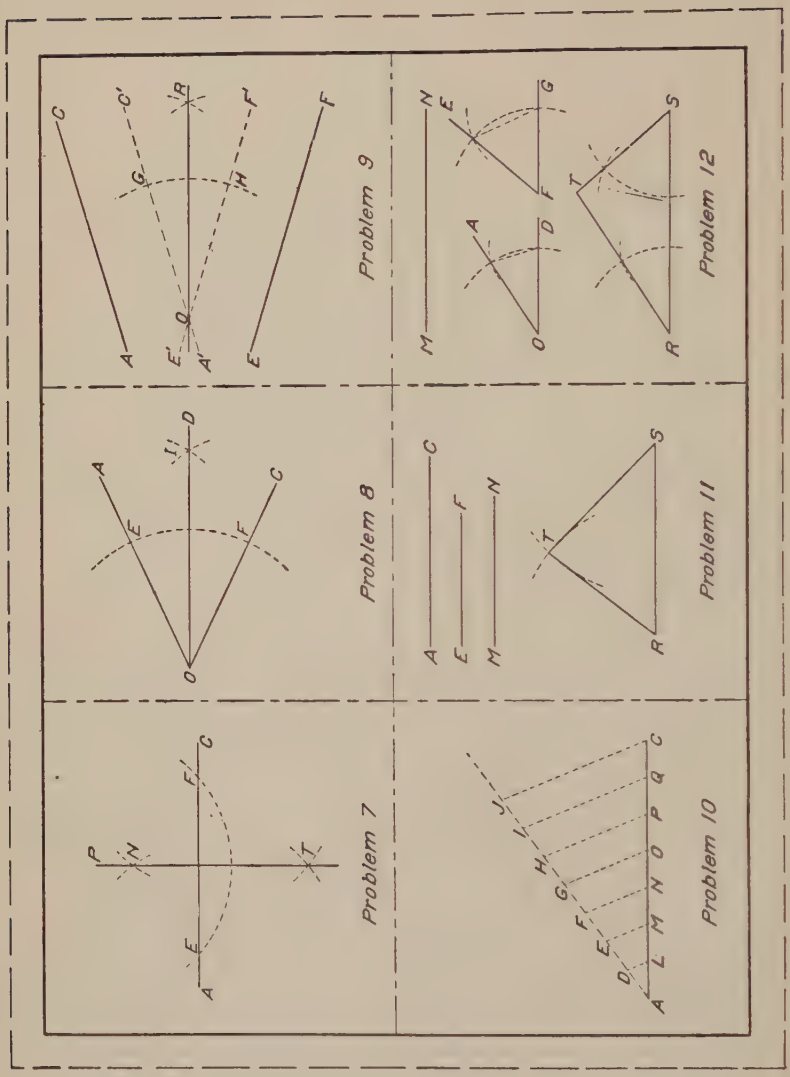
Let AC—about $3\frac{3}{4}$ inches long—be a given line. Suppose it is desired to divide it into 7 equal parts. First draw the line AJ at least 4 inches long, forming any convenient angle with AC. On AJ lay off, by means of the dividers or scale, points D, E, F, G, etc., each $\frac{1}{2}$ inch apart. (If dividers are used, the spaces need not be exactly $\frac{1}{2}$ inch.) Draw the line JC and through the points D, E, F, G, etc., draw lines parallel to JC. These parallels will divide the line AC into 7 equal parts.

Proof. If a series of parallel lines cutting two straight lines intercept equal distances on one of these lines, they also intercept equal distances on the other.

Problem. 11. *To construct a triangle having given the three sides.*

Draw the three sides—AC, $2\frac{3}{4}$ inches long; EF, $1\frac{15}{16}$ inches long; and MN, $2\frac{3}{16}$ inches long. Draw RS equal in length to AC.

With R as a center and a radius equal to EF describe an arc. With S as a center and a radius equal to MN draw an arc cutting


Problem 9
Problem 12
Problem 8
Problem 11
Problem 7
Problem 10

the arc previously drawn at T. Connect T with R and S to form the triangle.

Problem 12. *To construct a triangle having given one side and the two adjacent angles.*

Draw the line MN $3\frac{1}{4}$ inches long and draw two angles AOD and EFG about 30 degrees and 60 degrees, respectively. Draw RS equal in length to MN.

With R as a vertex and RS as one side construct an angle equal to AOD. In a similar manner construct at S an angle equal to EFG. Draw lines from R and S through the two established points until they meet at T. The triangle RTS will be the required triangle.

Problem 13. *To describe an arc or circumference through three given points not in the same straight line.*

Locate the three points A, B, and C, with a distance between A and B of about 2 inches and a distance between A and C of about $2\frac{1}{4}$ inches. Connect A and B and A and C. Erect perpendiculars to the middle points of AB and AC as explained in Problem 1. Now draw light pencil lines connecting the intersections I and J and E and F. These lines will intersect at O.

With O as a center and a radius equal to the distance OA describe the circumference passing through A, B, and C.

Proof. The point O is equally distant from A, B, and C, since it lies in the perpendiculars to the middle points of AB and AC. Hence the circumference will pass through A, B, and C.

Problem 14. *To inscribe a circle in a given triangle.*

Draw the triangle LMN of any convenient size. MN may be made $3\frac{1}{4}$ inches, LM, $2\frac{3}{4}$ inches, and LN, $3\frac{1}{2}$ inches. Bisect the angles MLN and LMN by the method used in Problem 8. The bisectors MI and LJ intersect at O, which is the center of the inscribed circle. The radius of the circle is equal to the perpendicular distance from O to one of the sides.

Proof. The point of intersection of the bisectors of the angles of a triangle is equally distant from the sides.

Problem 15. *To inscribe a regular pentagon in a given circle.*

With O as a center and a radius of about $1\frac{1}{2}$ inches describe the given circle. With the T-square and triangles draw the center lines AC and EF perpendicular to each other and passing through O. Bisect the radius OC at H and with this point as a center

and a radius HE describe the arc EP. This arc cuts the diameter AC at P. With E as a center and a radius EP draw arcs cutting the circumference at L and Q. With the same radius and centers at L and Q draw the arcs cutting the circumference at M and N.

The pentagon is completed by drawing the chords EL, LM, MN, NQ, and QE.

Problem 16. *To inscribe a regular hexagon in a given circle.*

With O as a center and a radius of $1\frac{3}{8}$ inches draw the given circle. With the T-square draw the diameter AD. With D as a center and a radius equal to OD describe arcs cutting the circumference at C and E. Now with C and E as centers and the same radius draw the arcs cutting the circumference at B and F. Draw the hexagon by joining the points thus formed.

To inscribe a regular hexagon in a circle mark off chords equal in length to the radius.

To inscribe an equilateral triangle in a circle the same method may be used, the triangle being formed by joining the opposite vertices of the hexagon.

Proof. Since the triangle OCD is an equilateral triangle by construction, the angle COD is one-third of two right angles and one-sixth of four right angles. Hence arc CD is one-sixth of the circumference and the chord is a side of a regular hexagon.

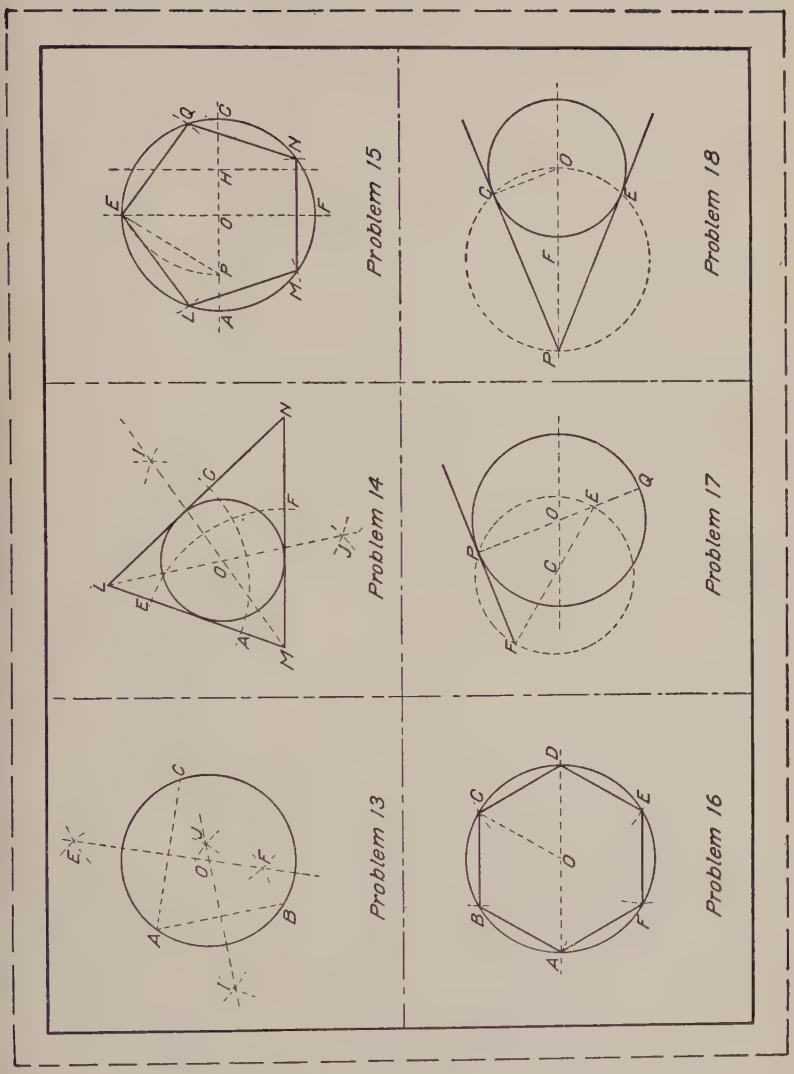
Problem 17. *To draw a line tangent to a circle at a given point on the circumference.*

With O as a center and a radius of about $1\frac{1}{4}$ inches draw the given circle. Assume some point P on the circumference and join the point P with the center O. By the method given in Problem 6, construct a perpendicular to PO, which perpendicular will be the desired tangent to the circle at the point P.

Proof. A line perpendicular to a radius at its extremity is tangent to the circle.

Problem 18. *To draw a line tangent to a circle from a point outside the circle.*

With O as center and a radius of about 1 inch draw the given circle. Assume P, some point outside of the circle, about $2\frac{1}{2}$ inches from the center. Draw a straight line passing through P and O. Bisect PO and with the middle point F as a center describe the circle passing through P and O. Draw a line from P through the


Problem 15
Problem 18
Problem 14
Problem 17
Problem 13
Problem 16

intersection of the two circumferences C. The line PC is tangent to the given circle. Similarly PE is tangent to the circle.

Proof. The angle PCO is inscribed in a semicircle and hence is a right angle. Since PCO is a right angle, PC is perpendicular to CO. The perpendicular to a radius at its extremity is tangent to the circumference.

MECHANICAL DRAWING

GEOMETRICAL PROBLEMS

These problems have been chosen because of their particular bearing on the work of the mechanical draftsman.

All of these problems should be drawn in order to become familiar with their construction. Problems 25, 26, 27, and 28 are worthy of special attention because the methods shown in them are used in the construction of gear teeth. Draw the construction lines light and use a heavier line for the outlines of the figure. In penciling, all intersections must be formed with great care as the accuracy of the results depend upon them. Keep the pencil points in good order at all times and draw lines exactly through intersections.

Problem 19. *To draw an ellipse when the axes are given.*

Draw the lines LM and CD about 3 and 2 inches long, respectively, making CD perpendicular to LM at its middle point P and having CP equal PD. The two lines LM and CD are the axes. With C as a center and a radius LP equal to one-half the major axis draw the arc, cutting the major axis at E and F. These two points are the foci.

Now locate several points on PM, such as A, B, and G. With E as a center and a radius equal to LA draw arcs above and below LM. With F as a center and a radius equal to AM describe short arcs cutting those already drawn, as shown at N. With E as a center and a radius equal to LB draw arcs above and below LM as before. With F as a center and a radius equal to BM draw arcs intersecting those already drawn, as shown at O. The point R and others are found by repeating the process. Find at least 12 points on the curve —6 above and 6 below LM—and these 12 points with L, C, M, and D will enable one to draw the curve.

After locating these points, draw a free-hand curve passing through them lightly and finish with the French curve.

Problem 20. *To draw an ellipse when the axes are given.*

Draw the two axes AB and PQ. With O as a center and a radius equal to one-half the major axis describe a circle. Similarly

with the same center and a radius equal to one-half the minor axis describe another circle. Draw any radii, such as OC, OD, OE, OF, etc., cutting both circumferences. These radii may be drawn with the 60- and 45-degree triangles. From C, D, E, and F, the points of intersection of the radii with the large circle, draw vertical lines; and from C', D', E', and F', the points of intersection of the radii with the small circle, draw horizontal lines. The intersections of these lines are points on the ellipse.

Draw a free-hand curve passing through these points; about five points in each quadrant will be sufficient.

Problem 21. *To draw an ellipse by means of a trammel.*

As in Problems 19 and 20, draw the major and minor axes, UV and XY. Take a slip of paper having a straight edge and mark off CB equal to one-half the major axis, and DB equal to one-half the minor axis. Place the slip of paper in various positions, keeping the point D on the major axis and the point C on the minor axis. If this is done, the point B will mark various points on the curve. Find as many points as necessary and sketch the ellipse.

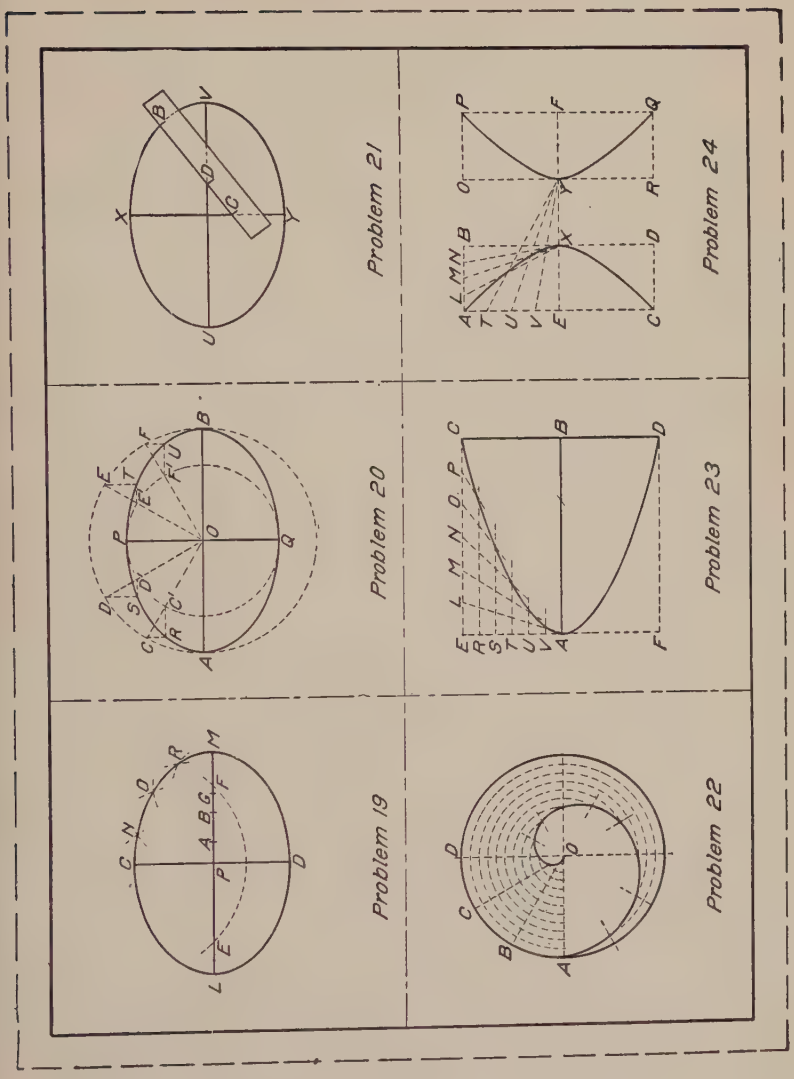
Problem 22. *To draw a spiral of one turn in a circle.*

Draw a circle with the center at O and a radius of $1\frac{1}{2}$ inches. Locate twelve points $\frac{1}{8}$ inch apart on the radius OA and draw circles through these points. With the 30-degree triangle, draw radii OB, OC, OD, etc., 30 degrees apart, thus forming 12 equal parts.

The points on the spiral are now located; the first is at the center O; the next is at the intersection of the line OB and the first circle; the third is at the intersection of OC and the second circle; the other points are located in the same way. Sketch in pencil a smooth curve passing through these points.

Problem 23. *To draw a parabola when the abscissa and the ordinate are given.*

Draw the straight line AB—about 3 inches long—as the axis or abscissa of the parabola. At A and B draw the lines EF and CD perpendicular to AB, and with the T-square draw EC and FD $1\frac{1}{2}$ inches above and below AB, respectively. Let A be the vertex of the parabola. Divide AE and EC into the same number of equal parts. Through R, S, T, U, and V, draw horizontal lines and connect L, M, N, O, and P, with A. The intersections of the horizontal lines with the oblique lines are points on the curve. For


Problem 21
Problem 24
Problem 20
Problem 23
Problem 19
Problem 22

instance, the intersection of AL and the line V is one point and the intersection of AM and the line U is another.

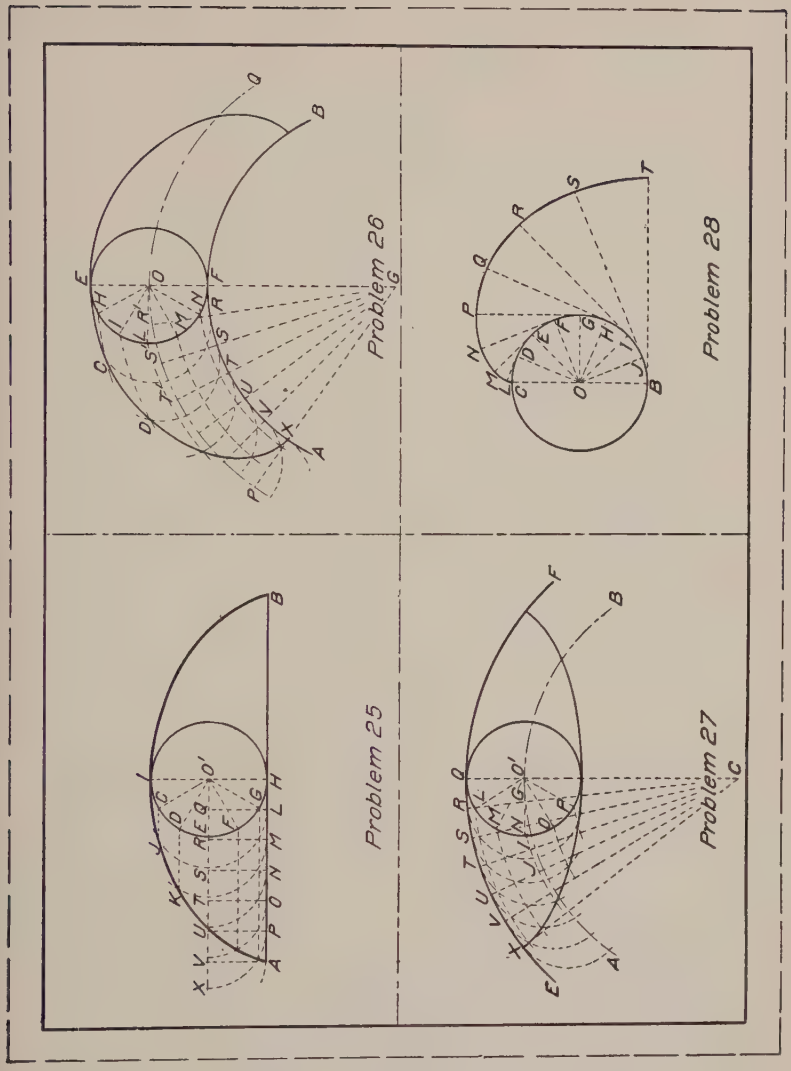
The lower part of the curve AD is drawn in a similar manner.

Problem 24. *To draw a hyperbola when the abscissa EX, the ordinate AE, and the diameter XY are given.*

Draw EF about 3 inches long and mark the point X 1 inch from E, and the point Y 1 inch from X. With the triangle and T-square draw the rectangles $ABDC$ and $OPQR$ such that AB is 1 inch in length and AC is 3 inches in length. Divide AE and AB into the same number of equal parts. Connect Y with the points T, U, and V on AE, and connect X with L, M, and N on AB. The first point on the curve is at A; the next is at the intersection of TY and LX; the third is at the intersection of UY and MX. The remaining points are found in the same manner. Repeat the process for XC and the right-hand curve PYQ.

Problem 25. *To construct a cycloid when the diameter of the generating circle is given.*

With O' as a center and a radius of $\frac{7}{8}$ inch draw a circle and, tangent to it, draw the indefinite horizontal straight line AB. Divide the circle into any number of equal parts—12 for instance—and through these points of division C, D, E, F, etc., draw horizontal lines. Now with the dividers set so that the distance between the points is equal to the chord of the arc CD mark off the points L, M, N, O, P on the line AB, commencing at the point H. At these points erect perpendiculars to the center line XO', which is the line of centers of the generating circle as it rolls along the line AB. With the intersections Q, R, S, T, etc., as centers describe arcs of circles as shown. The points on the cycloid will be the intersections of these arcs and the horizontal lines drawn through the points C, D, E, F, etc. Thus the intersection of the arc whose center is Q and the horizontal line through C is a point J on the curve. Similarly, the intersection of the arc whose center is R and the horizontal line through D is the point K on the curve. The remaining points on the left, as well as those on the right, are found in the same manner. To obtain great accuracy in this curve, the circle should be divided into a large number of equal parts, because the greater the number of divisions the less the error due to the difference in length between a chord and its arc.


Problem 26
Problem 28
Problem 25
Problem 27

Problem 26. *To construct an epicycloid when the diameter of the generating circle and the diameter of the director circle are given.*

The epicycloid and the hypocycloid may be drawn in the same manner as the cycloid if arcs of circles are used in place of the horizontal lines. With O as a center and a radius of $\frac{3}{4}$ inch describe a circle. Draw the diameter EF of this circle and produce EF to G such that the line FG is $2\frac{3}{4}$ inches long. With G as a center and a radius FG describe the arc AB of the director circle. With the same center G draw the arc PQ, which will be the path of the center of the generating circle as it rolls along the arc AB. Now divide the generating circle into any number of equal parts—12 for instance—and through the points of division H, I, L, M, and N draw arcs having G as a center. With the dividers set so that the distance between the points is equal to the chord HI, mark off distances on the director circle AFB. Through these points of division R, S, T, U, etc., draw radii intersecting the arc PQ in the points R', S', T', etc., and with these points as centers describe arcs of circles as in Problem 25. The intersections of these arcs with the arcs already drawn through the points H, I, L, M, etc., are points on the epicycloid. Thus the intersection of the circle, whose center is R', with the arc drawn through the point H is a point upon the curve. Also the intersection of the arc, whose center is S', with the arc drawn through the point I is another point on the curve. The remaining points are found by repeating this process.

Problem 27. *To draw a hypocycloid when the diameter of the generating circle and the radius of the director circle are given.*

With C as a center and a radius of 4 inches describe the arc EF, which is the arc of the director circle. Now with the same center and a radius of $3\frac{1}{4}$ inches describe the arc AB, which is the line of centers of the generating circle as it rolls on the director circle. With O' as a center and a radius of $\frac{3}{4}$ inch describe the generating circle. As before, divide the generating circle into any number of equal parts—12 for instance—and with these points of division L, M, N, O, etc., draw arcs having C as a center. Upon the arc EF lay off distances QR, RS, ST, etc., equal to the chord QL. Draw radii from the points R, S, T, etc., to the center of the director circle C and describe arcs of circles having a radius equal to the radius of the generating circle, using the points G, I, J, etc., as centers. As

in Problem 26, the intersections of the arcs are the points on the hypocycloid. By repeating this process, the right-hand portion of the curve may be drawn.

Problem 28. *To draw the involute of a circle when the diameter of the base circle is known.*

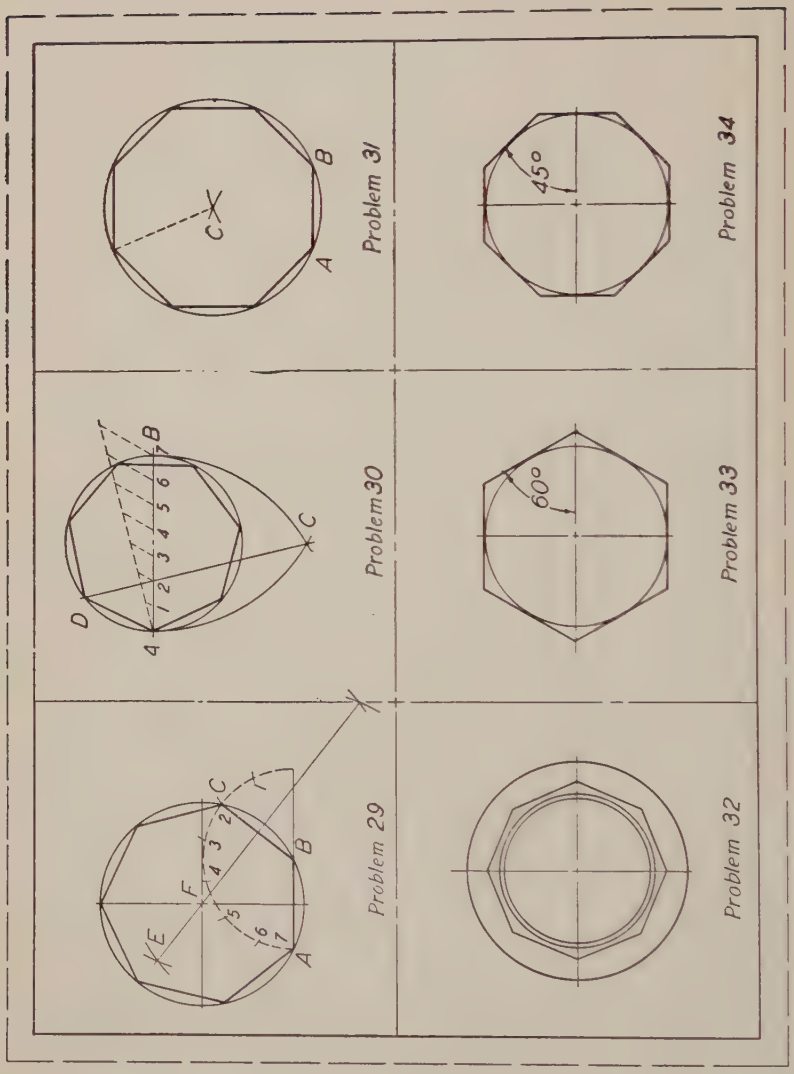
With the point O as a center and a radius of 1 inch describe the base circle. Divide the circle into any number of equal parts—16 for instance—and draw radii to the points of division. At the point D draw a light pencil line perpendicular to OD. This line will be tangent to the circle. Similarly at the points E, F, G, H, etc., draw tangents to the circle. Set the dividers so that the distance between the points will be equal to the chord of the arc CD, and measure this distance from D along the tangent. From the point E measure on the tangent a distance equal to two of these chords; from the point F, three divisions; and from the point G, four divisions. Similarly, measure distances on the remaining tangents, each time adding the length of the chord. This will give the points L, M, N, P, etc., to T. The curve drawn through these points will be the involute of the circle.

Problem 29. *To construct a polygon* (any number of sides) *on a given line.*

The figure used in this problem is a heptagon. Extend the line AB beyond B a distance greater than the line AB. Line AB should be about $1\frac{5}{8}$ inches long. With center at B and a radius BA describe a semicircle. By trial divide this semicircle into the number of parts equal to the number of sides of the required polygon—in this case 7. Draw the line BC through point *2* on the semicircle. BC is equal to AB. Bisect AB and BC. The intersection of these lines will give the center of the circle. Draw the circle touching the three points A, B, and C. Step off the other sides on the circle and complete the figure.

Problem 30. *To construct a polygon* (any number of sides) *given the diameter of the circle which will circumscribe the figure.*

The illustration shows a heptagon. Describe a circle 3 inches in diameter and draw the diameter AB. Divide the diameter AB into the same number of parts as the required figure—in this case seven. With center at B and a radius BA draw the arc AC. With center at A and a radius AB draw the arc BC. Draw the line CD passing through C and the point *2* on the diameter of the circle so that it cuts


Problem 29
Problem 30
Problem 31
Problem 32
Problem 33
Problem 34
A
B
C
D
E
F
1
2
3
4
5
6
7
45°
60°

the circle at D. Draw the line AD. This line gives the length of one of the sides of the figure. Step this distance around the circle and complete the figure.

Problem 31. *To construct an octagon within a given circle, two sides being parallel to a horizontal line.*

On a piece of scratch paper lay out a circle with the required diameter—in this case 3 inches. Divide the circle into four equal parts by drawing two diameters at right angles to each other. Bisect the four angles to divide the circle into eight equal parts. Draw the horizontal line AB equal to one of the parts of the figure already drawn. Set the compass to the radius of the circle already drawn and with centers at A and B describe the arcs cutting each other at C. With center at C and a radius CA describe the circle. Complete the figure by setting off the distance AB around the circle.

NOTE: Problems 29, 30, and 31 are extremely important because of their wide application to all kinds of drafting and shop problems.

Problem 32. *Given the front view of an automobile hub cap. Diameter of outside circle 3 inches. Diameter of octagon across the corners $2\frac{1}{2}$ inches. Diameter of inner circle 2 inches. Diameter of circle next to octagon $2\frac{1}{8}$ inches.*

Required to draw the figure showing all construction lines. The method which is shown in Problem 30 may be used for drawing the octagon.

NOTE: It is often necessary to circumscribe a hexagon or an octagon around a circle, and it is often necessary to form a hexagon where the distance across the flat of the hexagon is the only dimension given, and the same thing applies to the octagon.

Problem 33. *To circumscribe a circle with a hexagon.*

Draw a circle equal in diameter to the distance across the sides of the hexagon, which in this case is $2\frac{1}{2}$ inches. With the T-square draw lines tangent to the top and bottom of the circle. With the 60° triangle complete the hexagon by drawing the other four sides tangent to the circle as shown.

Problem 34. *To circumscribe a circle with an octagon.*

Draw a circle equal in diameter to the distance across the sides of the octagon, which in this case is $2\frac{1}{2}$ inches. With the T-square draw horizontal tangent lines at top and bottom of circle. With triangles draw vertical tangent lines to circle, and complete the octagon by using the 45° triangle as shown.

MECHANICAL DRAWING

SYMBOLS AND CONVENTIONS

Like in all crafts, there has developed in drafting a system of symbols and conventions that helps to eliminate written instruc-

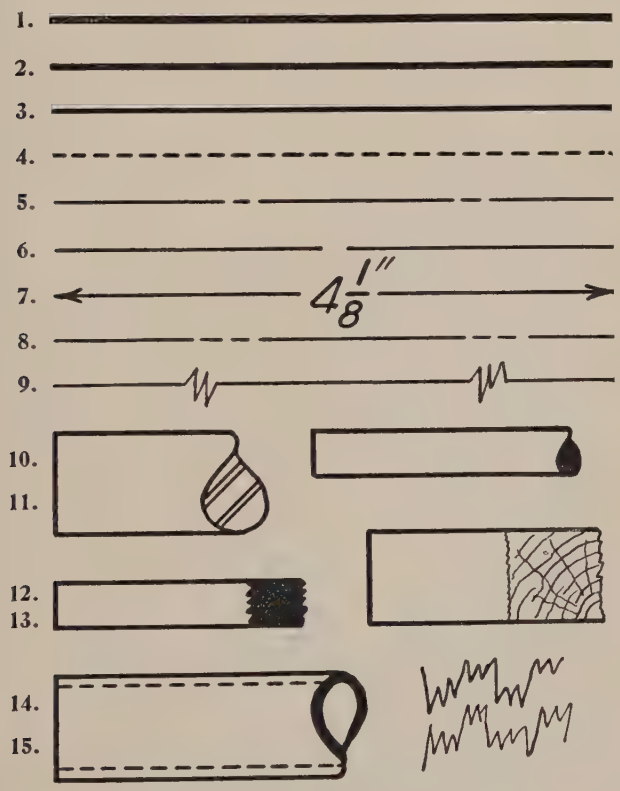


Fig. 1. Alphabet of Lines in General Use

1. Border line
2. Shade line
3. Visible line
4. Invisible line
5. Center line
6. Extension line
7. Dimension line
8. Cutting plane line
9. Limiting break
10. Break for large solid shaft
11. Break for small solid shaft
12. Break for small wood beam
13. Break for large wood beam
14. Break for tube
15. Break for wood

tions with regard to the making of an article or machine. The common symbols and conventions used in standard drafting practice are illustrated in Figs. 1, 2, 3, and 4.

Use of Lines. In Fig. 1, numbers 1 to 9, are shown the different classes of lines used in working drawings, and each one of these lines has a definite use. It is readily seen that these lines are not all of the same weight or thickness, and in drawings this same variance in thickness for the different types of lines should be evident to the eye.

A good drawing has snap and character and it is given to the drawing by the clear cut variation in the weight of the different lines. A drawing in which the lines are all the same thickness is flat and lacks character.

The border line is the heaviest line always. It is one of the first lines to be drawn and the last to be inked in. It entirely surrounds all of the views or drawings on the sheet. Being a heavy line it is not always possible to draw it with one stroke of the ruling pen. Sometimes it is necessary to use two or three strokes to draw two or three lines which touch each other in order to complete the line to the necessary width or thickness. A clear cut line of equal width its entire length can be obtained much more easily by drawing two or three lines than by trying to draw it with one pen setting. Draw the lines close together so that they join or touch each other without any spaces in between them.

The shade line is lighter in weight or thickness than the border line, but it is heavier than the full line which is used to show the outline of the object.

While the shade line has been quite extensively used in the past on all drawings, owing to the fact that it takes more time to shade line a drawing, it is not used in general practice today. However, it is used to some extent on illustrative drawings for magazines and on drawings which are submitted to the government with applications for patents on certain objects or machines.

The full or visible line is a very important line because it shows the outline or shape of the exterior of the object. It is about half the weight or thickness of the shade line. This line should always be clean cut and distinct.

The dotted or invisible line is also an important line, because it shows the shape of that part of the object which is invisible or hidden to the eye. The dots or short dashes should be uniform in length and about $\frac{1}{16}$-inch long with about a $\frac{1}{64}$-inch space between them.

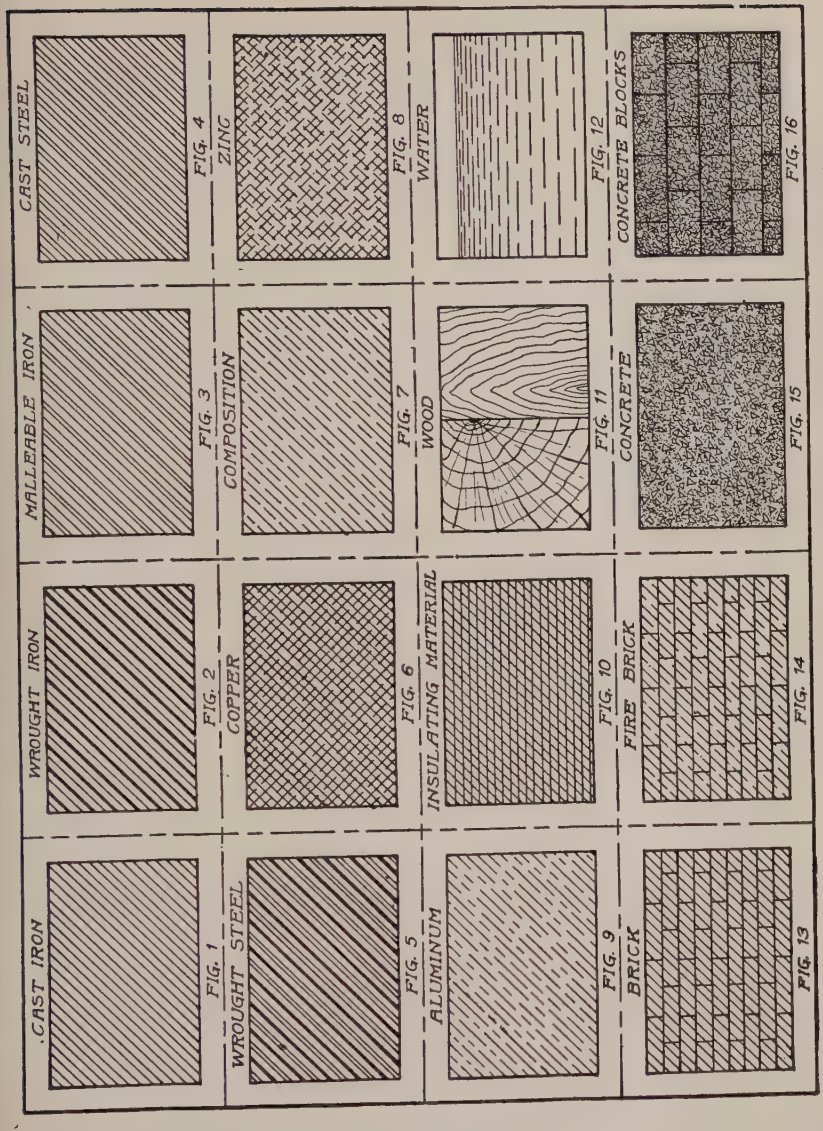
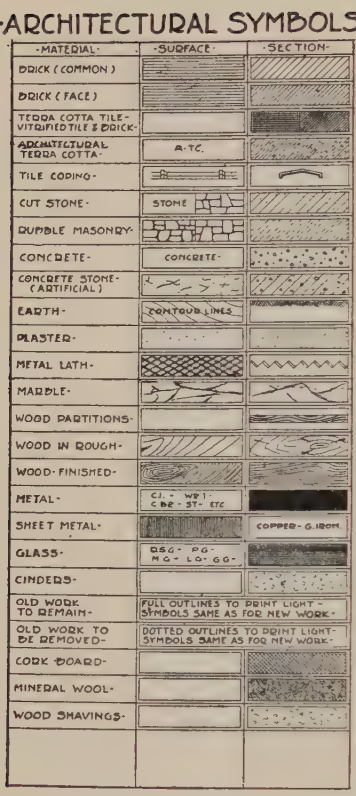


Fig. 2. Symbols for Representing the Cut Surfaces of Sectional Views of Various Materials



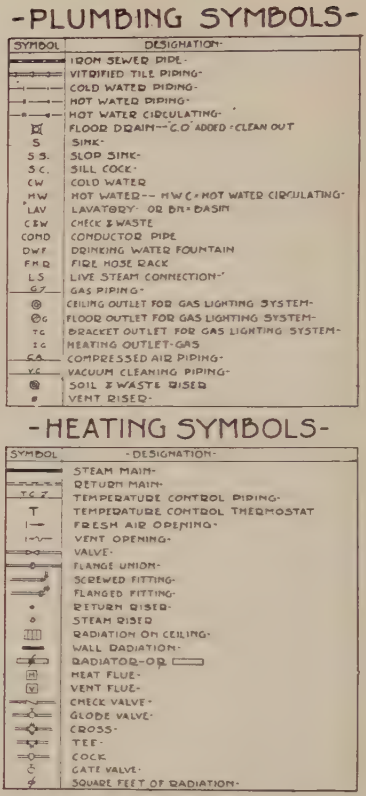


Fig. 3. Symbols Used in Architectural, Plumbing, and Heating Trades
Courtesy of H. A. Rogers & Company, Minneapolis

Note—The symbols shown here are the ones most generally used. However, as these symbols are varied somewhat by different architects, a key to materials is sometimes shown on the plans.

-ELECTRICAL SYMBOLS-

SYMBOL	-DESIGNATION-	SYMBOL	-DESIGNATION-
100	CEILING OUTLET ELECTRIC ONLY- NUMBER INDICATES WATTAGE OF LAMPS-	G	GALVANOMETER-
100/2	CEILING OUTLET ELECTRIC & GAS- NUMBERS INDICATE 100 WATTS CAP. & 2 GAS BURNERS.	V	VOLTMETER-
100	BRACKET OUTLET - ELECTRIC ONLY- NUMBER INDICATES WATTAGE OF LAMPS-	R	FIELD RHEOSTAT-
100/2	BRACKET OUTLET - ELECTRIC & GAS- NUMBERS INDICATE 100 WATTS CAP. & 2 GAS BURNERS.	A	AMMETER-
60	WALL OR BASE BOARD RECEPTACLE- NUMBER INDICATES CAPACITY IN WATTS-	S	MOTOR OUTLET. NUMBER INDICATES HORSE POWER.
60	FLOOR OUTLET - NUMBER INDICATES CAPACITY IN WATTS.		MOTOR CONTROL OUTLET.
60	OUTLET FOR OUTDOOR PEDESTAL- ELECTRIC ONLY. NUMBER INDICATES CAPACITY IN WATTS-		TRANSFORMER-
60/6	OUTLET FOR OUTDOOR PEDESTAL-ELECTRIC & GAS. NUMBERS INDICATE 60 WATTS CAP. & 6 GAS BURNERS.		TELEPHONE OUTLET - PRIVATE-
D	DROP CORD OUTLET-		TELEPHONE OUTLET- PUBLIC-
	ONE LIGHT OUTLET FOR LAMP RECEPTACLE-		BELL OUTLET-
	SPECIAL OUTLET FOR LIGHTING - HEATING OR POWER CURRENT - AS PER SPECIFICATION -		BUZZER OUTLET.
	CEILING FAN OUTLET-	2	PUSH BUTTON OUTLET NO. INDICATES NUMBER OF PUSHES.
S	SINGLE POLE SWITCH OUTLET.	8	ANNUNCIATOR OUTLET. NO. INDICATES NUMBER OF POINTS-
S^2	DOUBLE POLE SWITCH OUTLET.		SPEAKING TUBE -
S^3	3 WAY SWITCH OUTLET.	C	WATCHMANS CLOCK OUTLET-
S^4	4 WAY SWITCH OUTLET.		WATCHMANS STATION OUTLET-
S^D	AUTOMATIC DOOR SWITCH OUTLET.	TC	MASTER TIMECLOCK OUTLET.
S^E	ELECTROLIER SWITCH OUTLET.	D	SECONDARY TIME CLOCK OUTLET-
	METER OUTLET.		DOOR OPENER-
	DISTRIBUTION PANEL.	SC	SPECIAL OUTLET FOR SIGNAL SYSTEM-
	JUNCTION OR PULL BOX.		BATTERY OUTLET-

	MAIN OR FEEDER RUN CONCEALED UNDER FLOOR-
	MAIN OR FEEDER RUN CONCEALED UNDER FLOOR ABOVE-
	MAIN OR FEEDER RUN EXPOSED.
	BRANCH CIRCUIT RUN CONCEALED UNDER FLOOR.
	BRANCH CIRCUIT RUN CONCEALED UNDER FLOOR ABOVE.
	BRANCH CIRCUIT RUN EXPOSED.
	POLE LINE.
	RISER.
	CIRCUITS UNDER FLOOR CONCEALED.
	CIRCUITS UNDER FLOOR ABOVE CONCEALED.

Fig. 4. Symbols Used in Electrical Trade
Courtesy of H. A. Rogers & Company, Minneapolis

A dotted line which has its dashes and spaces uniform will add much to the neatness and appearance of a drawing.

The center line, as its name implies, is the center line of the drawing of an object. In practically every drawing the center line is the first line drawn and in many cases is the line from which dimensions are taken. The center line is not only used for the actual center of the drawing but it is also used wherever there are circles or where holes are shown represented by circles or squares.

The extension line is drawn opposite the points on the drawings from which measurements are taken or indicated and between these extension lines is placed the dimension line with the arrowheads on the end.

The dimension line is the line on which the dimension is placed. The arrowheads should touch the extension lines, as they indicate the exact points from which the measurement or dimension is taken.

The cutting plane line, sometimes known as a reference line, is used to indicate the point at which a section is taken or shown. It is used by some draftsmen as a center line. It is also used in Perspective Drawing.

DIMENSIONING DRAWINGS

The most important thing on a drawing are the dimensions, because these are the directions to the workman. In placing the dimensions on a drawing one should aim for clearness, consequently, the numerals should not be crowded and should be large enough so that they are plain and easy to read. Neatness in dimensioning will not only add greatly to the appearance of the drawing but it will also make it easy to read.

It is most important that one be accurate, in that the points from which measurements or dimensions are taken must be clearly and accurately indicated by the correct placement of extension lines, dimension lines, and arrows. The dimension must also be correctly given and the same dimension should not be repeated on the different views.

All necessary dimensions should be given so that the workman does not have to add or subtract one dimension from another in order to get a dimension which should have been given by the drafts-

man but was left off the drawing. Wherever possible the dimension should be placed outside the figure or drawing of the object. This eliminates confusion and it is much easier to show the dimension.

Extension lines, while drawn opposite the point from which dimensions are taken, should not touch the figure or drawing, see Fig. 5.

The dimension line should be placed at about $\frac{5}{16}$-inch from the drawing outline as shown in Fig. 5. Where there is more than one dimension to be shown on the same side of the drawing, $\frac{5}{16}$-inch should be allowed between each of the dimension lines and the dimensions should be staggered as shown in Fig. 6.

The largest dimension should always be the farthest away from the drawing, so that the crossing of the dimension lines with extension lines can be avoided. Horizontal fraction lines should be used and as a general rule are in line with the dimension line, see Fig. 6.

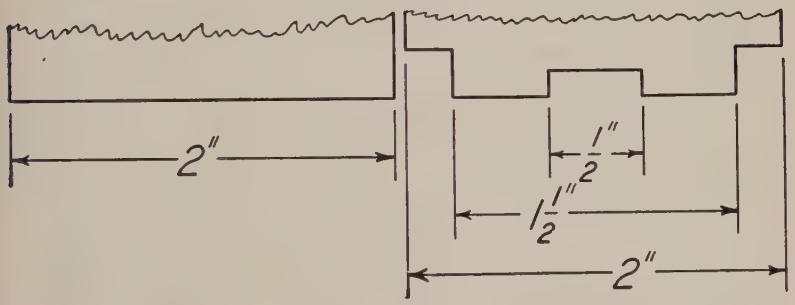


Fig. 5 and Fig. 6. Methods of Showing and Placing Dimensions

When showing the dimensions for an arc or a radius, the dimension line and arrow should point from the center for the arc to the arc as in Fig. 7.

Arrows neatly formed and of uniform size add greatly to the appearance and neatness of the drawing. The arrowhead should be bold, sharp, and narrow or with very little flare, see Fig. 8. The dimension line is drawn first, and in forming the arrowhead the line should follow the dimension line and then gradually come away from it.

Drawing Arcs and Fillets. Sharp corners on metal objects, except in the case of cutting tools, should be avoided wherever possible. This will prevent cracking to a large extent, and in many

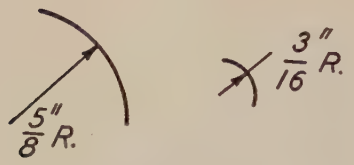


Fig. 7. Methods of Indicating Dimensions and Centers for Arcs

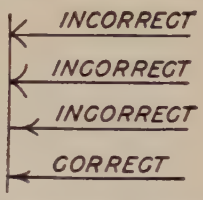


Fig. 8. Incorrect and Correct Shape for Arrows on Dimension Lines

cases it gives strength with very little increase in weight. Corners should be rounded with a radius as shown in Fig. 9. In this figure the corner is rounded with a $\frac{1}{4}$-inch radius. With center at A and compass set to the required $\frac{1}{4}$-inch radius, draw an arc so that it cuts the lines as at B and C, Step 1. Then with center at C, Step 2, draw a short arc having the required $\frac{1}{4}$-inch radius. With center at B, draw a short arc with the $\frac{1}{4}$-inch radius, Step 3, so that it cuts

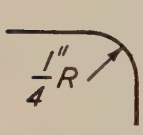


Fig. 9. Method of Indicating Dimension and Center for Radius

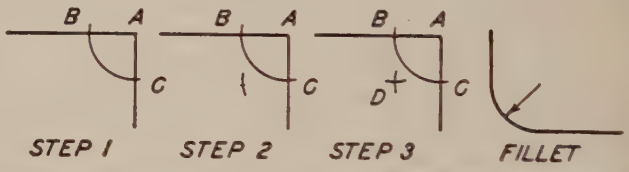


Fig. 10. Steps Used When Drawing a Radius or Fillet

the other short arc as at D. Then D is the center for the required radius. In Fig. 10 is shown a fillet, which is also often called a radius, and the same method as for Fig. 9 can be used.

General Hints on Dimensions

Dimensions must read on the line and either at the right-hand side or the lower side of the plate.

Do not repeat same dimensions unless there is a good reason.

Never use a center line as a dimension line.

Give the diameter of circles.

Give the radius of arcs.

The abbreviation of "R" or "Rad" is used for radius.

Extension lines should not be a continuation of the side of an object. Leave a small space between the side of the object and the end of the extension line.

HOW TO INK A DRAWING

Inking. The pen should not be dipped in the ink but should be filled by means of a common steel pen or quill, to a height of about $\frac{1}{4}$ or $\frac{3}{8}$ inch, by touching the quill, which is found in the cork of the bottle, to the end of the space between the points of the pen, and the ink will flow from the quill into the pen. If too much ink is placed in the pen, it is likely to drop out and spoil the drawing. Upon finishing the work, wipe the pen with chamois or a soft cloth.

Fig. 11. Using the Ruling or Lining Pen

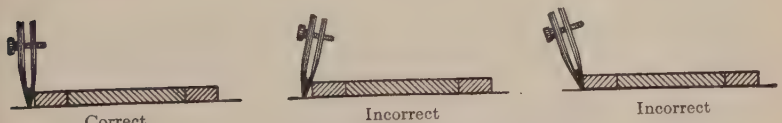


Fig. 12. Correct and Incorrect Method of Holding the Ruling or Lining Pen in Relation to the T-Square

In using the pen, care should be taken that both blades bear equally on the paper in order that the line may be smooth. The pen is usually inclined slightly in the direction in which the line is drawn and should touch the triangle or T-square lightly so as not to press the blades together and thereby change the width of the line, Figs.

11 and 12. The pen must not be tipped outward, however, as the danger of blotting is greatly increased when the line is drawn so close to the guide.

To ink a drawing well requires great care and some experience. One should not attempt to ink his work until he can make a clear-cut, strong line. It is well to practice inking in straight pencil

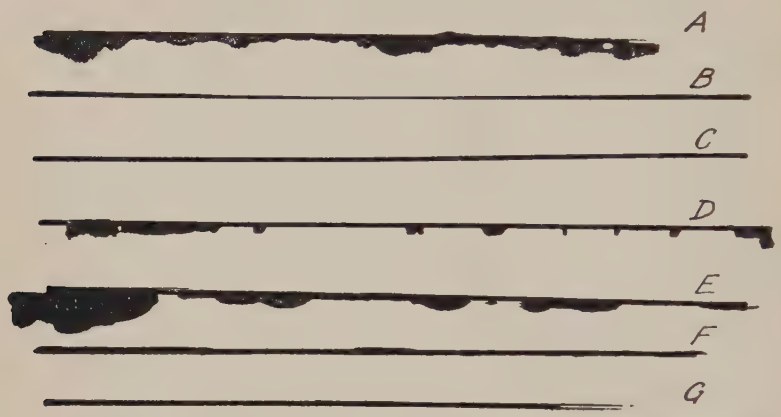


Fig. 13. Samples of What Careless or Inaccurate Use of the Ruling or Lining Pen Will Do

A. Pen point tilted inwards
B. Pen pressed against T-square too hard
C. Pen not parallel to T-square all way
D. T-square touched wet line
E. Ink on outside of pen run under T-square
F. Pen point tilted outwards
G. Not enough ink in pen to finish line

lines, rectangles, and intersections. These lines should be definite, each line stopping exactly at the right point. Before starting to ink in, adjust the pen by means of the thumb screw until a good clean line of desired width is obtained, making frequent test lines on a piece of material similar to that which has to be used. Keep the pressure of the pen on the paper uniformly light, remembering that different weights of lines are not obtained by pressure as with the ordinary writing pen but only by adjusting the nibs of the pen. If the lines are ragged, the pen should be put in order. Sometimes when the ink does not flow regularly, moisten the end of the finger and touch the point of the pen. Care should be taken not to put too much ink in the pen, but on the other hand there must be enough to draw the next line as it is difficult to continue a line after re-filling the pen. The only way to draw fine lines well is to frequently clean and

refill the pen. If the amount of the ink in the pen is small, it is likely to thicken in the point and cause clogging. When this occurs, draw a small strip of paper between the nibs to clean out the clogged ink.

When drawing, the pen should be held with the thumb screw out and should be inclined slightly in the direction in which it is moved. Be careful, however, not to incline it too much, as the best of pens when incorrectly held will produce poor lines. It is therefore advisable at the start to acquire the correct method of holding the pen. Do not press the sides of the pen point too heavily against the ruling edge as this will vary the width of the line; after a little practice the pen can be lightly and firmly brought in contact with the paper and ruling edge at the same time. The pen should be drawn from left to right, the hand being steadied by sliding it on the end of the little finger.

Always try to get into the easiest position when inking a line, even if it becomes necessary to walk around the drawing. The average draftsman prefers the standing position while inking as he can usually obtain much better results. Keep the ruling edge between the line and the body so that the pen will be drawn against the ruling edge for if this is not done, the pen is liable to be pulled off at an angle, making a crooked line. Be careful after inking a line to draw the ruling edge toward the body away from the line in order to avoid blotting. Where lines meet at a point, always ink towards the point, being sure to allow one line to dry before inking another. Always ink in the top and left-hand lines first, gradually working down to the right, thus saving time that otherwise would be lost in waiting for the lines to dry. When the pen is set at the proper width, draw all the lines of that width before making a change. Never push the pen backward over a line. If a good line is not drawn the first time, it is better to go over it again in the same direction, taking great care not to widen the original line.

Ink dries very quickly and should not be left in the pen on account of its corrosive effects. The celluloid triangles should be washed frequently in water and all ink spots removed.

In using the compass, bend both legs so that each will be perpendicular to the paper or cloth when the arc or circle is drawn. When the pen attachment is used, special care must be exercised on this point for in no other way can the nibs of the pen be made to

bear evenly on the surface. In drawing arcs, hold the cylindrical handle at the top of the compass loosely between the thumb and the forefinger and let it roll between the two during rotation; allow the compass to lean slightly in the direction of rotation, pressing down the pen point slightly but not the needle point. Be sure to fix the needle point firmly in its proper place on the paper before touching the pen to the paper, as otherwise a slip is likely to occur. In setting the needle down on any particular center, guide it with a finger of the left hand. Avoid making a noticeable hole in the paper.

Ink in the circumference of a circle with one continuous motion, giving an even pressure to the pen throughout the operation and stopping it sharply at the end of one revolution. Since straight lines can be more easily drawn tangent to curves than the reverse, it is always advisable to ink in all arcs or circles first. When a number of circles are to be drawn from one center, the smaller should be inked first while the center is in the best possible condition.

The main outlines of an object should be drawn so that the drawing will stand out against the lighter center, dimension, and invisible lines. A drawing in which the lines are all of the same weight has a very flat appearance and is hard to read.

Inking the Border. 1. Adjust the ruling pen to about one-third the thickness of the border line and make two or three trial lines on the edge of the sheet as it is difficult to make a clean start in a broad line.

2. Place the edge of the T-square so that it coincides with the left-hand line. In making this line, the head of the T-square is not to be used as a guide. In starting the line, do not allow the pen to rest on the paper as the ink begins to flow as soon as the pen touches the paper and if left there for a second or two will soon cause a blot.

3. Place the T-square to draw the lower border line. In this case, the head of the T-square should be against the edge of the drawing board. Start this line with the pen dry in order to draw the ink that has accumulated at the end of the left-hand line. Continue the line as far as the ink will go, then refill the pen and finish the line.

4. Draw the top border line. Start this line with the pen filled with ink. Be sure that the line at the left is dry before placing the T-square upon it.

5. Draw the right-hand border line. Start this line with the pen filled with ink.

Ink. India ink is always used for drawing as it makes a permanent black line; it is obtainable in solid stick or in liquid form. The liquid form is most convenient but as it contains acid which corrodes steel, it is necessary to keep the pen perfectly clean.

The pen should be cleaned by drawing a piece of linen through the space between the points, and it is a good plan to clean it each time before filling it.

How to Sharpen the Ruling Pen. When it is impossible to make a smooth line with the ruling pen, it should be sharpened. Screw the blades together and grind them to a parabolic shape by drawing the pen back and forth over a small, flat, close-grained oilstone. This process, of course, makes the blades dull but insures their being of the same length. Now separate the points slightly and rub one of them on the oilstone, keeping the pen at an angle of from 10° to 15° with the face of the stone, and giving it a slight twisting movement. This part of the operation requires great care as the shape of the ends must not be altered. After one point has become fairly sharp, grind the other in a similar manner, grinding always on the *outside* of the blades and removing the burr from the inside with leather or pine wood. Test the pen by filling it with ink and drawing several lines. Unless the lines are smooth, the grinding must be continued.

Essential "Dont's"

Don't screw up the nibs of the pen too tight.
Don't use a blotter on lines that have been inked.
Don't run the pen or pencil backward over a line.
Don't fill a pen over a drawing.

General Hints for Making Working Drawing

Pencil Work

1. Use a 4H or 6H pencil.
2. Have the point sharp.
3. Do not sharpen the pencil over the drawing board.
4. Clean the pencil after sharpening it.
5. Lines must be fine, light, and clear.
6. Inaccurate pencil work cannot be corrected in inking.

Order of Pencil Work in Making Plates

1. Lay out border and title spaces.
2. Plan arrangement of views.
3. Draw primary center and base lines.
4. Lay off principal measurements.
5. Block in the views.
6. Lay off detailed measurements.
7. Draw the center lines for details.
8. Draw all circles, parts of circles, and round corners.
9. Draw all lines that could not be previously drawn.
10. Draw all extension and dimension lines.
11. Crosshatch sections.
12. Put on the title.
13. Check the work.

Lettering

1. Do not mix styles of letters.
2. For all notes to be placed on the drawing, use capital and lower-case letters.
3. Always draw guide lines before lettering.
4. Follow carefully the directions for forming each letter.
5. Good lettering can only be obtained by constant practice.

Order for Inking

(Start at the top of the drawing and work down.)

1. Center lines.
2. Small arcs and circles.
3. Large arcs and circles.
4. Irregular curves.
5. Dotted circles and arcs.
6. Full horizontal lines.
7. Full vertical lines.
8. Full inked lines.
9. Dotted lines.
10. Extension and dimension lines.
11. Arrowheads and figures.
12. Section lines.
13. Title and notes.
14. Border.
15. Check the drawing.

Cautions

1. Keep the pen clean.
2. Wipe the pen carefully after using.
3. Do not put over $\frac{1}{4}$ of an inch of ink in the pen.
4. Never dip the pen in the ink bottle.

MECHANICAL DRAWING

PRINCIPLES OF THE WORKING DRAWING

The working drawing is the universal language of the draftsman, designer, craftsman, and engineer. From the working drawing the mechanic learns the details of size, shape, material, accuracy required, and finish of the part or whole of the machine he is making. The tinsmith in making a commonplace article like a stove pipe, the blacksmith in forging a simple crank, the carpenter in making a box, all use drawings instead of the printed or spoken word for their information.

The working drawing has played a very important part in the development of the modern production methods. Parts for machines, bridges, buildings, etc., are often made in different plants, and yet from the information given by the working drawings these parts are so accurately made that locomotives and automobiles, bridges and skyscrapers are assembled without special fitting of their parts.

Relation of Shape of an Object to the Drawing. A photograph of an object gives us a general idea of how it looks, but it does not tell us all the facts, because some of the details of the picture are foreshortened and do not show their true shapes. The true shape of every side of a figure is needed to tell all the facts.

The construction or shape of the object is the controlling factor as to the number of views required of the object to give all the necessary details and dimensions for the mechanic. The different views of an object are referred to as the top or plan view; the front or elevation view; and the side or end elevation view.

The top or plan view is a drawing of the top view of an object; the front or elevation view is a drawing of the front view of an object; while the end view is a drawing of the end view of an object.

Fig. 1 is a picture of a simple square block. If the draftsman placed this block flat on the top of a table so that he could look down

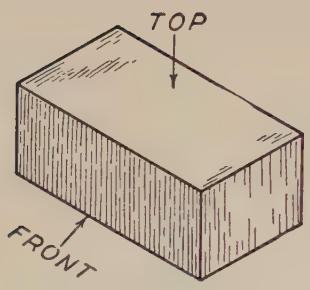


Fig. 1. Plain Block

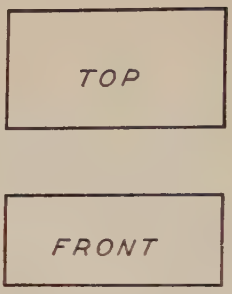


Fig. 2. Top and Front View of Plain Block Shown in Fig. 1

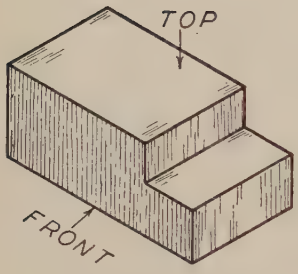


Fig. 3. Block with Step Cut in Right End

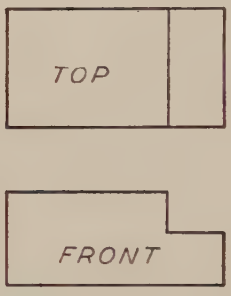


Fig. 4. Top and Front View of Block Shown in Fig. 3

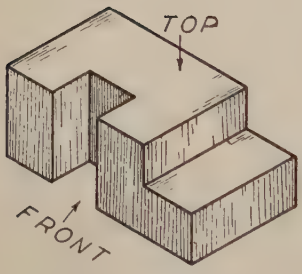


Fig. 5. Block with Groove Cut in Front Face and Step in Right End

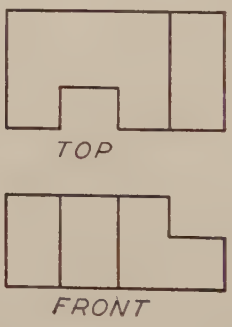


Fig. 6. Top and Front View of Block Shown in Fig. 5

on it, he would see only the flat top of the block and its four edges, therefore, this view would be a rectangle the same size as the top of the block, Fig. 2.

To get the front view the draftsman would place the block directly in front of and in line with his eyes, and in this view he would see the front surface of the block and its four edges. This view would be a rectangle the same size and shape as the front of the block, Fig. 2.

Fig. 3 shows the same block with a step cut in one end. As we look down on the block, we still see the outline of the shape of the top and we also see the line which forms the edge of the step, Fig. 4. The front view shows the width and the height of the step, Fig. 4. Note that the top view shows that the step is cut clear across the block. Cover up Fig. 3 and see if you can visualize the shape of the block from the top and front views.

In Fig. 5 we still see the same block but another change has been made in its shape by cutting a square groove in the front face of the block. As we look down on the top of this block to get the top view, we see just what was shown in the top view of Fig. 4, but in addition to this we also see the top of the square groove. In this case the top view will show the depth of the groove. In looking at the front to get the front view, we see all that was seen in the front view in Fig. 4, also two vertical edges or lines which represent the groove in the front face, Fig. 6.

In Fig. 7 is a picture of a round bar or cylinder, standing on end vertically. In looking down on the top of this bar, we see a circle the same size as the bar itself. If this bar is held in front of the eye so that only the top edge can be seen, it would appear as a straight line and must be drawn as such. To test this out take a round disc, such as a half dollar piece, and hold it up in front of the eye so that you can only see the edge. The thickness of the metal is seen and appears as a straight line. Fig. 8 shows the top and front views of this round bar. In the top view is shown the size and shape of the bar while the elevation view shows the height of the bar. Both plan and elevation show the diameter of the bar in this case.

If the round bar is so placed that it is in a horizontal position as in Fig. 9, so that we see the end of the bar from in front, the position of the views would then be reversed, as shown in Fig. 10, because in

looking down on the bar we would see its length and width in the top view and in the front view we would see the end of the bar, showing its diameter.

Fig. 11 shows this same bar placed vertically but with a pin showing on one end. In looking down at the bar to get the top view,

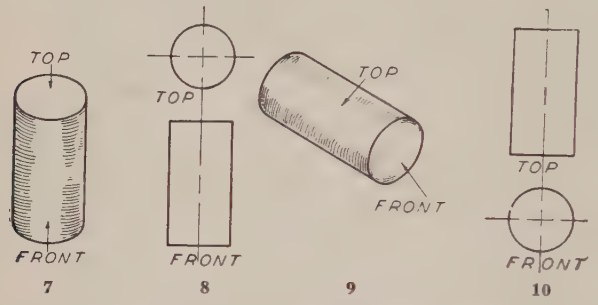


Fig. 7. Plain Round Bar or Cylinder Standing Vertically
Fig. 8. Top and Front Views of Round Bar Shown in Fig. 7
Fig. 9. Plain Round Bar or Cylinder in Horizontal Position
Fig. 10. Top and Front Views of Round Bar Shown in Fig. 9

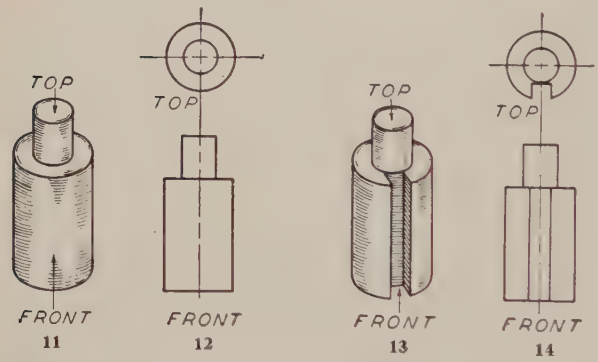


Fig. 11. Round Bar or Cylinder with Pin on Top
Fig. 12. Top and Front Views of Round Bar Shown in Fig. 11
Fig. 13. Round Bar or Cylinder with Pin on Top and Groove Cut in Front
Fig. 14. Top and Front Views of Round Bar Shown in Fig. 13

we will see two concentric circles having the same centers, Fig. 12. The small circle will be the top view of the pin, the larger circle, the top view of the bar. In the front view we see the length and thickness of the pin and also the length and thickness of the bar. Remember that the edge of a circle will appear and be drawn as a straight line.

The same bar is shown in Fig. 13, but a groove or keyway has been cut the full length of the bar to a depth which is just in line with the pin on the end of the bar. If we look down on the top of this bar, we again see the small circle complete, but the large circle shows this groove or keyway, and gives its depth, Fig. 14. The front is like the front view on Fig. 12, but we see two vertical lines, which is the front view of the groove and shows that the groove runs the full length of the bar. The top view and the front view show that the keyway is slightly narrower than the pin on the end of the bar. Cover up Fig. 13 and try to visualize the actual shape of the bar.

Principles of Projection. The method used to find the different views of an object is called "Orthographic Projection" because the drawings or views are considered as being projected on to planes, which may be represented as sheets of paper placed in certain positions above and in front of the object. This method is the basis of all working drawings and unless it is thoroughly understood complicated drawings and peculiar shaped objects are difficult to draw.

The most common planes of projection are the vertical and the horizontal. The top or plan view is termed the horizontal projection, and the front or elevation view is termed the vertical projection.

The vertical plane may be considered as a sheet of paper placed in a vertical position. See Step 1, Fig. 15.

The horizontal plane may be considered as another sheet of paper in a horizontal or flat position. See Step 1, Fig. 15.

In Step 2 of Fig. 15 these two planes are brought together, forming a right angle and joining each other at line *GL* (ground line).

In Step 3, Fig. 15, we have swung the horizontal plane so that it is now in line with the vertical plane, and now we have these two planes as one sheet of paper separated by the ground line. See Step 4, Fig. 15.

Fig. 16 shows the four angles of projection. They are made by two planes intersecting each other at right angles. They are numbered 1, 2, 3, and 4, as shown in Fig. 16. The line made by the intersection of the planes is the ground line.

Study the drawing which is in the first angle of projection and also the position of the eye. Note how the narrow edge of the block is projected back on the vertical plane and the broad surface of the block down on the horizontal plane.

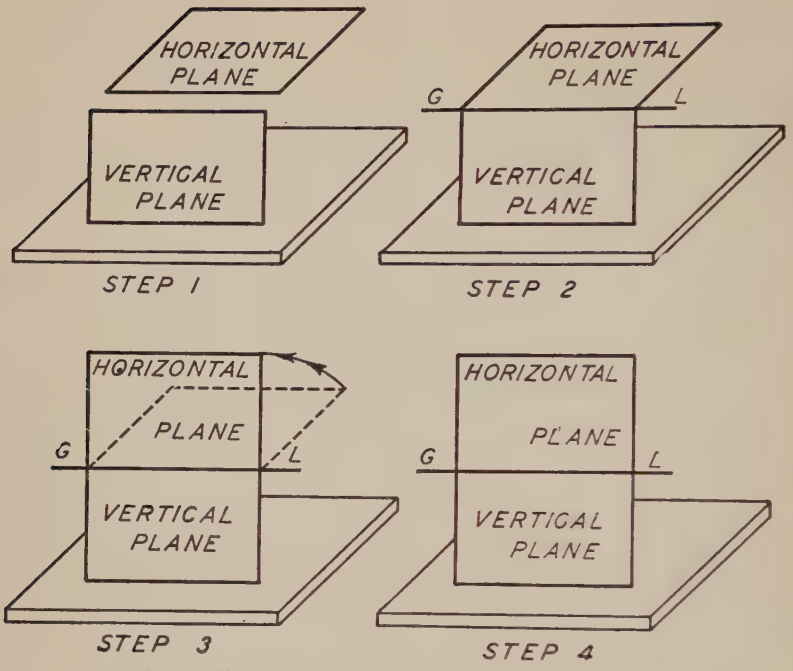


Fig. 15. Planes of Projection Represented by Sheets of Paper

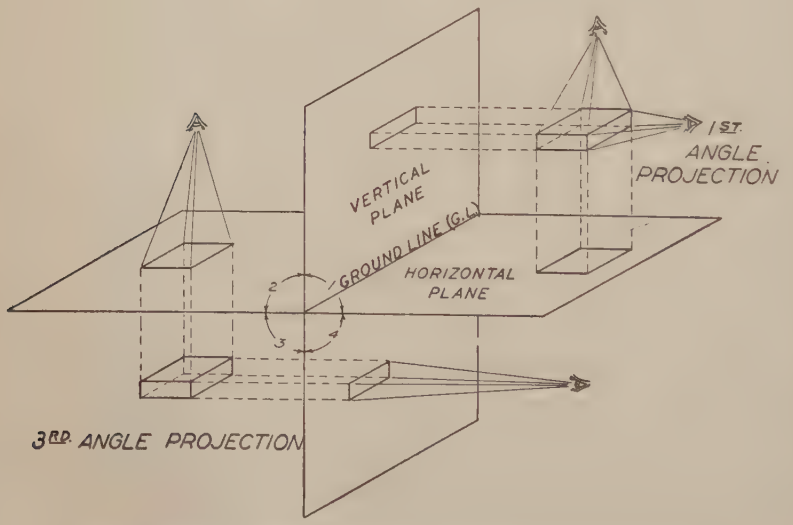


Fig. 16. Position of Planes of Projection in First and Third Angle Projection

Now study the third angle projection. Note in this angle of projection the observer is **looking through the plane in each case** and note that the narrow edge of the block is projected forward on the vertical plane and the broad surface of the block is projected upwards on the horizontal plane. Study this third angle of projection along with Fig. 15. Note how it corresponds to Step 2 of this figure.

Look at the sketch of the block in Fig. 17, and then refer to the projection of this block in Fig. 18. First of all, note the object behind the vertical plane and below the horizontal plane. Then follow the projection of this block on to the vertical plane and up to the horizontal plane. These two projections are shown by the words "Top" and "Front." Study Fig. 19 in connection with this movement and see how the horizontal plane has been swung up so that it is now in line with the vertical plane. In making this movement, the distances of the block behind the vertical plane and below the horizontal plane are shown by the dotted projection lines *A*, *B*, *C*, and *D*, above and below the ground line.

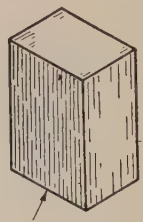


Fig. 17. Plain Block

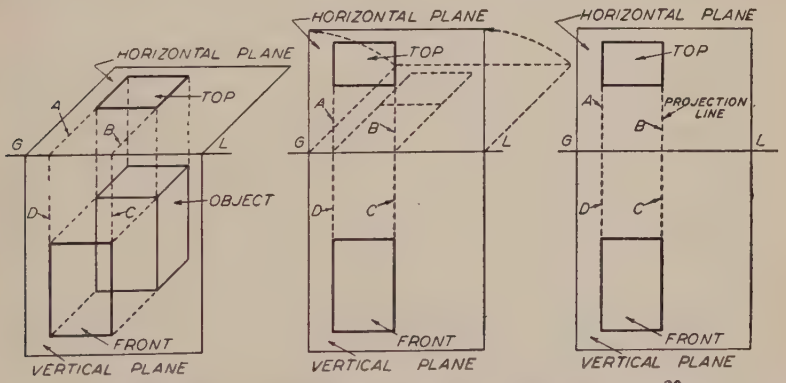


Fig. 18. Plain Block Placed in Projection Position behind Vertical Plane and below Horizontal
Projections of top and front are shown on the plane of projection.
Fig. 19. Horizontal Plane in Position as It Would Be on Sheet of Paper
Fig. 20. Showing Projections as They Would Appear in a Drawing

This is made clear in Fig. 20, which shows only the top and front views of the block with the connecting projection lines. Note how Fig. 20 coincides with Step 4 in Fig. 15, only we have the actual drawings now showing on the planes.

Study Fig. 21. Then refer to Figs. 22, 23, and 24. In Fig. 22 note the object behind the vertical plane and below the horizontal plane. Note how the front of the object is transferred to the vertical plane, and the top of the object, to the horizontal plane.

Fig. 23 again shows us the movement of the horizontal plane when it is swung up in line with the vertical plane. It also shows us the projection lines which indicate the position of the block behind the vertical plane and below the horizontal plane.

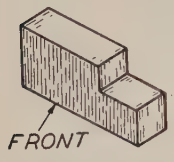


Fig. 21. Block with Step Cut in Right End

Fig. 24 shows the complete working drawing. Again refer to Fig. 16 and note how these series of drawings coincide with the third angle projection illustration.

Refer to Fig. 25. It is a flat block resting on its wide surface. Figs. 26, 27, and 28 show the steps in the projection of this block.

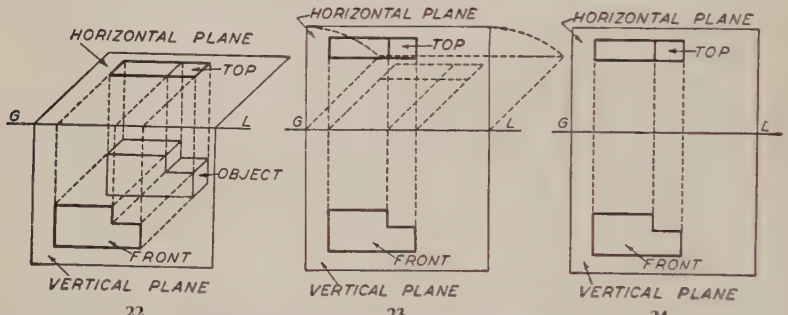


Fig. 22. Block Shown in Fig. 21 Placed in Projection Position
Projections of top and front are shown on the planes of projection.
Fig. 23. Horizontal Plane in Position as It Would Be on Sheet of Paper
Fig. 24. Showing Projections as They Would Appear on a Drawing

Fig. 26 shows the object in its relation to the projection planes. Fig. 27 again shows the horizontal plane swung up in line with the vertical plane. Fig. 28 shows the drawings as they now appear on a flat piece of paper. Again refer to Fig. 16 and note how this coincides with the illustration shown in the third angle projection.

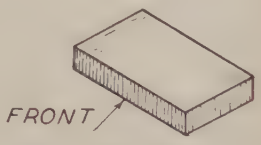


Fig. 25. Plain Flat Block

Figs. 17, 21, and 25 have been developed on the projection planes in order to show how the top or plan view always appears directly above the front or elevation view when drawings are made in the third angle projection.

Fig. 29 shows a cylinder with a circular and square projection cut on one end. A number of steps are necessary in making a working drawing of this. Refer to Step 6 of this series of drawings, which is the completed drawing.

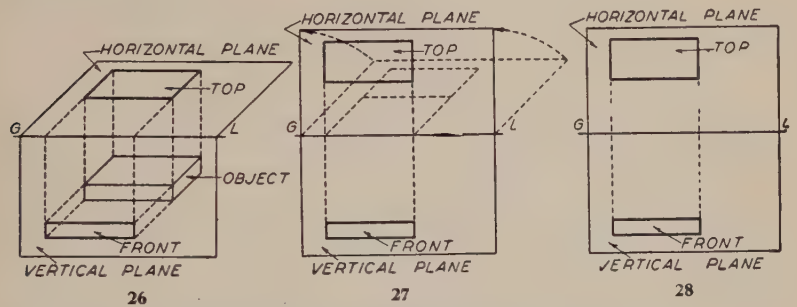


Fig. 26. Plain Flat Block Placed in Projection Position
Projections of top and front are shown on the planes of projection.
Fig. 27. Horizontal Plane in Position as It Would Be on Sheet of Paper
Fig. 28. Showing Projections as They Would Appear on a Drawing

Note that the top appears as two circles with a square in the center. Imagine you are looking down on the top of this object and see if it coincides with the drawing. If this object were put under the

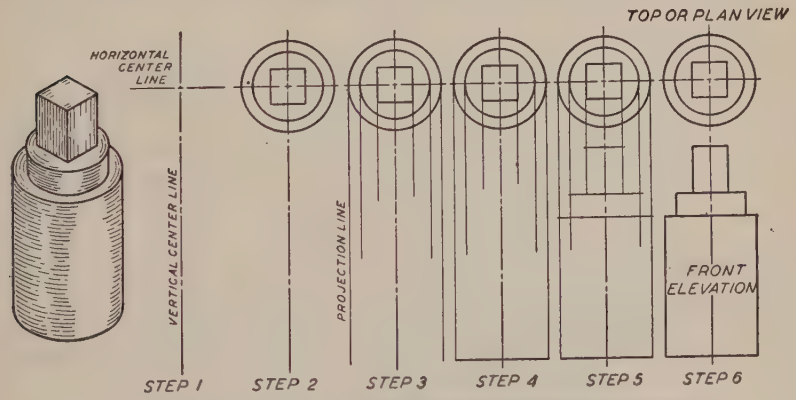


Fig. 29. Round Bar with Circular and Square Projection on One End
Steps required to make and project the top and front views.

projection planes, this view would appear on the horizontal projection plane.

Next look at the lower part of Step 6 which shows the shape of the cylinder from the front. This is the drawing that would appear on the vertical plane of projection.

Now go back to Step 1. The horizontal and vertical center lines are shown. In a figure of this kind these are always drawn first because all measurements are taken from these lines. Seeing this figure is circular in form, we start with the top view first and draw the circles and then the square.

Step 3 shows a series of projection lines being brought down to give us the shape of the front view.

Step 4 shows the base line of the object drawn in.

Step 5 shows the horizontal lines drawn to meet the projection lines which cut off the object into its shape. These lines are determined by actual measurement.

Step 6 shows the finished drawing.

These are the steps that the average draftsman would follow in drawing this object.

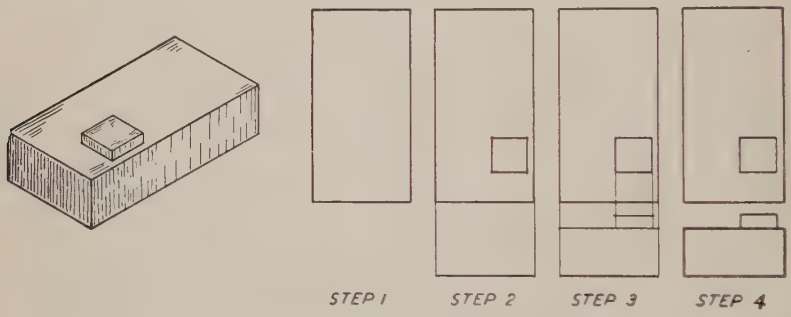


Fig. 30. Square Block with Square Projection on Top Face
Steps required to make and project the top and front views.

In Fig. 30 are shown steps in the layout of a rectangular shaped object, with a square pad on the top surface. Center lines are not necessary when making a working drawing of an object of this shape. In this case the drawing could be started with either top or front view.

To locate the position of the square pad it would be necessary to make the measurements from one end and one side of the object.

In all these drawings it will be noted that we have left out the ground line. However, it is still assumed to be there and therefore the top drawing is known as the top or plan view and the lower drawing as the elevation or front view.

The third angle projection is the method commonly used in the United States and therefore you will find most of the drawings in shops made according to this plan. Occasionally, however, one does find the first angle projection being used, and the main difference in the drawing is that the views are reversed in their position; that is, the front view or elevation appears above the top or plan view. By again referring to Fig. 16 you will see the reason for this.

PRACTICAL EXPERIMENTS IN PROJECTION

To help one visualize the views of an object it is suggested that he construct a projectograph and experiment with it continuously.

Three small wire screens similar to those used on the windows of residences during the summer months will make an excellent set of planes for experimental purposes. The fine mesh on the wire

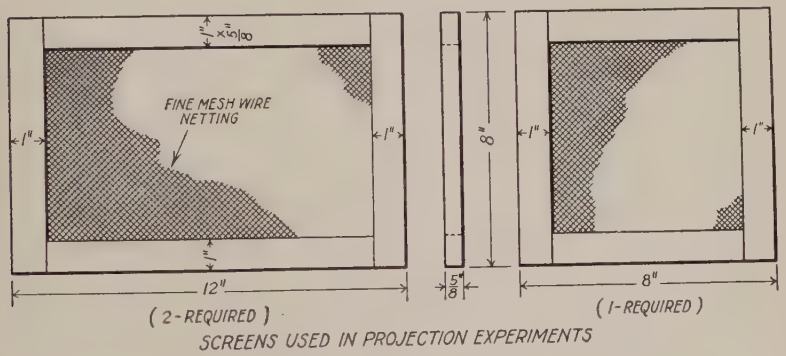


Fig. 31. Screens Used in Projection Experiments

screen makes it possible to see the object clearly through the planes and also makes a good surface for tracing a line on the planes in chalk.

Two screens 8″ by 12″ and one screen 8″ by 8″ are large enough to handle objects like match-boxes, ink wells, or small models of prisms, cones, or pyramids. The wood frame need not be made out of any heavier material than $\frac{5}{8}$″ by 1″. A simple butt joint and nails will be strong enough construction, as the netting will help to hold the screens together.

Hinges for the screens, made out of tape or webbing about $\frac{3}{4}$″

wide, will allow the planes to flatten out after the sketches have been made.

Fig. 31 shows the details of the construction of the screen.

Fig. 32 shows the screens set up with a figure inserted for drawing.

Fig. 33 shows the screens flattened out and the position of the chalk lines on the screen.

Note that these three pictures or views on the screen give all the facts regarding the true shape of the block.

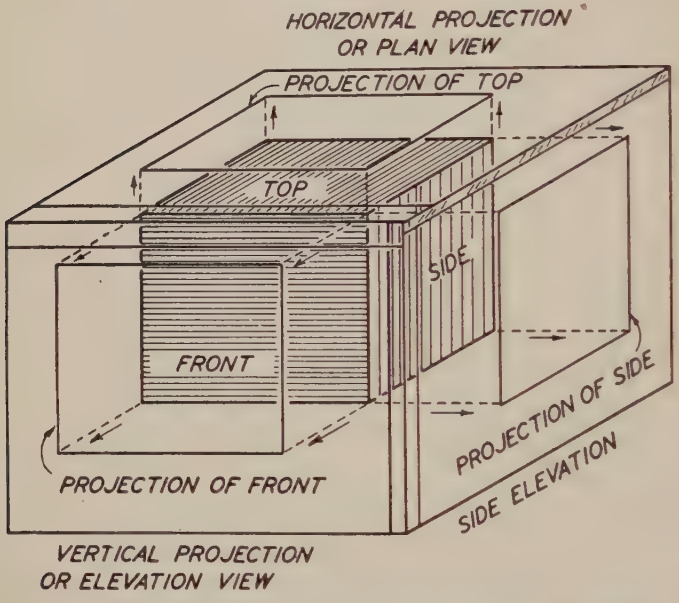


Fig. 32. Projectograph in Position for Experiment
Note projections of shaded box shown on top, front, and side of projectograph.

Note that the edges which form the hinges of the screen can be thought of as lines which make the divisions of the horizontal and vertical planes.

Note that the front view of the block is on the lower half of the picture; that the top view is on the upper half; and that the side view is shown in the lower right-hand part of the picture.

The objects mentioned above make very good subjects to experiment with, but if these things are not at hand, take a piece of soap or

some other fairly firm material and cut out some variously shaped objects.

After erecting the projectograph as shown in Fig. 32, place any object in the position as shown by the shaded box inside of the projectograph, and then make the sketches on the top and front of the screen of what is inside. A piece of chalk may be used to make the

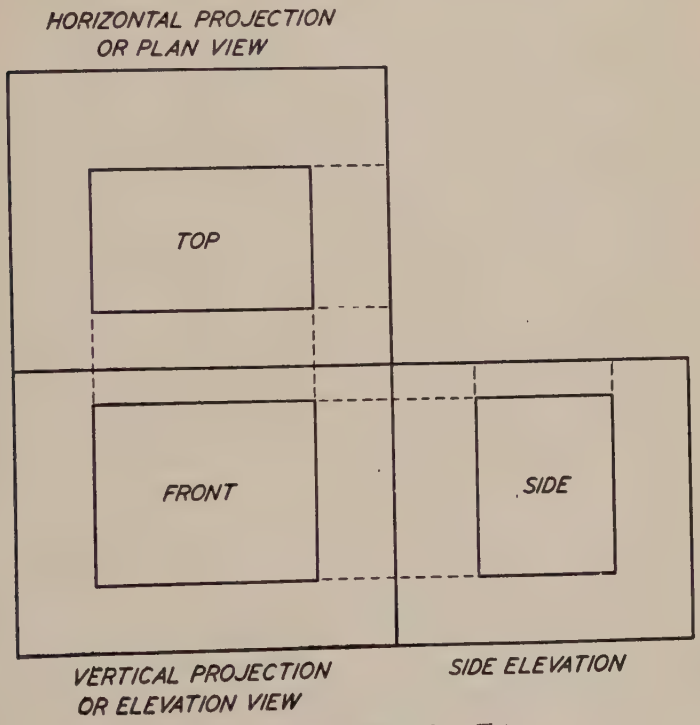


Fig. 33. Projectograph Laid out Flat
Note position of views.

rough drawings or sketches on the screening. The next step is to flatten out the projectograph, or planes as they may be termed, as shown in Fig. 33, and the position of the views and planes are clearly seen.

To help one visualize the shape of the object from its different views, one should draw all the orthographic projections not shown in Fig. 34. They should be drawn large enough so that they are clear. The different views should be labeled *H* for horizontal plane

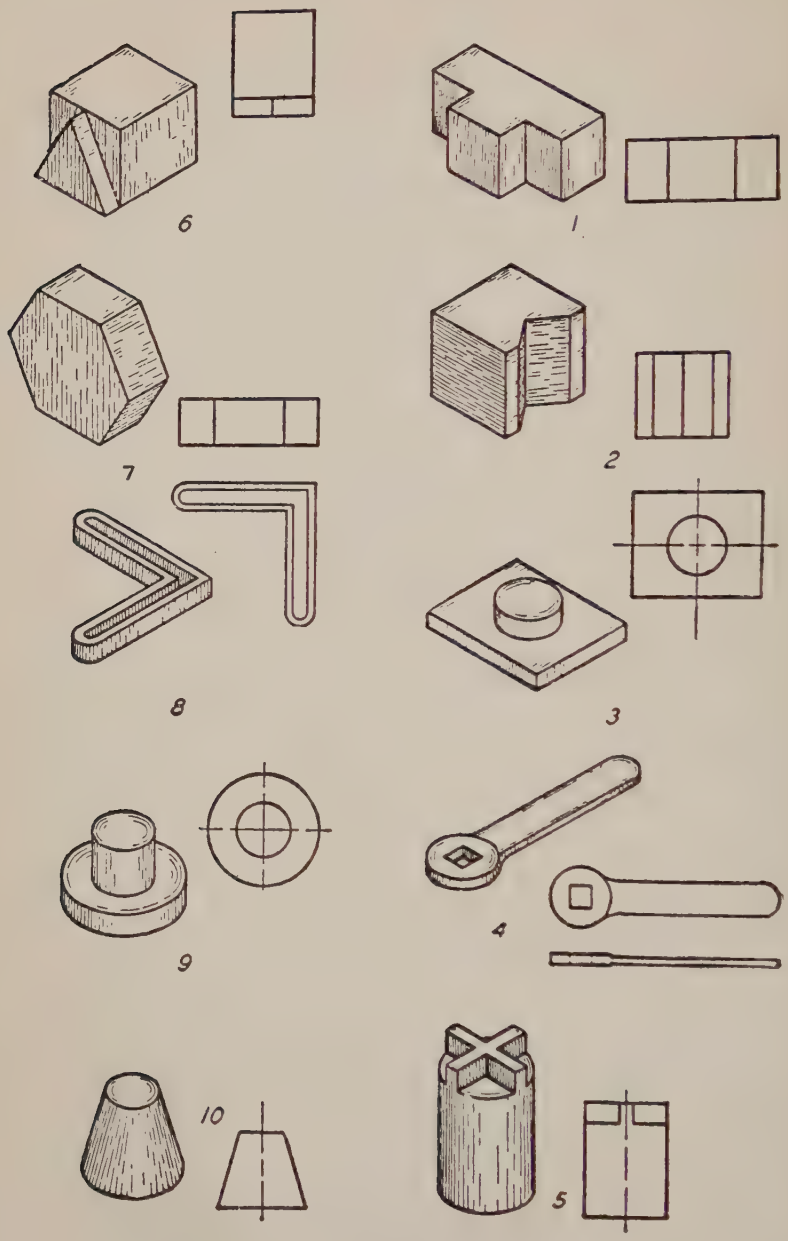


Fig. 34. Sketch in Missing views

and *V* for vertical plane, according to the projection, and each view should have written near it what is shown in the view or projection in regard to height, depth, diameter, etc.

It is not necessary to send this work in to the School, but we will gladly look the work over if it is sent in.

MECHANICAL DRAWING

HIDDEN PARTS

Hidden Lines. Thus far in projection the figures used or objects illustrated have been composed of solid figures, but practically everything the mechanic makes has either holes or other unseen parts indicated on the drawings, and to show these parts another method must be introduced into our projection work. In Fig. 36 is shown a sketch of a round bar with a hole drilled through it from end to end. If this bar were placed vertically and then viewed from the front for the vertical projection, we would see the top and bottom edges as a straight line and the sides of the bar as vertical lines; but we

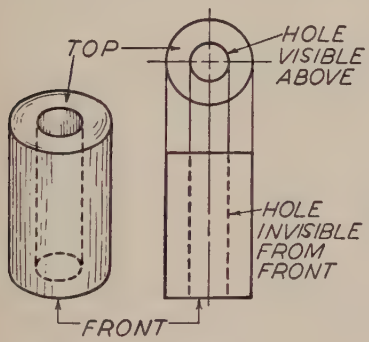


Fig. 36 Fig. 37

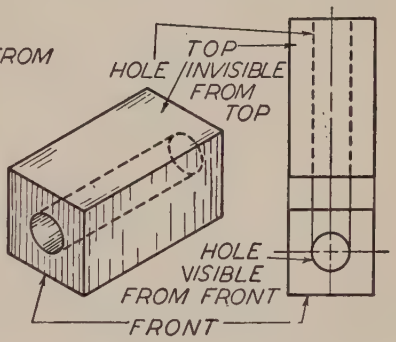


Fig. 38 Fig. 39

would not see the hole because it is hidden inside the bar. To indicate to the mechanic that there is a hole in this piece, two dotted lines will be drawn from top to bottom and as far apart as the diameter of the hole, as shown in Fig. 37. To get the horizontal projection, or top view, we look down from above on the top of the bar and see the large circle equal in diameter to the bar itself; inside this we see the small circle, which is the view of the hole through the bar. Note that in this view the hole is shown as a full circle because it is visible to the eye as we look down on it.

In Fig. 38 is shown a block with a hole drilled through it from one end to the other, and it is so placed that the end with the hole

in it is directly in front of the draftsman, so that for the vertical projection, or front view, the view is a rectangle with a hole in the center. The rectangle and the hole are shown in full line because they are visible to the eye from in front, Fig. 39. If we look down from above to get the horizontal projection, or top view, we will also see a rectangle, but in this view the hole will be invisible to the eye and, therefore, must be shown dotted. The circle in the front view shows that it is located on the center of the rectangle, but it does not show how far the hole goes through the block, while the plan view shows that the hole goes through the entire block from front to rear. As the block is placed behind the vertical plane, the lower or bottom edge shown in the plan view is always the front of the object, while the top line in the plan view is always the back of the object. In the elevation view, the top line is the top of the object and the bottom line is the bottom of the object.

Hidden Edges. Hidden edges of objects are also shown as dotted in their projection views. In Fig. 40 is shown a vertical tri-

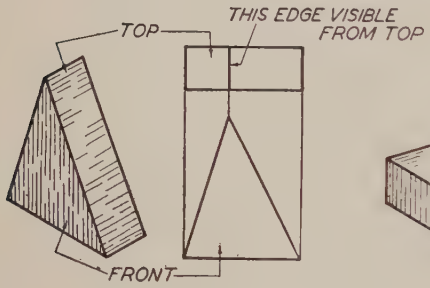


Fig. 40 Fig. 41

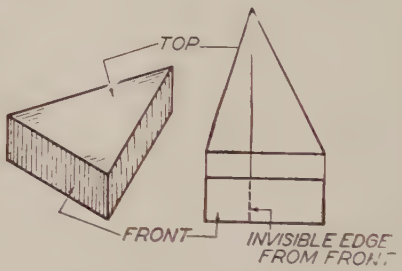


Fig. 42 Fig. 43

angular block. The projections of this block are shown in Fig. 41. In this view every edge is visible to the eye from either above or in front and therefore is shown in full line. If the block were laid down flat so that the top edge is away from us, as in Fig. 42, the top edge would be invisible to the eye from in front because we would be looking through the bottom of the block then, and the projections would be as in Fig. 43.

Hidden edges inside the object must also be shown dotted if these edges or lines cannot be seen from the different projection

positions whether they be inside the object or are a part of the outside shape of the object.

In Fig. 44, is shown the picture sketch of the base of a lathe dead center or tail stock. When this part is placed so that the small dovetail is in a horizontal position, and we look down on the top of it to get the top view, we will see the flat top of the object, also the flat top of the small dovetail, Fig. 45.

The bottom of the dovetail is narrower than the top, consequently, two horizontal dotted lines must be shown indicating this. In the plan view this illustrates the edge or line which, although actually part of the outside of the object, is invisible to the eye and must therefore be dotted.

The other dotted lines show that there are some invisible edges or lines somewhere else in the part. The elevation shows that there is a *V*-shaped slot which rests on the ways of the lathe, a recessed section at the center of the block, and then a flat section which rests on the bed of the lathe. These are invisible to the eye from above

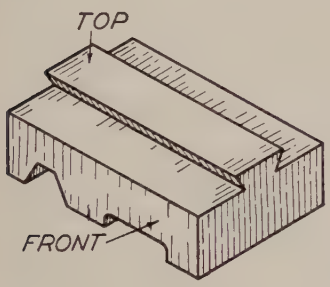


Fig. 44

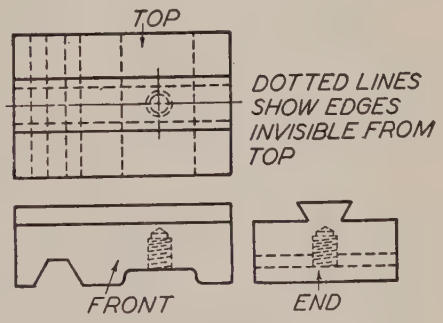


Fig. 45

and are therefore dotted in the plan view. In the elevation we see the front face of this base. The shape of the bottom of the base is clearly outlined, and the dotted lines shown indicate a threaded hole. This threaded hole is also shown dotted in the plan, being invisible to the eye in both views. The hole is left out of the object in Fig. 44 for clearness.

Fig. 46 shows the picture sketch of a hexagonal block with a hole through it. Fig. 47 shows the plan and elevation views of this block. The plan view shows the shape of the block, the size of the

hexagon across the flats and corners, or what we see when we look down on it. It also shows that there is a hole through it. The elevation shows the thickness of the block. The dotted lines in this view show that the hole goes through the block. Although the hole is indicated by the circle in the plan view, it does not indicate just

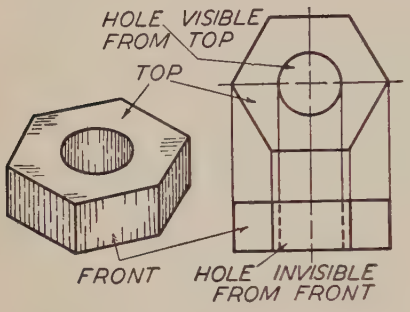


Fig. 46 Fig. 47

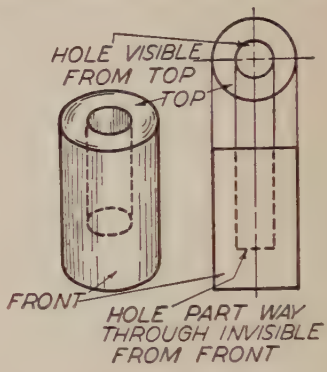


Fig. 48 Fig. 49

how far this hole goes through the hexagonal block, therefore the elevation must show this. Dotted lines are used because the hole would be invisible to the eye, if the block were held up directly in front of the eye. Notice the projection lines connecting points in line which are projected from one view to another.

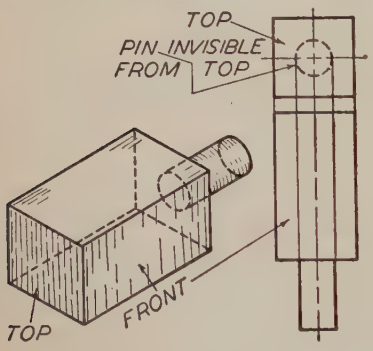


Fig. 50 Fig. 51

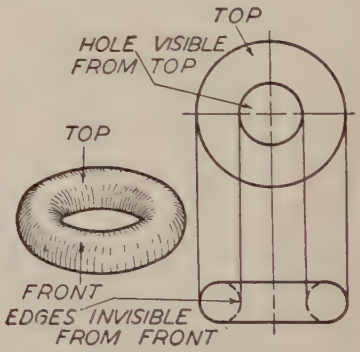


Fig. 52 Fig. 53

Fig. 48 shows the picture sketch of a cylinder which has a hole part of the way through it. The plan or top view, Fig. 49, shows the object to be round and of a certain diameter. The small circle in the center shows that there is a hole in the piece. The elevation,

or front view, shows the height of the object, while the dotted lines in this view show that the hole goes only part of the way through. The dotted lines are used because the hole in the object is invisible to the eye.

Fig. 50 represents a square bar with one end machined to a pin. The front elevation shows the length and width of the bar and also the length and width of the pin. The plan, Fig. 51, shows the square top. The dotted circle shows the shape of the pin and also that the pin is on the bottom and is, therefore, invisible from the top view. A bottom view would give a better idea of the complete form of the bar. Enough views should always be shown by the draftsman to give the workman a clear idea of what he is to make.

Fig. 52 is the plan view of a circular ring made from round stock or material. The front elevation, Fig. 53, shows the thickness and the diameter of the ring, and the dotted lines in the elevation show the shape of the material, which is hidden to the eye. The projection lines in Figs. 51 and 53 show the points projected from one view to another.

Fig. 54 represents a shaft with a number of different diameters. The shaft has been machined down in such a way that there are five different diameters, as shown. All these diameters, and the lengths between, may be shown in the front elevation. From this view alone the actual shape could not be ascertained. It might be square or circular, but the plan view shows that all parts are circular. The third and fifth diameters, counting from the top, would be invisible to the eye from above and are therefore dotted in the plan view.

Thus it is seen that dotted lines inside the object are used to indicate hidden holes, edges, or lines in the construction of the object, and the different views show the complete shape or construction of the object.

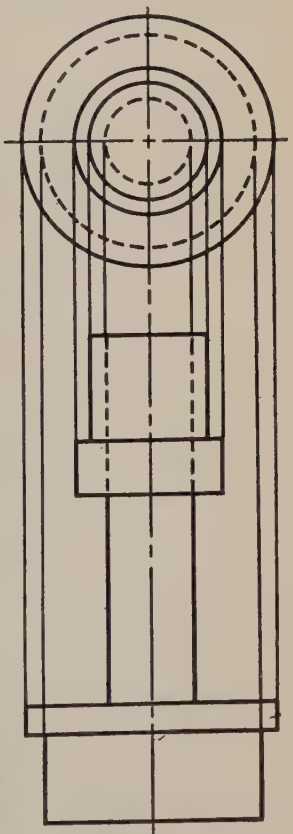


Fig. 54

THIRD VIEW

Method of Projecting. The number of views or projections required to show the shape, size, and construction of an object depends entirely on the shape of the object. Thus far only simple objects have been presented and have required only two views to show their size and shape.

As the plan and elevation views do not always show the shape or form of the end of the object, an end view or side elevation must be shown. The simple planes of projection consist of horizontal

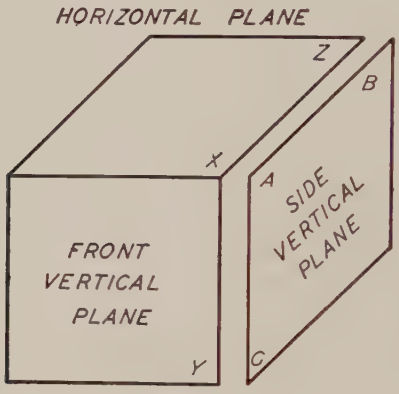


Fig. 55

and vertical planes, therefore to get the end view of an object, another plane of projection must be added to these simple planes in the form of another vertical plane, which is placed at the right-hand edges of the other two planes, as shown in Fig. 55.

Place the edges of the vertical side plane AC and AB under the horizontal plane edge XZ and behind the front vertical plane edge XY, and the plane is in the correct position. It will be seen that these planes now form the same construction as the projectograph. If an object is placed under the horizontal plane and behind the two vertical planes, we have the object placed in its projection position. If the object is seen through the vertical side elevation plane, the observer will always look at the right-hand side or end of the object. Therefore, what you see of the object from this position is what you draw in the end view or side elevation. In using the projectograph, use the side of the instrument for the third view.

In Fig. 56 is shown a rectangular block, and in Fig. 57 this block is in the projection position. The plan or top view shows the shape of the top of the block and its length and width; the elevation or front view shows the shape of the front face and its width and height; but as neither of these views show the shape as seen from the side, it is necessary to have the side vertical plane projection.

In Fig. 58 these planes have been moved into such a position that the top view is directly above the front view, and the side view is swung forward on the edge AB so that it is in line with the front

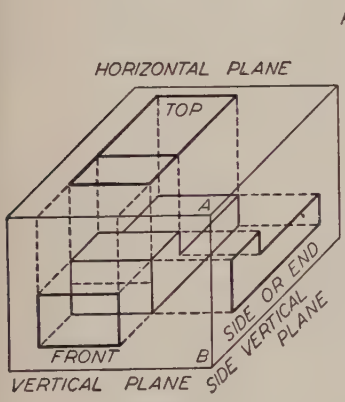


Fig. 57

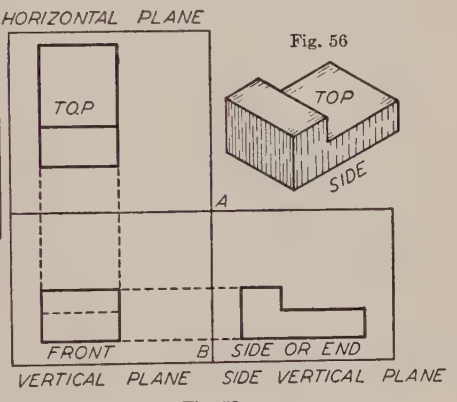


Fig. 58

Fig. 56

elevation and directly to the right of the front elevation. The planes are now in the position they would be in if they were placed on a sheet of drawing paper, the different views being unaltered in relation to each other and unchanged in shape.

Fig. 59 shows a picture of a part or detail of a desk drawer. Three views of the drawer must be shown in order to give all the necessary details in regard to its construction. The plan view, Fig. 60, gives the thickness of the piece of wood, and shows that there is some kind of construction at the two ends. The elevation gives the length and height and shows that the construction extends down from the top a certain distance. Neither of these two views shows the shape of the construction, but the end view shows that there are two dovetails to which the sides of the drawer are attached.

Projection Lines and Method of Revolving to Find Third View. As each projected point of an object shown in the plan view must be

directly over the projection of the same point in the front elevation, a thin vertical line will connect these points, as projected in pairs; and as each projected point in an end view must be directly opposite the projection of the same point in the front elevation, a thin horizontal line will connect these points, as projected in pairs. These lines are called projection or construction lines.

If these projection lines are used when making different projections or different views of objects, no difficulty will be found in keeping the points being projected in line with each other. These lines will also aid in making the views more accurate in regard to shape and dimensions. Where the top views only are concerned, the projection lines are vertical lines only, but where a third view

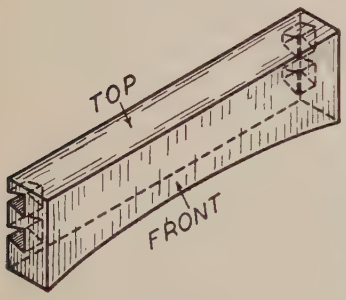


Fig. 59

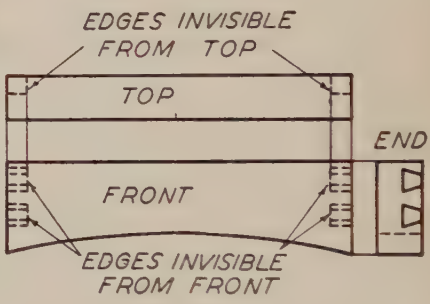


Fig. 60

must be shown, as in the case of an end view, projection lines should be used and taken from both views.

In Fig. 61, Step 1, is shown the plan and elevation view of a stand or stool. To find the end view, draw a horizontal line between the two views, Fig. 61, Step 2. Next, draw a vertical line so that it extends a little above and a little below the two projections already drawn and at any convenient distance between them, Fig. 61, Step 2. The next step is shown in Fig. 61, Step 3 The different points in the plan view are projected across with projection lines until they intersect the auxiliary vertical line. Next, revolve these points with the compass, as shown in Fig. 61, Step 3, until they intersect the auxiliary horizontal line; and then with a T square and triangle project the points vertically downward as shown. Fig. 61, Step 4, shows the different points projected across from the elevation view horizontally

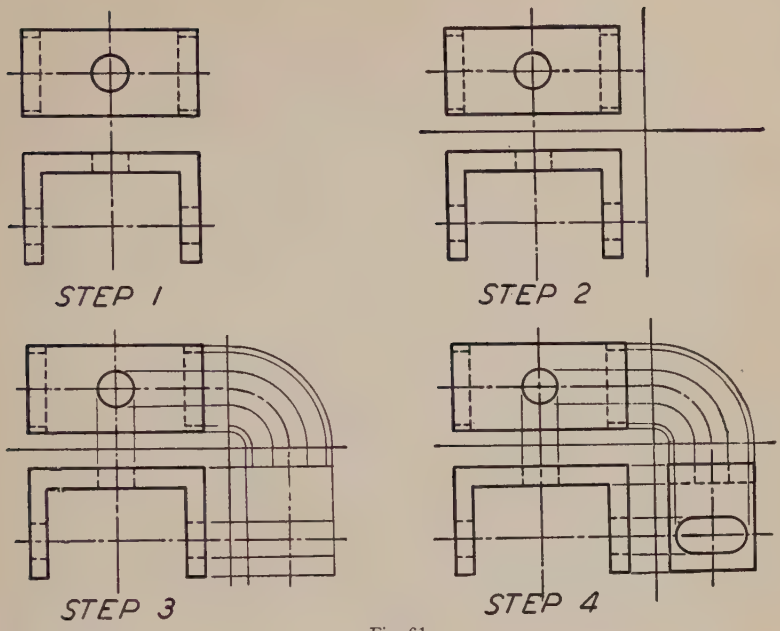


Fig. 61

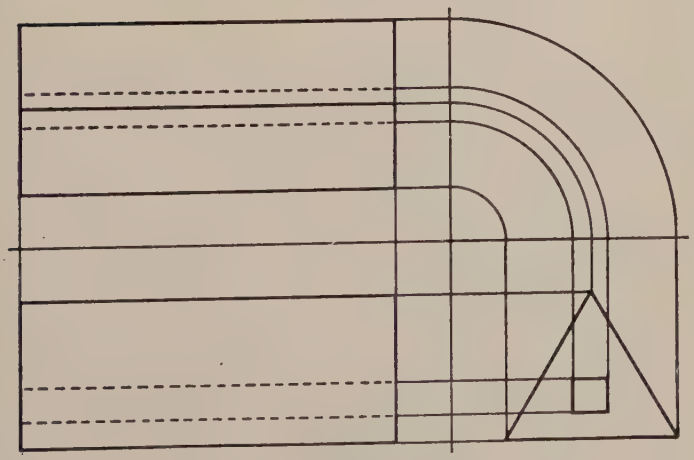


Fig. 62

until they intersect the thin vertical lines. Lines drawn between these intersections will give the shape of the end or side view.

This method can be used to find the projections of any object no matter how complicated it may be in shape or construction.

In Fig. 62 are shown the projections of a triangular block with a square hole through it. As neither the plan view nor the elevation indicates that the block is triangular, a third view must be shown to indicate the true shape of the object. The end view or side view in this case is represented by the triangle to the right of the elevation. While the dotted lines in the plan and elevation views indicate that there are some hidden edges or lines in this figure, they do not show the shape of this hidden part; but the small square in the triangle or end view shows immediately that this hidden part is square in shape, while the dotted lines in the plan view show that it is located directly below the upper edge of the triangle. The elevation shows its location in relation to the base of the triangular block and that the hole goes clear through the block.

Notice the thin projection lines. The method of projecting and revolving the different points to find the third view is exactly the same as shown by the steps in Fig. 61.

MECHANICAL DRAWING

SECTIONS AND DRAWING TO SCALE

Scale Drawing. Sometimes the object to be drawn is so large that it is impossible to make a drawing of it, its actual or full size, consequently the drawing must be reduced in size. To enable the draftsman to make the drawing so that all parts of it will be reduced in like proportion, a certain scale of reduction must be used. This scale should be large enough for the drawing to be clear and easily read by the mechanic.

When making a drawing to scale, a certain length or distance on the object is represented by a line of a certain length on the drawing. There are ten different scales found on the three-cornered Architects' Rule, and they are marked at the end of the rule as follows: 3, $1\frac{1}{2}$, $1\frac{3}{4}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{3}{16}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{32}$. When any of these scales are used, they represent 1 foot on the object. For instance, if the $1\frac{1}{2}''$ scale is used, we would say that this is the scale of $1\frac{1}{2}''=1'$, or one and one-half inches equal one foot; if the $\frac{3}{32}''$ scale is used, we would say that this is the scale of $\frac{3}{32}''=1'$, or three thirty-seconds of an inch equals one foot.

The scale usually used by the architect for residences and the average size job is $\frac{1}{4}''=1'$; and for very large construction the scale used is $\frac{1}{8}''=1'$.

The mechanical engineer may use any of the following scales $6''=1'$, or half size; $3''=1'$, or quarter size; $1\frac{1}{2}''=1'$, or one-eighth size.

Fig. 69 shows the part of the scale where $\frac{3}{8}$ of an inch equals one foot. To the right of the line marked *O*, Fig. 69, is given the scale in inches. You will notice there are 12 small divisions, each small division representing $1''$.

Fig. 70 shows the reduction of $8'4''$ on the $\frac{3}{8}''$ scale. Notice that the measurements start at the eighth division to the left of the *O* line and end at the fourth line to the right of the *O* line.

Fig. 71 is an example of a reduction of $3'9''$ on the $\frac{3}{8}''$ scale. Notice the measurements start at the third division to the left of the *O* line and end at the ninth division to the right of the *O* line.

The scale that is used by the mechanical draftsman for general work is the one marked 16. This means that the small divisions of

the rule are equal to $\frac{1}{16}$ (one sixteenth) of an inch, and of course drawings made to this scale will be full or actual size.

When using a scale such as 3″ = 1 foot, or quarter size, the actual or true dimension should not be divided by 4 and then laid off directly from the full size scale, but the actual scale measurement should be used and the reduction made directly from it.

When indicating the dimension on the drawing, the full size of the object should be given and not the reduced or scale dimension.

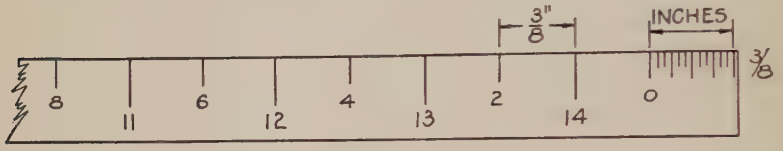


Fig. 69

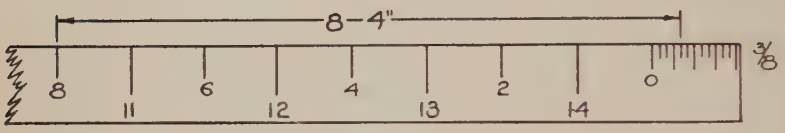


Fig. 70

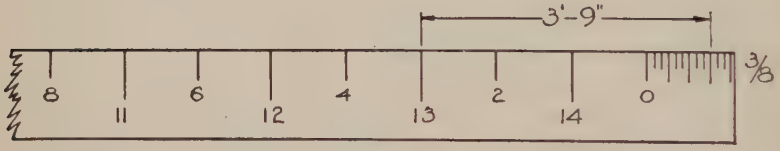


Fig. 71

For instance, if a line 12 inches long is drawn to the scale of 3″ = 1′ the dimension placed by the line will be 12″ and not 3″. The draftsman should always indicate somewhere on the drawing the scale to which the drawing was made.

It is general drafting room practice to show measurements that are two feet or under in inches, but anything over this should be shown in feet and inches.

Before drawings are allowed to go out of the drafting room they are carefully checked by a checker, as a general rule, but even under this system drawings will sometimes pass through with missing and

indistinct dimensions. These dimensions should be obtained whenever possible from the drafting room, for when a drawing is sent a considerable distance to some outside manufacturer, these dimensions cannot be obtained without some delay, which is costly. Under these conditions a drawing is sometimes scaled. This means that the rule is placed on the drawing and a direct measurement made of the drawing. This is not considered a safe or good practice, especially where a drawing is made to scale. It is not only hard to get the exact measurement but sometimes dimension figures are changed without making a new drawing and this would of course make the dimensions found by measuring the drawing incorrect.

The unwritten rule which is a good one to remember is, never measure a drawing.

Cross Sections or Cutaway Views. It sometimes happens that the construction of an object is so complicated that it is difficult to

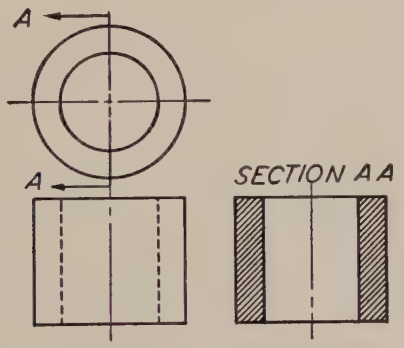


Fig. 72

show in the regular projection views. If too many dotted lines are used to show the hidden edges in the object, the views become confusing and hard to read. To show the construction clearly, the object is cut into sections as it could be in reality with a saw. A simple bushing is shown in Fig. 72 in plan and elevation views. It really would not be necessary to show a cross section of this, but it will serve as a means of illustration.

When considering the cross section of this simple bushing, it is supposed to be cut down through the center on the line *AA*. The arrows indicate the direction from which the section should be

viewed. Let the bushing stand in a vertical position, and then take the saw and cut it through downward, on the line indicated; if the portion nearest to the observer is taken away, the remaining portion would be seen from the direction indicated by the arrows and it would be like the section shown. The section itself is also indicated as section *AA*. It is seen that the saw would cut the edges of the hole only, and these are the edges which should be crosshatched. Therefore, surfaces or parts actually cut are the parts to be crosshatched.

In Fig. 73 is shown the picture sketch of a rectangular piece of metal with a piece cut out of one end, a round hole drilled through one side, and a square hole through the other. Two sections are to be taken on lines *AA* and *BB*.

The part is cut on line *AA* and the front portion removed, then the section shows the edges of the part cut at the left end. The two lines representing the square hole are at the left end; in the center is

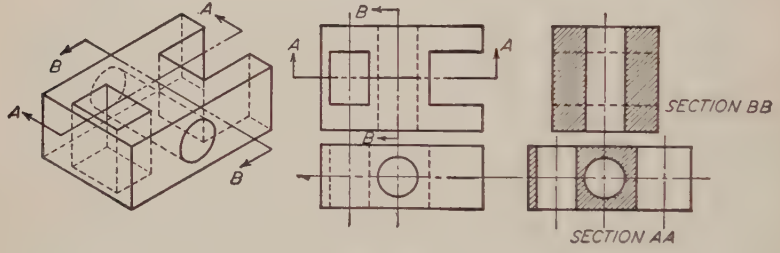


Fig. 73

the circle which is the round hole; while the plain part is the right-hand end or portion which has a piece cut out of it which would be untouched by the saw. This part untouched by the saw when cutting the object is not crosshatched.

The section *BB* is taken down through the center of the round hole and the portion left is viewed in the direction of the arrows. The remaining section will be shown as in section *BB*. The full lines show the edges of the hole. The dotted lines show the hidden edges of the portion which has a piece cut out of it.

In many instances machine parts are made of castings. If these castings were made of solid metal, they would be much too heavy and expensive. The drawing is made in the usual way and then sent to the pattern maker and he makes the pattern from the drawing.

Where castings are made in which the parts are to be light, cores are used. To show these cored parts the draftsman uses sections. Care must be taken when indicating these sections so that the pattern maker will know exactly how to make his patterns from which the castings will be made.

Fig. 74 shows a casting which is cored. If the section is taken on AA, we would see the edge of the round hole and the peculiar

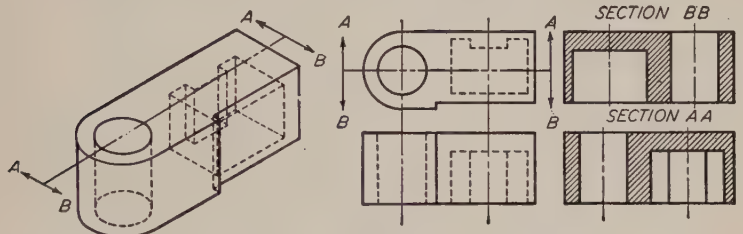


Fig. 74

shape of the cored part at the back. If the section is taken on BB and the part brought around so that it can be seen in the direction of the arrows, we will again see the edges of the round hole. The full lines indicate the rectangular part of the core, which is quite plain in construction.

Fig. 75 shows a pulley. The section is taken as indicated and we see the edge of the rim of the pulley, while the pulley arm is left

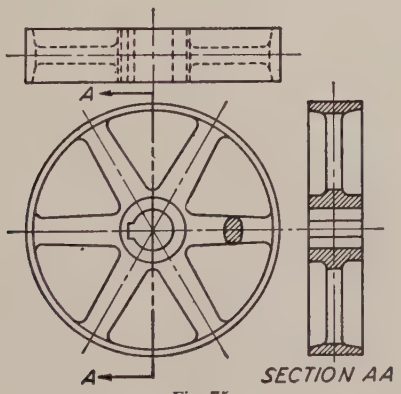


Fig. 75

full and not crosshatched because it is not cut. The hub is cut and we see the edge of the keyway and hole edges and the hole edges

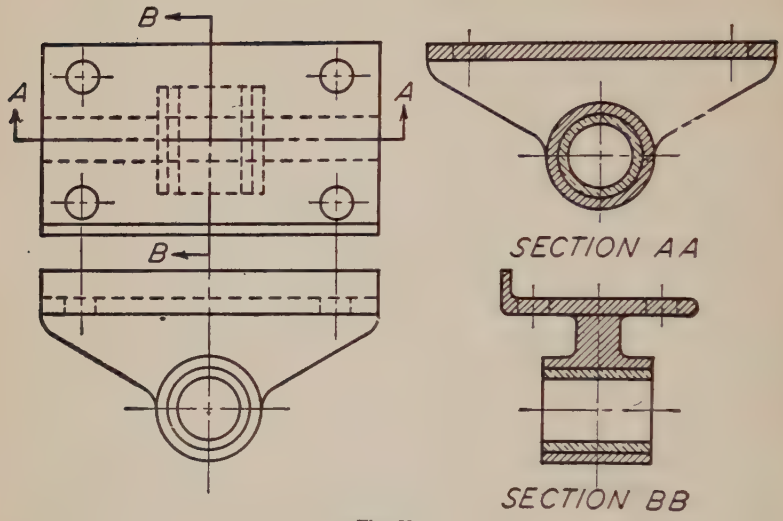

B
A
A
B
SECTION AA
SECTION BB

Fig. 76

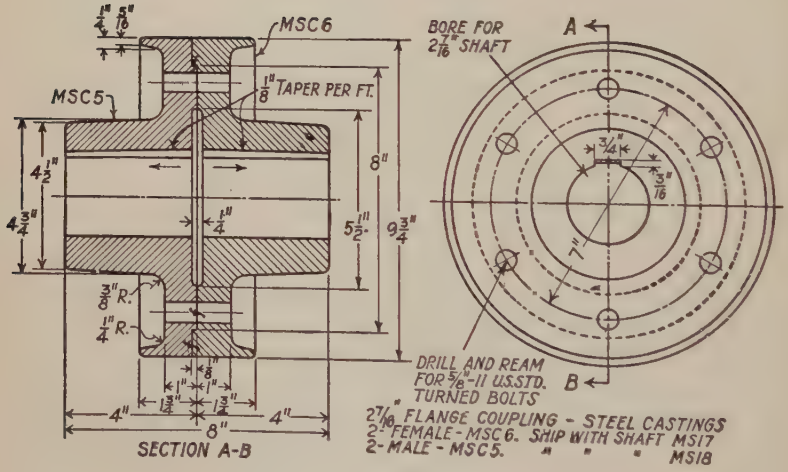

MSC6
MSC5
1" TAPER PER FT.
BORE FOR 2 7/16" SHAFT
A
B
DRILL AND REAM FOR 5/8"-11 U.S.STD. TURNED BOLTS
SECTION A-B
2 7/16" FLANGE COUPLING - STEEL CASTINGS
2-FEMALE - MSC6. SHIP WITH SHAFT MS17
2-MALE - MSC5. " " " MS18

Fig. 77

being cut would be crosshatched. To show the shape of the pulley arm a small revolved section is shown in the arm, which indicates the section that would be seen if the arm was cut.

In Fig. 76 is shown a spring bracket which has a web to give strength without a large amount of weight. A section taken on the center line *AA* would also actually cut the web as well. It is standard drafting room practice not to crosshatch the rib, to avoid confusion, when the cut is taken in the same direction as the rib is located. If, on the other hand, the section was taken on line *BB*, it would cut across the web and this section would be crosshatched.

In the circular portion of the bracket a bronze bushing or bearing is inserted, and as this is of a different metal than the bracket itself, it will, of course, have a different crosshatch convention. When two different parts of the object come together and are crosshatched, the crosshatch lines change in direction, Fig. 77.

Sections are not always taken on straight lines. Sometimes the section is taken on a zigzag line. The draftsman should always make his drawing so that it will indicate as plainly as possible the construction of the object.

Sections are not only used by the mechanical draftsman but also to a very large extent by the architectural draftsman. To show the details of a window frame construction would be very confusing if shown in straight plan and elevation views, but it can be very clearly shown in sectional views. Such exterior work as building construction around windows and such interior work as doors or partitions and interior walls can be shown very clearly by section views.

In Fig. 78 are shown a door, a basement window, and a kitchen cabinet drawn in elevation and cross section. On each elevation you will note line *AA*. This line indicates that the elevation has been cut in half along that line and one-half of it moved away so the true shape of the object can be seen. The sections, under each elevation, show an enlarged view of the objects as they actually appear along the cutting lines.

To one who is especially interested in architectural work, these drawings will be very interesting. Note how the different types of crosshatching are used to show different kinds of materials, also study how these sections are taken and shown.

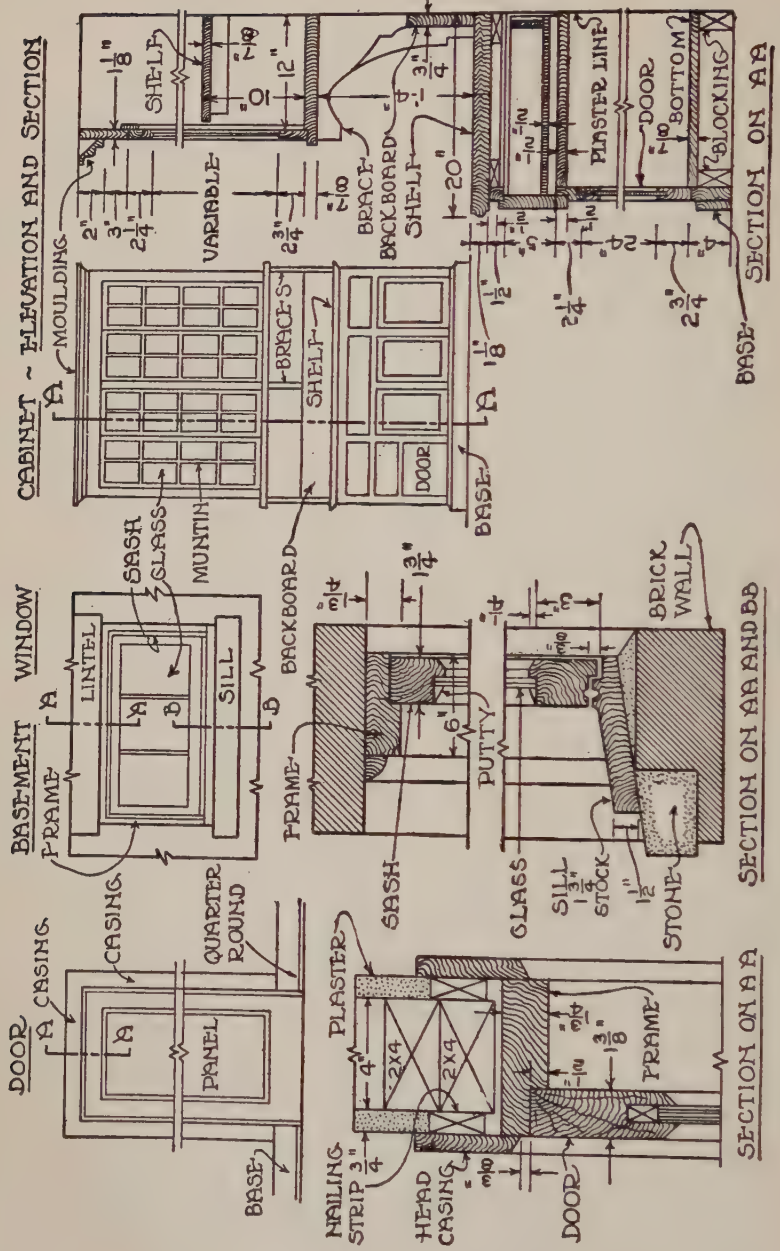

CABINET ~ ELEVATION AND SECTION
MOULDING
SHELF
VARIABLE
BRACE
BACK BOARD
SHELF
PLASTER LINE
DOOR
BOTTOM
BLOCKING
BASE
SECTION ON A A
BRACES
SHELF
DOOR
BASE
SASH
GLASS
MUNTIN
BASEMENT WINDOW
FRAME
LINTEL
SILL
BACKBOARD
FRAME
SASH
PUTTY
GLASS
SILL
STOCK
BRICK WALL
STONE
SECTION ON AA AND BB
DOOR
CASING
PANEL
BASE
CASING
QUARTER ROUND
NAILING STRIP
PLASTER
2X4
2X4
HEAD CASING
DOOR
FRAME
SECTION ON A A

Fig. 78

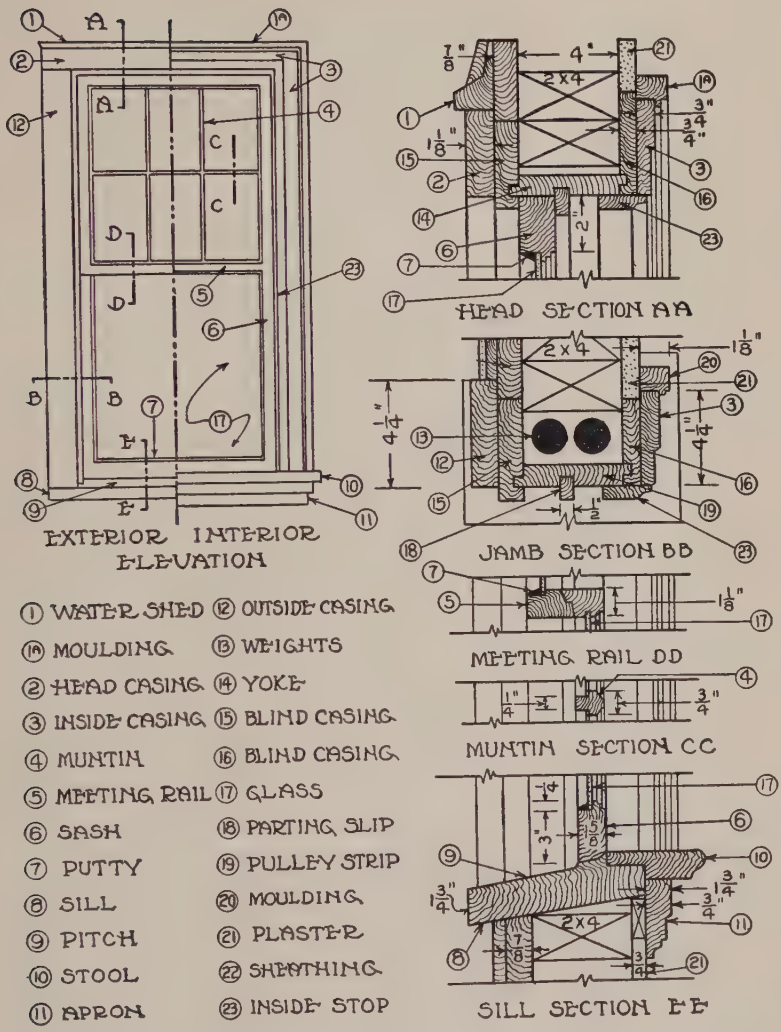

EXTERIOR ELEVATION
INTERIOR
HEAD SECTION AA
JAMB SECTION BB
MEETING RAIL DD
MUNTIN SECTION CC
SILL SECTION EE
2x4
4"
1/8"
3/4"
1 1/8"
1 1/4"
3/4"
1/2"
1/4"
3/8"
7/8"
1 3/4"
3"
5/8"
① WATER SHED
② HEAD CASING
③ INSIDE CASING
④ MUNTIN
⑤ MEETING RAIL
⑥ SASH
⑦ PUTTY
⑧ SILL
⑨ PITCH
⑩ STOOL
⑪ APRON
⑫ OUTSIDE CASING
⑬ WEIGHTS
⑭ YOKE
⑮ BLIND CASING
⑯ BLIND CASING
⑰ GLASS
⑱ PARTING SLIP
⑲ PULLEY STRIP
⑳ MOULDING
㉑ PLASTER
㉒ SHEATHING
㉓ INSIDE STOP

Fig. 79

In Fig. 79 an ordinary double hung window, such as you may have in your home, has been shown so that both the interior and exterior can be seen in elevation. Numerous parts of the window are shown in section and designated by the letters *AA*, *BB*, etc., which

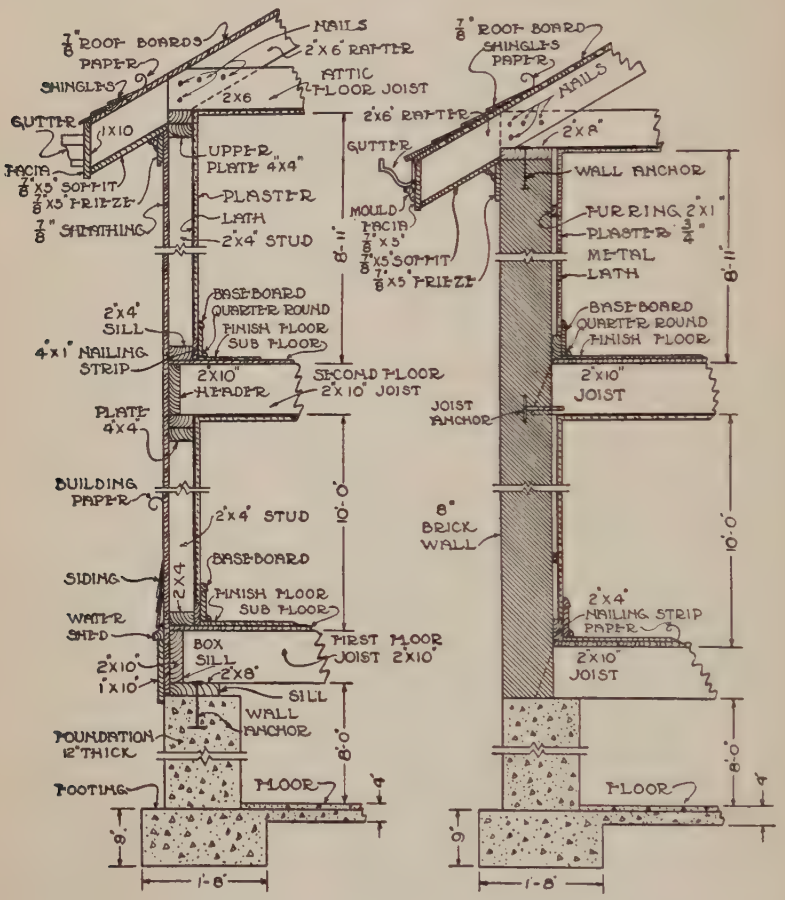


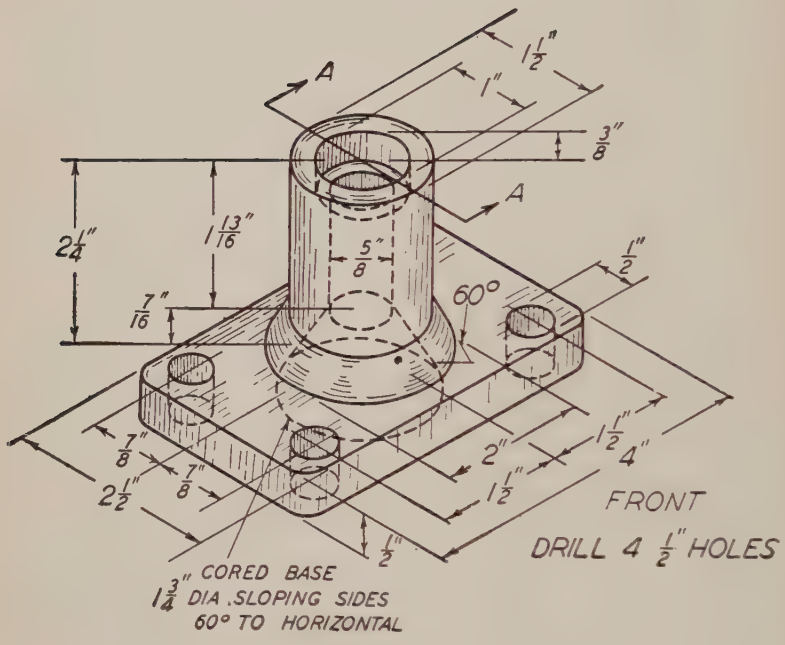
Fig. 80

correspond to the same letters on the reference lines in the elevation. The schedule gives the name of each and every part of the whole window and shows the location of same on the elevation and section.

It is very often necessary to show a vertical section of a house to show wall construction, such as attachment of beams, joists, or the

method of fastening a frame of a building to the foundation. These details could not very clearly be shown in the usual plan and elevation, so sections are used which will give all the necessary details for the bricklayer, carpenter, structural iron worker, etc.

In Fig. 80, at the left, are shown wall sections for a frame house and at the right a brick house. To better understand these sections, imagine this frame house and this brick house actually being sawed or cut in half so you could remove one-half and look at the cut surface of the remaining half. The outside walls in the remaining half would appear as shown in this drawing, giving the complete details of the construction clearly.


A
A
1 1/2"
1"
3/8"
2 1/4"
1 13/16"
5/8"
7/16"
60°
1/2"
1 1/2"
2"
4"
1 1/2"
1/2"
7/8"
7/8"
2 1/2"
FRONT
DRILL 4 1/2" HOLES
1 3/4" CORED BASE
1 3/4" DIA. SLOPING SIDES
60° TO HORIZONTAL

MECHANICAL DRAWING

WORKING DRAWINGS AND RULES OF PROJECTION

In general, there are two classes of drawings—the assembly drawing, which is a drawing showing the parts assembled into a whole; and the detail drawing, which is the one showing a drawing of each distinct part. The draftsman should remember at all times that his working drawing is to be an instruction sheet to the workman, therefore, every necessary dimension must be clearly and accurately given, both in regard to the points between which the dimension is taken and in the stating of the dimension, but should not be repeated or crowded. The drawings must be clear and easy to read.

All detail measurements of different parts should be given so that it will not be necessary for the mechanic or workman to subtract or add in order to find a missing dimension. Overall dimensions should cover detail dimensions and the total of the detail dimensions should equal the overall dimensions.

Dimensions should always be taken from center lines or from machined or finished surfaces, and plenty of space should be allowed between the different views for putting in the dimensions.

Laying Out the Drawing. Every draftsman at the start has difficulty in deciding which surface of the object should be used for the plan or elevation view, and no definite rule can be laid down as to the choice. The draftsman should try to visualize the object in its simplest position. This position might be one in which the object could stand alone, but this is not always possible. Therefore the views should be selected that will require the fewest invisible edges or lines to make the drawing clear and easy to read.

If the object is part of a machine, then the views should be placed in the same relative positions as the object itself would be on the machine or in the assembly drawing. Having decided upon the views of the object, the next step is to lay out the center or base lines from which dimensions are to be taken.

In Fig. 84, is shown a crank. It might be placed on a machine vertically, horizontally, or at an angle. In its simplest position

this object would be placed so that it would rest on the flat back. Then in the plan we would see the round or circular part from above.

To lay out this crank, start with the vertical center lines placed $5\frac{1}{4}$ inches apart, as shown in Fig. 85, Step 1. Note how they are drawn with reference to the amount of space each view will require. The reason for the position of the horizontal center line will be seen in Step 2.

The horizontal and vertical center lines are the basis for the plan dimensions. The base line, which is the full line at the bottom of Fig. 85, Step 1, is the basis for the measurements which will give the height of the crank in the front elevation.

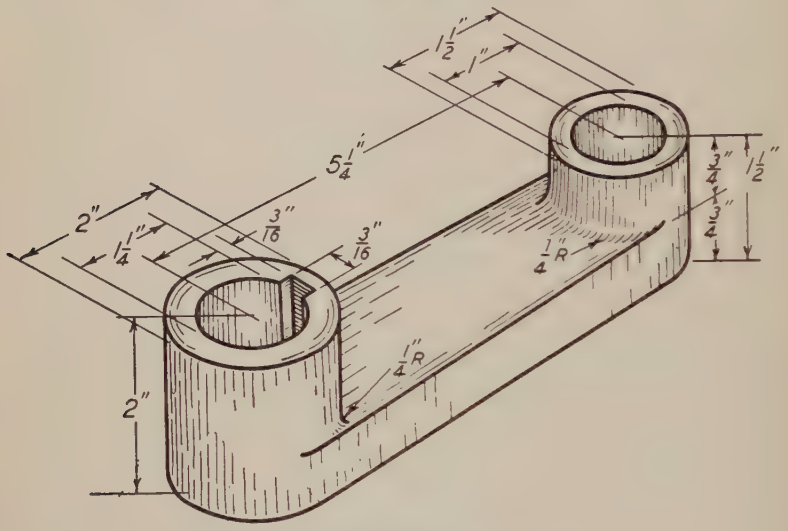
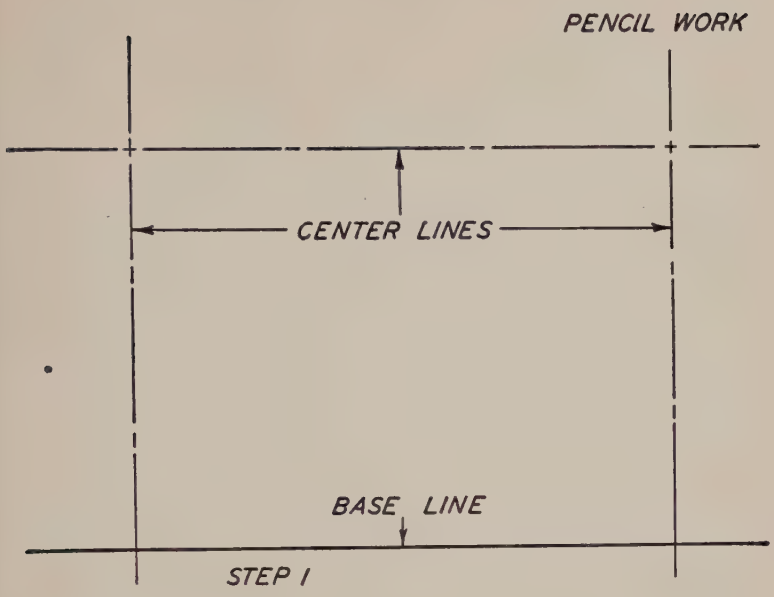


Fig. 84

Now take the plan view Fig. 85, Step 2, and draw the circles which represent the outside of the round ends of the crank and the holes in the center of these round parts. The reason for starting with the circles is because it is easier to project from circles than to draw circles between projected lines—which would have to be done in this case if the elevation view was the first view to be drawn.

Having drawn these circles, draw the lines which are shown tangent to the large circles at both ends and which represent the sides of the crank in Fig. 85, Step 2. This completes the plan or top view.



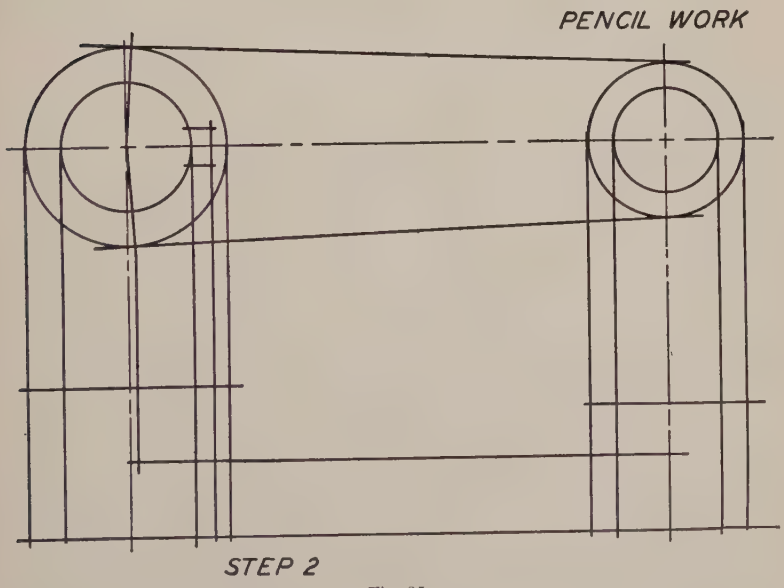


Fig. 85

Start the elevation view by measuring from the base line up the distances which will give the height of both ends of the crank and the distance for the thickness of the back part of the crank, and draw these horizontal lines as shown in Fig. 85, Step 2. Project the circles down until they intersect the lines drawn in the elevation. The rough outline for the elevation is now completed. It will be noticed that the line which gives the thickness of the back of the crank does not extend to the ends of the crank. The edge or line would gradually taper off into the rounded part of the crank. To find the approximate ending point of this edge, we must find the

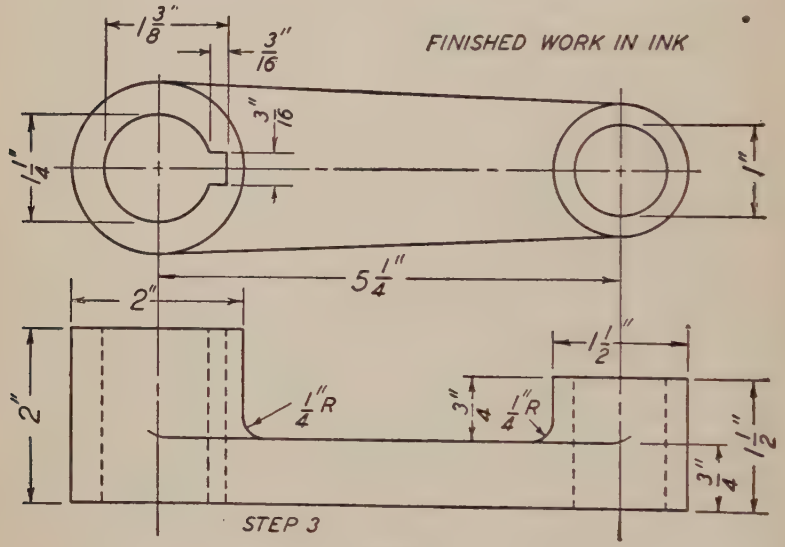


Fig. 85

approximate point of tangency as shown in the plan view, Fig. 85, Step 2. Draw a line at right angles to the tangent lines so that it passes through the center of the circle. Mark the points on the circles, then project these points downward until they intersect this edge in the elevation view. This will be the approximate end of this edge.

Fig. 85, Step 3, shows the finished drawing in ink. When inking in a tracing or drawing, be sure not to overrun the lines. The only time that lines are allowed to overlap is in architectural drawings where they are drawn so as to slightly cross each other

at the ends. Follow closely the order of inking as previously given. Note how the dimensions are placed on the dimension lines. From these different views, visualize the shape of the crank with its hidden holes and note the general neatness and appearance of the work.

To be a successful designing draftsman it is very necessary to know how to visualize objects in their different shapes. A good designer can visualize or see in his mind's eye the finished object or parts of that object. Sometimes it is not possible for the draftsman to see the object he is to draw, and oftentimes he has only a sketch or picture to work from. If he can visualize the object and its completed views before he starts to make the drawing, it will help him to place the views in the best and most logical manner.

The following sketches of different objects and their orthographic or working drawings should be carefully studied.

In Fig. 86 is shown the picture of a pillow block. This object is placed in its simplest position, standing on its base with the circular portion vertical. Now if we look down on the object, we will see the round part and the rectangular base with a hole in each corner, Fig. 87. This is the top or plan view. In the front view or elevation of Fig. 87 we see the thickness of the base and the height above the base of the circular part of the object.

In Fig. 88 is shown a camshaft bracket. This object could be placed so it would be either in a vertical or a horizontal position. As the top of the object is round, its simplest position would be vertical. The front view is the best one to start out with in this case. In the front view of Fig. 89, we see the length of the base and its thickness. We also see the width of the vertical part with the rounded top and the hole and the height of the object. In the top view of Fig. 89, we see the depth of the object as a whole with the small holes in the base.

The corresponding parts as projected in the different views should be carefully studied in regard to shape and construction so that the shape of the object can be easily and quickly visualized, for the draftsman must be able to do this in detailing a drawing where the shape, size, and construction of each part, large or small, must be drawn and shown.

Just as a mechanical draftsman must draw the plan, or top view, and the elevation, or front view, to give the workman a clear under-

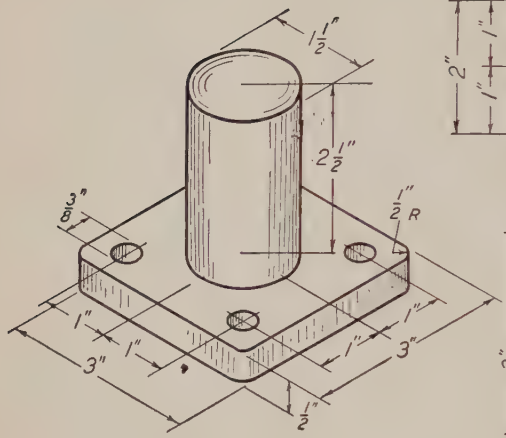


Fig. 86

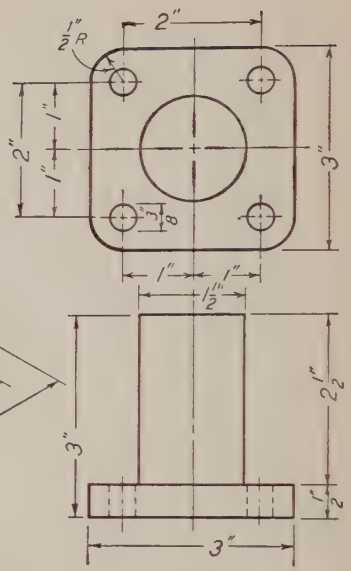


Fig. 87

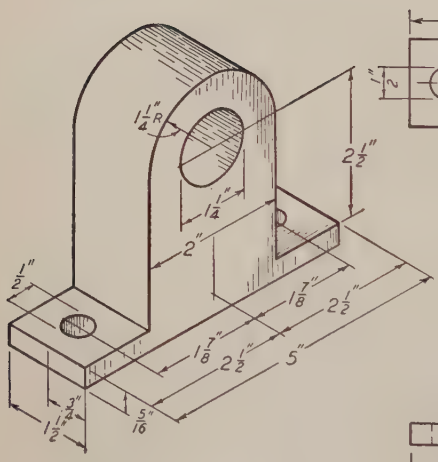


Fig. 88

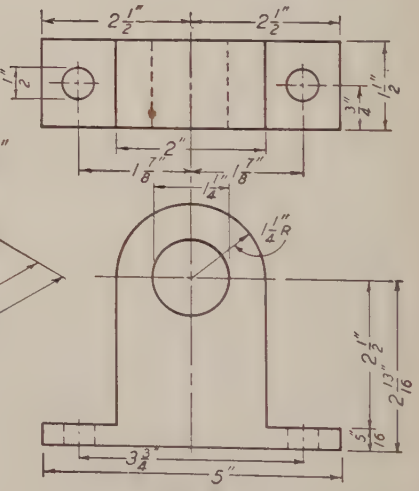


Fig. 89

standing of the shape of the object, so must the architect draw plan and elevation views to show the layout of the house, arrangements of rooms, etc., and also the shape of the different sides of the house. Dimensions must be given on these architectural drawings.

Fig. 90 shows the plan and elevation views for a small frame residence. The plan tells in detail the size and shape of walls, rooms, porches, etc. The front and side elevations show how the house will appear when completed. To the left of the plan is an isometric drawing. To understand this, imagine that some one took a huge saw and completely sawed through the upper part of the house, starting midway between the first and second floors. Then imagine the top part being removed so you could look down on the cut surface of the lower part. Compare this with the plan and the elevations.

The difference in technique in architectural drawings is seen in Fig. 90. The ends of the lines cross each other slightly, and there is a greater freedom in the style of lettering.

Drawing Arcs. It is often necessary to connect two lines with an arc or radius, and when doing so the following rule applies. *To find the center for any arc or radius, draw lines parallel to the lines which are to be joined by the arc and at a distance from them which is equal to the radius of the arc.* Fig. 91.

In Section 1 of Fig. 91 you have two lines shown at *A* which are to be joined by an arc, as shown at *B*. Let us suppose that the arc has a $\frac{1}{2}$-inch radius. Draw a line parallel to the horizontal line and $\frac{1}{2}$ inch from it, as in Section 2. Next draw a line parallel to the diagonal line, and $\frac{1}{2}$ inch from it, as shown in Section 3. Where these lines intersect is the center for the arc, as shown in Section 4. The flat dog shows the application of this drawing detail.

Sometimes it is necessary to join an arc or a circle to a straight line with an arc or radius. The steps for doing this are shown in Fig. 92.

Step 1, draw a line parallel to the straight line at a distance from it equal to the radius of the arc.

Step 2, draw an arc which is concentric to the circle and at a distance from it equal to the radius of the arc.

Where the arc and straight line intersect is the center for the arc or radius by which the lines are to be joined.

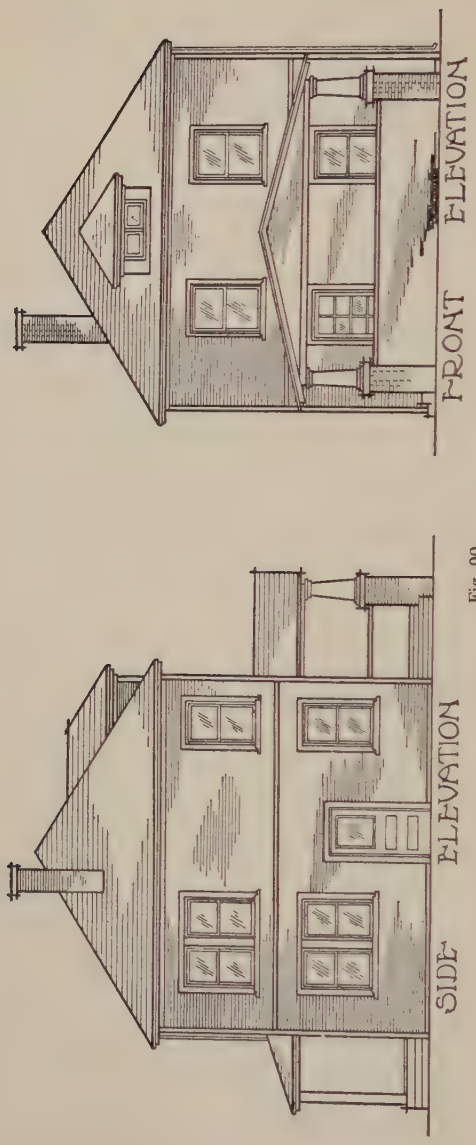


Fig. 90

26'-0"
6'-0"
12'-9"
7'-3"
8"
23"
23"
26'-0"
7'-4"
7'-7"
14'-10"
DINING ROOM
LIVING ROOM
6"
7'-4"
11'-9"
PORCH
22'-3"
11'-11"
14'-1"
KITCHEN
HALL
9'-9"
11'-2"
6'-11"
3"
10"
PLAN
7½"
5"
5'-5"
8'-1"
7'-11"
3'-4"
5'-11"
PORCH
LIVING ROOM
HALL
DINING ROOM
KITCHEN
ISOMETRIC

Fig. 90

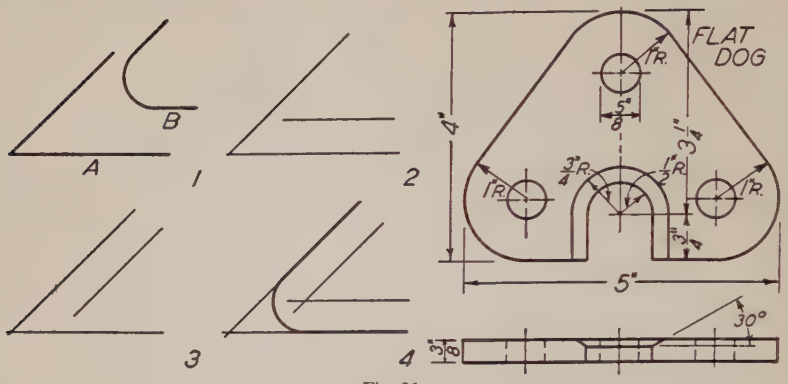

A
B
1
2
3
4
FLAT DOG
1R.
5/8"
4"
3 1/4"
3/4"R.
1/2"R.
1R.
1R.
3/4"
5"
30°
3/8"

Fig. 91

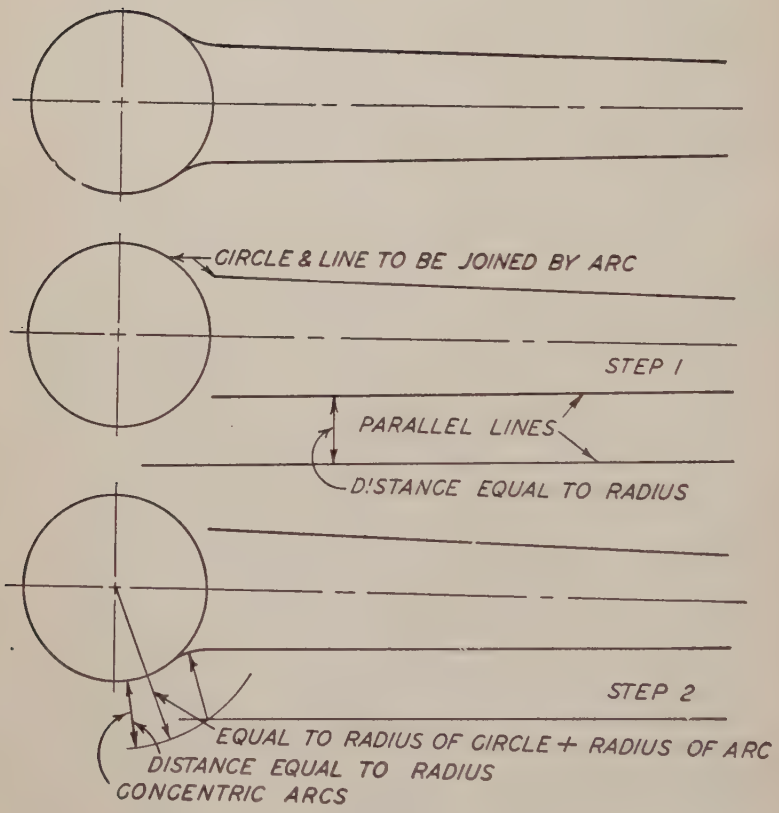

CIRCLE & LINE TO BE JOINED BY ARC
STEP 1
PARALLEL LINES
DISTANCE EQUAL TO RADIUS
STEP 2
EQUAL TO RADIUS OF CIRCLE + RADIUS OF ARC
DISTANCE EQUAL TO RADIUS
CONCENTRIC ARCS

Fig. 92

To join two circles or two arcs with an arc draw concentric arcs to the circles or arcs to be joined and at a distance from them which is equal to the radius of the joining arc, Fig. 93.

Rules of Projection. The student is strongly urged to carry out the following experiments with the projectograph step by step as given here.

In the following illustrations the letter *H* indicates the horizontal plane of projection as represented by the top of the pro-

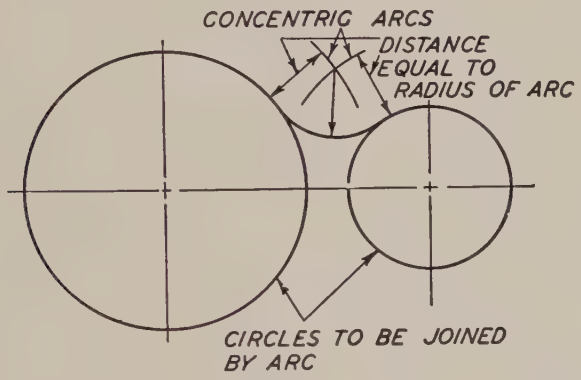


Fig. 93

jectograph. The letter *V* indicates the vertical plane of projection as represented by the front or vertical part of the projectograph.

When doing the experiments remember that a line or surface perpendicular to a plane of projection forms a right angle with that plane. Also a line or surface parallel to a plane of projection is an equal distance from that plane in its entire length.

In the illustrations the surfaces representing the planes *H* and *V* have been placed at a slight angle so that the lines and surfaces being projected can be seen clearly.

Study each step shown in the illustration carefully before doing the experiment.

Experiment 1

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 94

Let a pencil represent a line.

Place the pencil inside of the projectograph so that it is perpendicular to H and parallel to V.

Step 2—Fig. 94

Imagine that a line is drawn from the top end of the pencil up to H (horizontal plane).

The horizontal projection of the pencil or line is a point on H.

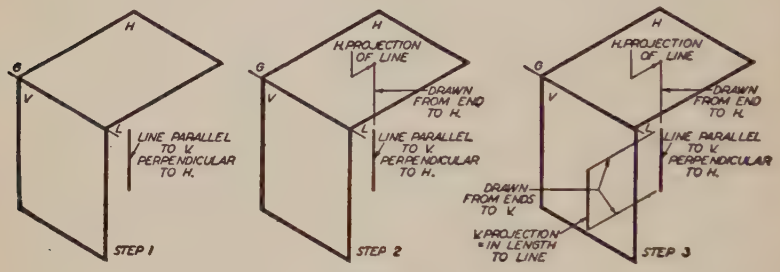


Fig. 94

Step 3—Fig. 94

Imagine that a line is drawn from each end of the pencil forward to V (vertical plane).

Draw a line between these two points on V.

The vertical projection of the pencil or line is a line on V.

Experiment 2

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 95

Let a pencil represent a line.

Place the pencil inside of the projectograph so that it is perpendicular to V and parallel to H.

Step 2—Fig. 95

Imagine that a line is drawn from the front end of the pencil forward to V (vertical plane).

The vertical projection of the pencil or line is a point on V.

Step 3—Fig. 95

Imagine that a line is drawn from each end of the pencil upward to H (horizontal plane).

Draw a line between these two points on H.

The horizontal projection of the pencil or line is a line on H.

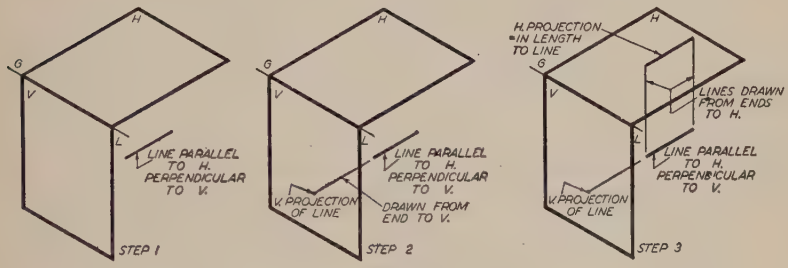


Fig. 95

If a straight line is perpendicular to the vertical plane, its projection on that plane is a point.

If a straight line is perpendicular to the horizontal plane, its projection on that plane is a point.

If a straight line is parallel to the vertical plane, its projection on that plane is a line equal in length to the line itself and makes the same angle with the ground line.

If a straight line is parallel to the horizontal plane, its projection on that plane is a line equal in length to the line itself and makes the same angle with the ground line.

When a line is perpendicular to the planes of projection, it forms a right angle with that plane.

Principle

If a straight line is perpendicular to a plane, its projection on that plane is a point; if a straight line is parallel to a plane of projection, its projection on that plane is a line equal in length to the line itself, and makes the same angle with the ground line; if a line is perpendicular to a plane of projection, one of its projections will be perpendicular to the ground line.

Experiment 3

Assemble the projectograph and place it so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 96

Let a pencil represent a line.

Place the pencil inside of the projectograph so that it is parallel to H and parallel to V.

Step 2—Fig. 96

Imagine a line drawn up to H from each end of the pencil or line. These lines will be perpendicular to H (horizontal plane.)

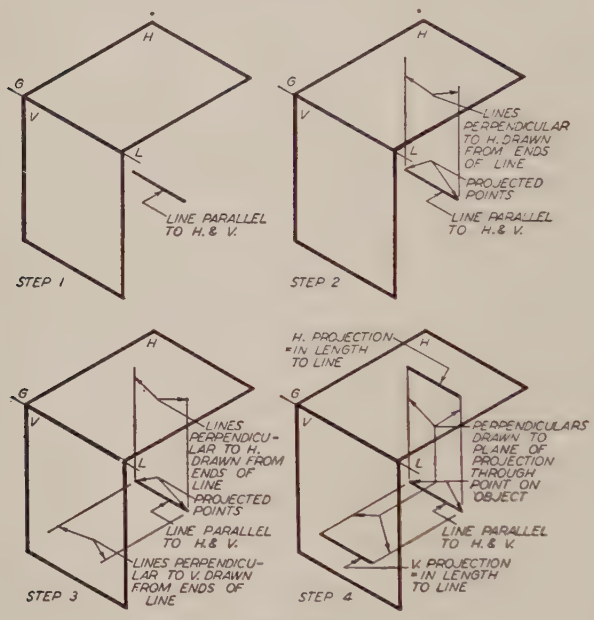

H
G
V
L
LINE PARALLEL TO H. & V.
STEP 1
LINES PERPENDICULAR TO H. DRAWN FROM ENDS OF LINE
PROJECTED POINTS
LINE PARALLEL TO H. & V.
STEP 2
LINES PERPENDICULAR TO H. DRAWN FROM ENDS OF LINE
PROJECTED POINTS
LINE PARALLEL TO H. & V.
LINES PERPENDICULAR TO V. DRAWN FROM ENDS OF LINE
STEP 3
H. PROJECTION = IN LENGTH TO LINE
PERPENDICULARS DRAWN TO PLANE OF PROJECTION THROUGH POINT ON OBJECT
LINE PARALLEL TO H. & V.
V. PROJECTION = IN LENGTH TO LINE
STEP 4

Fig. 96

Step 3—Fig. 96

Imagine a line drawn from each end of the pencil forward to V. These lines will be perpendicular to V (vertical plane).

The ends of the line or pencil are points projected to V.

The projected view of the ends of the line or pencil is where the lines drawn from the ends of the pencil enter the planes of projection.

Step 4—Fig. 96

Draw lines between these projected views of the ends of the pencil.

These lines will be the projected views of the line or pencil.

Principle

The projected view on a plane of projection of any point on an object is in a perpendicular drawn to the plane of projection through the point on the object. The projection of any line will be drawn between the points where the perpendiculars enter the planes of projection.

Experiment 4

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

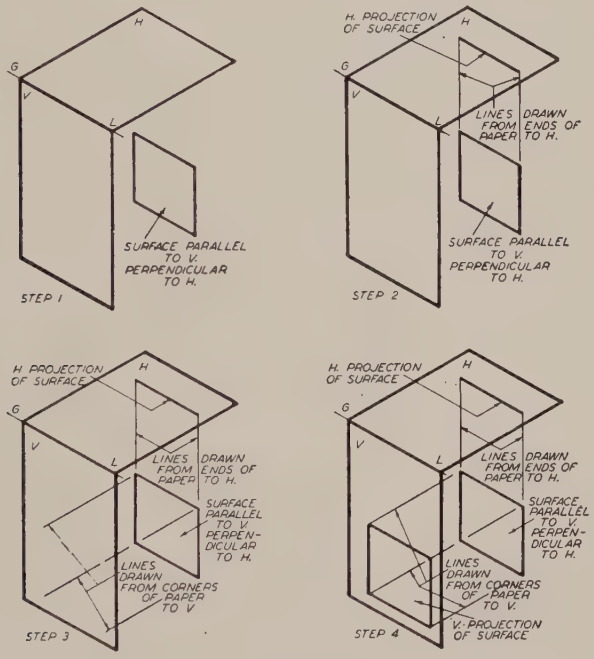


Fig. 97

Step 1—Fig. 97

Let a sheet of paper represent a surface.

Place the sheet of paper inside the projectograph so that it stands perpendicular to H and parallel to V.

Step 2—Fig. 97

Imagine lines drawn from the ends of the paper up to H (horizontal plane).

The ends of the paper are the projected points to H.

Draw a line between these projected points on H.

The straight line on H is the horizontal projection of the sheet of paper or surface.

Step 3—Fig. 97

Imagine lines drawn from the four corners of the sheet of paper forward to V (vertical plane).

The corners of the paper are the projected points to V.

Step 4—Fig. 97

Draw lines between these projected points on V.

Area enclosed in these lines is the projected view of the sheet of paper.

The projection of the sheet of paper is exactly the same size as the sheet of paper.

Principle

When a straight or flat surface is perpendicular to a plane of projection, its projection on that plane is a straight line equal in length to the surface. If a straight or flat surface is parallel to a plane of projection, its projection on that plane will be true size.

Experiment 5

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 98

Let a sheet of paper that has been slightly curved represent a curved surface.

Place the sheet of paper so that it is perpendicular to H (horizontal plane) with the two perpendicular edges parallel to V (vertical plane).

Step 2—Fig. 98

Imagine a number of points being laid off on the top of the sheet of paper and imagine lines drawn from these points up to H.

These points will be the projected points on H.

The curved line on H is the horizontal projection of the curved surface.

Step 3—Fig. 98

Imagine lines being drawn from the four corners of the paper forward to V (vertical plane).

The corners of the paper are the projected points to V.

Step 4—Fig. 98

Draw lines between these projected points on V.

The area enclosed in these lines is the projected view of the curved surface.

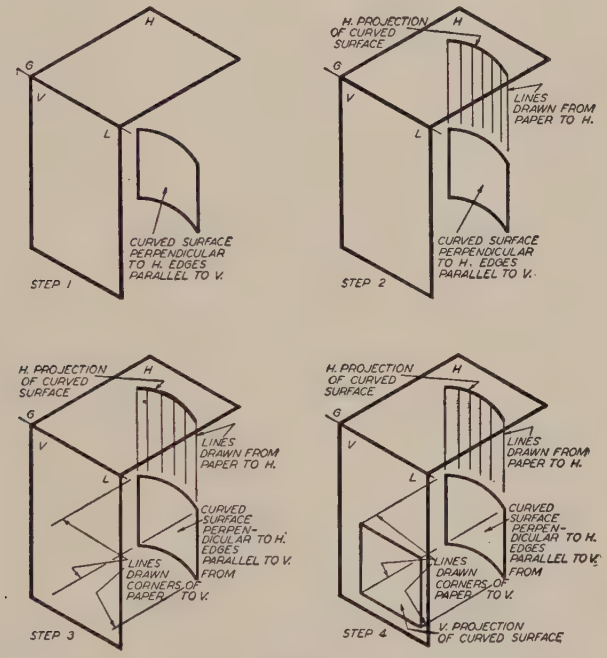


Fig. 98

Principle

If a curved surface is perpendicular to a plane of projection its projection on that plane will be a curved line equal in length to the curved surface.

Experiment 6

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 99

Let a pencil represent a line.

Place the pencil inside the projectograph so that it is not parallel to H (horizontal plane) but is parallel to V (vertical plane).

Step 2—Fig. 99

Imagine lines drawn from the ends of the pencil up to H (horizontal plane).

The ends of the pencil are the projected points to H.

Draw a line between these projected points on H.

The line on H is the horizontal projection and is shorter than the pencil itself.

The line on H is parallel to GL.

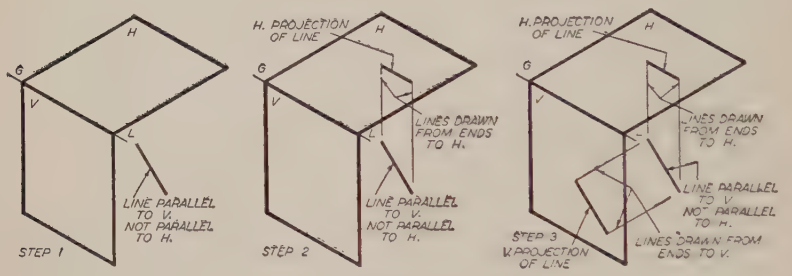


Fig. 99

Step 3—Fig. 99

Imagine lines drawn from the ends of the line or pencil forward to V (vertical plane).

The ends of the pencil are projected points to V.

Draw a line between these projected points on V.

The line on V is the vertical projection and is the same length as the line itself.

The line on V is not parallel to GL (ground line).

Principle

If a line is parallel to either the vertical or the horizontal plane of projection, its true length is shown on that plane. The other projection will be parallel to GL (ground line).

A line that is oblique to either plane of projection has its projection on that plane shorter than the line itself. The other projection will be oblique to GL (ground line).

Experiment 7

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 100

Let a pencil represent a line.

Place the pencil inside the projectograph so that it is at an angle to both H and V.

Step 2—Fig. 100

Imagine lines drawn from the ends of the pencil up to H (horizontal plane).

The ends of the pencil are the projected points to H.

Draw a line between these projected points on H.

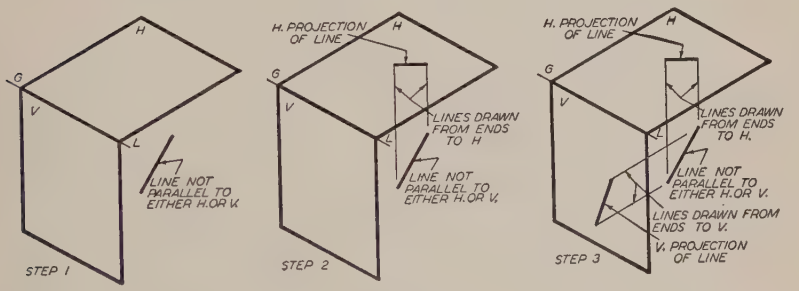


Fig. 100

The line on H is the horizontal projection and is shorter than the pencil or line itself.

The line on H is not parallel to GL (ground line).

Step 3—Fig. 100

Imagine lines drawn from the ends of the pencil forward to V (vertical plane).

The ends of the pencil are the projected points to V.

Draw a line between these projected points on V.

The line on V is the vertical projection and is shorter than the pencil or line itself.

The line on V is not parallel to GL (ground line).

Principle

Lines oblique to both planes of projection have their projections shorter than the line itself and both projections will be oblique to the ground line.

Experiment 8

Assemble the projectograph so that the front or vertical part is directly in front of you.

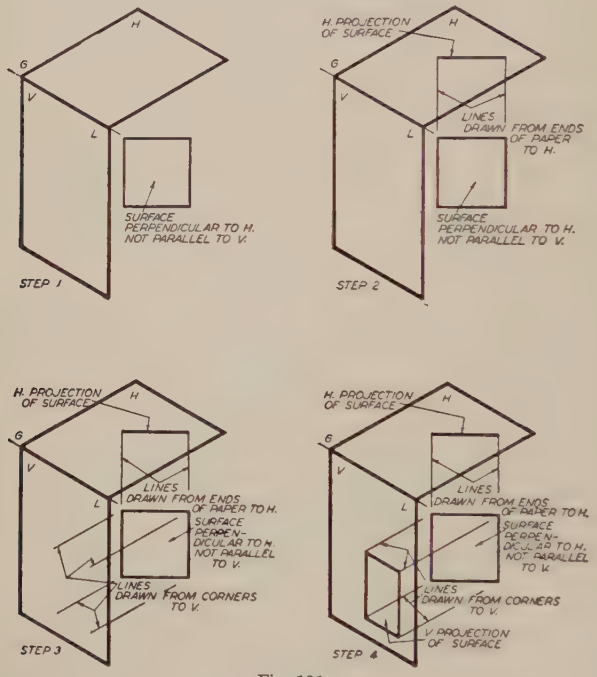


Fig. 101

Step 1—Fig. 101

Let a sheet of paper represent a surface.

Place the sheet of paper so that it is perpendicular to H (horizontal plane) and with the right-hand edge farthest away from the front of the projectograph. In this position the surface will not be parallel to V (vertical plane).

Step 2—Fig. 101

Imagine lines drawn from the top corners of the paper up to H. The corners of the paper are the projected points to H.

Step 3—Fig. 101

Draw a line between these projected points on H.

The line on H is the horizontal projection and is the same length as the sheet of paper but the projection is not parallel to GL (ground line).

Step 4—Fig. 101

Imagine lines drawn from all four corners of the sheet of paper forward to V.

The corners of the paper are the projected points to V.

Draw lines between these projected points on V.

The top and bottom lines on V are parallel to GL (ground line).

The two vertical lines on V show the paper is perpendicular to H. The area enclosed between these lines is the vertical projection of the sheet of paper.

The area is smaller than the sheet of paper.

Principle

When a plane or flat surface is oblique to a plane of projection, its projection on that plane will be smaller than the true size.

Experiment 9

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 102

Let two pencils represent two lines.

Place the two pencils inside the projectograph so that they are parallel to both H and V, one pencil to be slightly above and behind the other.

Step 2—Fig. 102

Imagine lines drawn from the ends of both pencils up to H (horizontal plane).

The ends of the pencils are the projected points to H.

Step 3—Fig. 102

Draw lines between these projected points on H.

The lines on H are the horizontal projections and are parallel to each other and parallel to GL (ground line).

Step 4—Fig. 102

Imagine lines drawn from the ends of the pencils forward to V (vertical plane).

The ends of the pencils are the projected points to V.

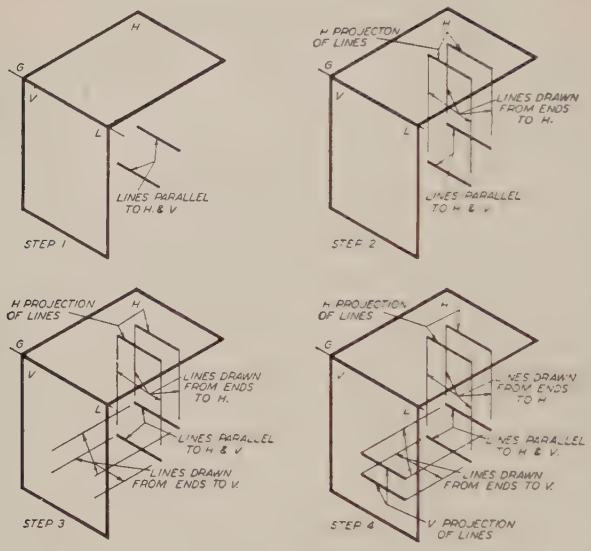


Fig. 102

Draw lines between these projected points on V.

The lines on V are vertical projections and are parallel to each other and to GL (ground line).

Principle

Lines that are parallel on an object have their vertical projections parallel and their horizontal projections parallel.

Experiment 10

Assemble the projectograph so that the front or vertical part is directly in front of you and the horizontal part is pointing away from you.

Step 1—Fig. 103

Take a small match box and let it represent a rectangular block.

Place the box so that the top is parallel to H (horizontal plane) and the front is parallel to V (vertical plane).

Step 2—Fig. 103

Imagine lines drawn from the four corners up to H.

The corners of the box are the projected points to H.

Step 3—Fig. 103

Draw four lines through these projected points on H.

The area enclosed in these lines is the horizontal projection of the box. The projections of the top front edge and the top back edge are parallel to each other and parallel to GL, therefore are true length.

The horizontal projection of the top front edge is the same dis-

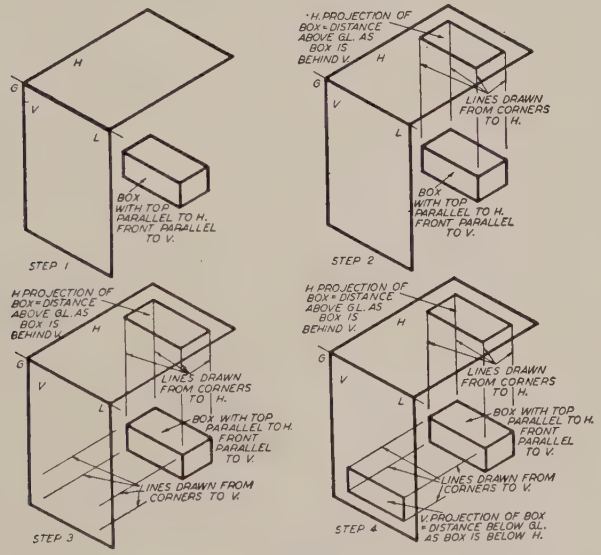


Fig. 103

tance above the ground line as the top front edge of the box itself is behind the front part of the projectograph.

The horizontal projection of the top back edge is the same distance above the ground line as the top back edge of the box itself is behind the front part of the projectograph.

Step 4—Fig. 103

Imagine lines drawn from the four corners forward to V.

The corners of the box are the projected points to V.

Draw four lines through these projected points on V.

The area enclosed in these lines is the vertical projection of the box.

The projections of the top front edge and the bottom front edge are parallel to each other and parallel to GL, therefore are true length.

The vertical projection of the top front edge is the same distance below the ground line as the top front edge of the box itself is below the horizontal plane.

The vertical projection of the bottom front edge is the same distance below the ground line as the bottom front edge of the box itself is below the horizontal plane.

Principle

Points on any object at the same height above its base must appear in the front elevation (or vertical projection) at the same distance below the ground line.

All points at the same distance back of the front face must appear in the plan (or horizontal projection) at the same distance above the ground line.

A thorough understanding of the principles of projection is very essential because many instances will be found where just the projections alone will not be sufficient to obtain a correct solution for certain drafting problems such as those met with in sheet metal and development work.

It will be seen from the foregoing experiments that the projections of a line or object and their relations to the ground line give us the actual position of the object in relation to the planes of projection.

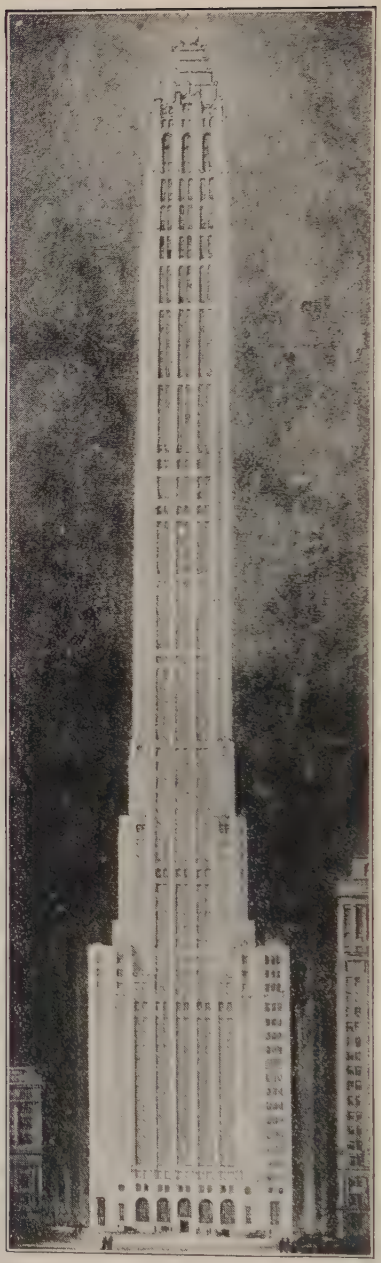
The true length of a line and the true size of a surface can only be found from its projections when its projections show that the line or object is parallel to at least one plane of projection.

A study of the horizontal projection in relation to the ground line will show the position of the object in relation to the vertical plane of projection.

If the horizontal projection is parallel to the ground line, then the line or object is parallel to the vertical plane and the true length can be taken directly from the vertical projection.

If the vertical projection is parallel to the ground line, it shows that the line or object is parallel to the horizontal plane of projection and the true length can be taken directly from the horizontal projection.

If both the horizontal and the vertical projections are not parallel to the ground line, this shows that the line or object is not parallel to either the horizontal or the vertical plane and its true length or true size cannot be taken from its projections.



ARCHITECT'S DRAWING OF THE CITY BANK—
FARMERS TRUST COMPANY BUILDING,
NEW YORK CITY

Courtesy of Underwood & Underwood

MECHANICAL DRAWING

INTERSECTIONS

One of the most interesting branches of drafting is that of sheet metal pattern drawing. The work consists of making drawings of objects which are made of sheet metal.

Such objects as stove pipes, cornices, and gutters, and such decorations as cupolas are all made from sheet metal. The drawings for these objects must be made before the tinsmith can set to work and cut out the different shapes or patterns from the metal from which the objects are formed.

As in all manufacturing, the cost of making these metal shapes must be kept down, therefore as little waste of metal as possible is desired when cutting out these shapes or patterns.

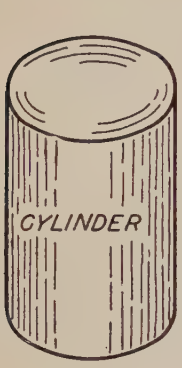
The average draftsman does not understand this phase of drawing and, in fact, very often avoids it; but if a few simple principles are remembered, the subject is not hard nor difficult to understand. In the study of this work the student should constantly keep in mind the planes of projection and their relation to each other. A thorough understanding of the rules of projection will help the student to visualize the position of surfaces and edges in the objects under discussion in this text.

In practically all sheet metal work the problem of intersection of one part with another is met with and it is this intersection of surfaces and edges which makes the knowledge of intersections necessary.

Intersecting Planes

Throughout the discussion of this subject the term cutting plane is used. This cutting plane should not be confused with the planes of projection. The cutting plane may be imagined as being a sheet of paper which is passed into or through the object and the point at which this sheet of paper or cutting plane cuts into the object is the point of intersection. The cutting plane is usually represented as a line.

Fig. 1 shows a cylinder and a sheet of paper. Now if the paper is moved over so that it cuts into the cylinder, as shown in Fig. 2, we have an intersection of the sheet of paper with the cylinder. The paper being perfectly vertical is perpendicular to the horizontal



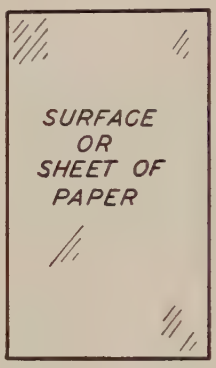


Fig. 1

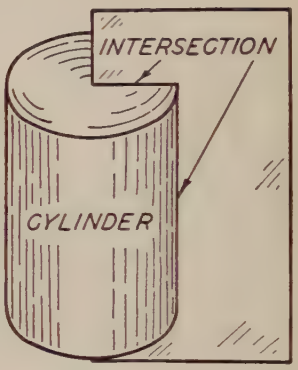


Fig. 2

plane and parallel to the vertical plane. We can therefore say that the cylinder is intersected by a cutting plane which is perpendicular to *H*—the horizontal plane—and parallel to *V*—the vertical plane.

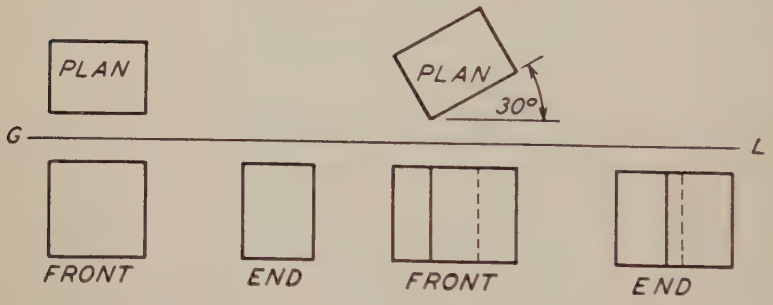


Fig. 3

Fig. 4

In sheet metal pattern drafting the student will, to a very large extent, deal with surfaces and edges which are not parallel to the planes of projection, and the ability to immediately recognize such conditions will help materially in a quick and accurate solution of the problems involved.

Lines not Parallel to the Planes of Projection

It has been shown that the ground line divides the horizontal and vertical planes. It is true that this line is not actually used in working drawings, but the thought that this line represents the horizontal and vertical planes, where projections are concerned, should be present all the time because it helps the visualization of the object, line, or part being drawn.

The position of the plan view, or horizontal projection, in relation to the ground line shows the position of the object in relation to the vertical plane.

The position of the front view, or vertical projection, in relation to the ground line shows the position of the object in relation to the horizontal plane.

When considering these two views, the student should remember that the planes of projection are at right angles to each other.

In Fig. 3 are shown the projections of a rectangular block placed in its simplest position, that is, with all surfaces or faces parallel to the planes of projection. The fact that the front edge in the plan view is parallel to the ground line shows that the object is parallel to the vertical plane.

The top edge of the front view, or elevation, is parallel to the ground line, therefore, the object is parallel to the horizontal plane.

In Fig. 4 the plan view shows that the right-hand side of the object is farthest away from the ground line, and in fact is placed at 30 degrees to the ground line. This then indicates that the object is placed at an angle to the vertical plane. The front view shows that the top of the object is still parallel to the horizontal plane because it is parallel to the ground line. This shows that the object has been rotated on an axis perpendicular to the horizontal plane and parallel to the vertical plane.

In Fig. 5 is shown a square pyramid. In the plan view the square is the horizontal projection of the base and the diagonal lines are the projections of the edges of the pyramid.

The plan view being parallel to the ground line shows that this figure has two of its base edges parallel to the vertical plane and two perpendicular to the vertical plane; while in the elevation view, the base line being parallel to the ground line shows that the base is also parallel to the horizontal plane.

In Fig. 6 the plan view shows that two edges in the base are still parallel to the vertical plane, because the horizontal lines in the projection are parallel to the ground line; but in the elevation we see the base line is so placed that the left end is nearest the ground line and the base line is at an angle of 30 degrees to the ground line. This shows that the base of the pyramid is now at an angle to the horizontal plane.

In Fig. 7 the plan view, or horizontal projection, shows that no line or edge on the base of the pyramid is parallel to the ground line, therefore, the base of the object is not parallel to the vertical plane. The front view shows that no front or back line or edge in this projection of the base is parallel to the ground line, therefore, the base is not parallel to the horizontal plane. From this it is seen that

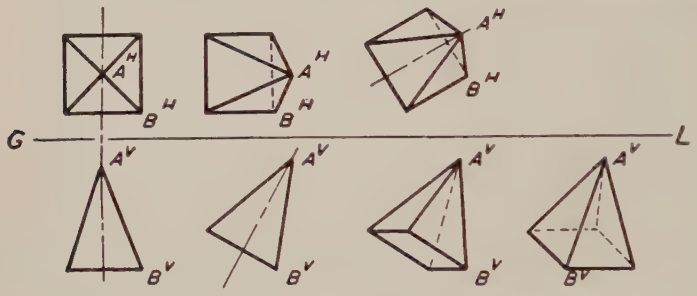


Fig. 5 Fig. 6 Fig. 7

the pyramid is at an angle to both planes of projection and is actually tilted toward the right and rotated away from the vertical plane.

Although the base of the pyramid in Fig. 5 is parallel to both planes of projection, the faces and their corners or edges are not parallel to either plane of projection. Line $A^V B^V$ is the vertical projection of the front right edge or corner and line $A^H B^H$ in the plan view is its horizontal projection. These projections are not parallel to the ground line, showing that this corner or edge is not parallel to either plane of projection; and as all four corners are alike, it indicates that no face of the pyramid is parallel to either plane of projection.

Most of the figures used in the explanation of the Principles of Projection have been straight line problems in which all the lines

in the figures have been parallel to either one of the planes of projection, that is, the vertical and horizontal planes of projection; but in sheet metal pattern drafting the projections or views as laid out on the drawing board will not give all the details or dimensions that are needed to actually lay out the pattern.

We have already learned that when projections are parallel to the ground line, the object is parallel to the planes of projection and

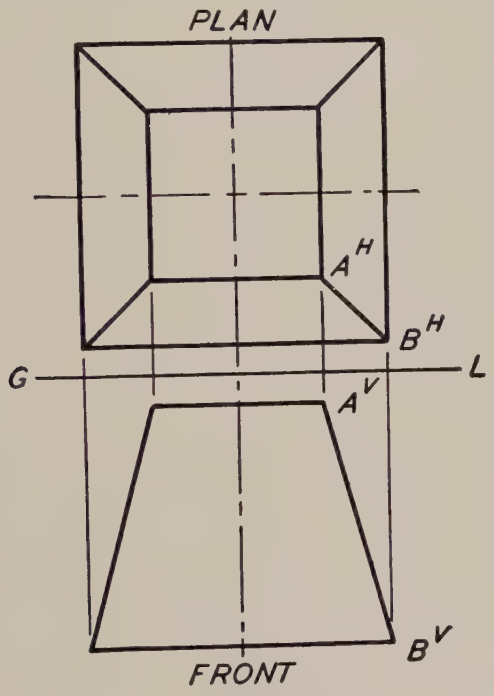


Fig. 8

the true size and shape can be found from the projections; but where projections are not parallel to the ground line, the true dimensions cannot be found directly from the projections because the object is not parallel to the planes of projection.

In Fig. 8 we have the projections of a frustum of a pyramid. The line $A^H B^H$ in the plan or top view is the horizontal projection of the front right-hand edge while line $A^V B^V$ is the vertical projection of this edge.

The horizontal projection shows that one end is farther above the ground line than the other end, showing that the edges on this object slope back or away from the observer. The vertical projection shows that the top end is nearest the ground line, therefore, the edge slopes upward as well as backward, showing immediately that no edge in this pyramid is parallel to either plane of projection. Therefore, the true length of the pyramid edges cannot be found from these projections.

It will also be seen that a line which is perpendicular to the vertical plane will also be parallel to the horizontal plane, but its horizontal projection will be at right angles or perpendicular to the ground line.

Surfaces at an Angle to Planes of Projection

If the projections of lines or edges which are placed at an angle to the planes of projection are shorter than the lines themselves,

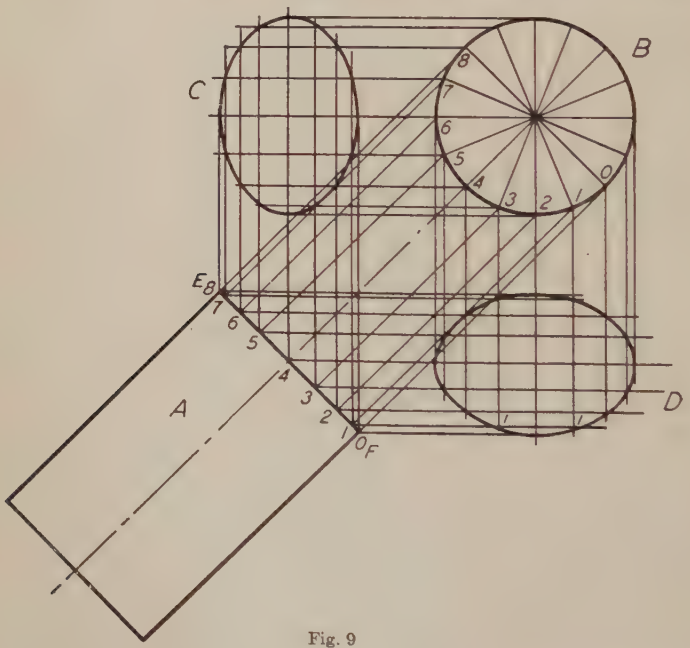


Fig. 9

then it must be true that the projections of surfaces will also be smaller and different in shape to the object when the object is placed at an angle to the planes of projection.

If a cylinder is placed so that the top face forms an angle with the horizontal or vertical planes, its projections will not be true circles but will be ellipses. A method for finding these ellipses is shown in Fig. 9.

Draw the elevation at the required angle, as shown at *A*. Then from this project the auxiliary plan view, as shown at *B*. Divide half of the plan view into any number of equal parts, as shown by numbers *1* to *8*. Project these points down until they intersect the line *EF*. From these points, project lines vertically upward to *C* and horizontally to *D*. From the points on the circle, draw lines horizontally across to *C* and downward to *D*. The curves which form the ellipses are drawn through the points of intersection of the horizontal and vertical lines, as shown in *C* and *D*.

If a vertical cylinder is intersected by a plane at an angle to the base of the cylinder, the resulting section will also be elliptical in shape.

Typical Examples of Intersections

If one surface meets another at some angle, an intersection is produced. The surfaces may be plane or curved. If both are plane, the intersection is a straight line; if one is curved, the intersection is a curve, except in a few special cases; and if both are curved, the intersection is usually curved. In the latter case, the entire curve does not always lie in the same planes. If all points of any curve lie in the same plane, it is called a **plane curve.** A plane intersecting a curved surface must always give either a plane curve or a straight line.

Planes with Pyramids. In Fig. 10 a square pyramid is cut by a plane which is parallel to the horizontal and parallel to its base. The cutting plane is also perpendicular to the vertical. The plane cuts from the pyramid a four-sided figure, the four corners of which will be points where the plane cuts the four slanting edges of the solid. The plane intersects the edge *OB* at point 4^V in the elevation. This point must be found in plan vertically above on the horizontal projection of line *OB*, that is, at point 4^H. Line *OE* is directly in front of *OB* so is shown in elevation as the same line, and the plane intersects *OE* at point 1^V in elevation and is found in plan at 1^H. Points *3* and *2* are obtained in the same way. The intersection is

shown in plan as the square $1^H2^H3^H4^H$, which is also its true size as it is parallel to the horizontal plane.

In a similar way the intersections are found in Fig. 11, which shows the horizontal and vertical projections of a plane intersecting a triangular pyramid. Note how the intersections of the cutting plane in elevation at points 1^V, 2^V, and 3^V have been projected up into the plan view at points 1^H, 2^H, and 3^H. It will be seen that in

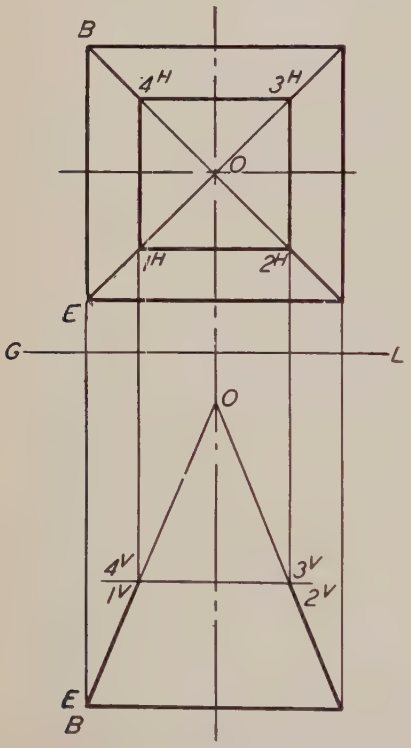


Fig. 10

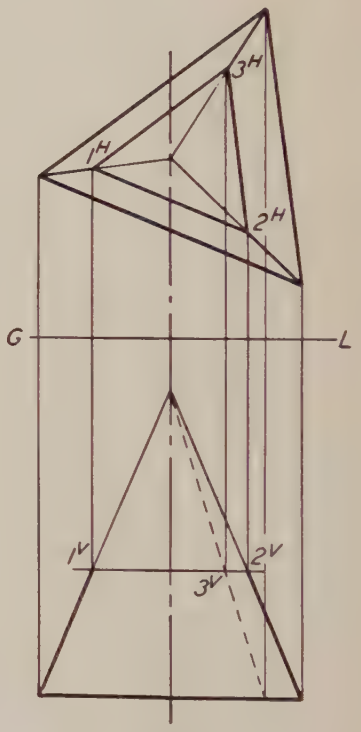


Fig. 11

these two cases where the planes are parallel to the bases, the sections are of the same shape as the bases and have their edges parallel to the edges of the base.

It is a uniform rule that when such a solid is cut by a plane parallel to its base, the section found is a figure of the same shape as the base but not always the same size. If then in Fig. 12 a right or true cone is intersected by a plane parallel to the base, the section

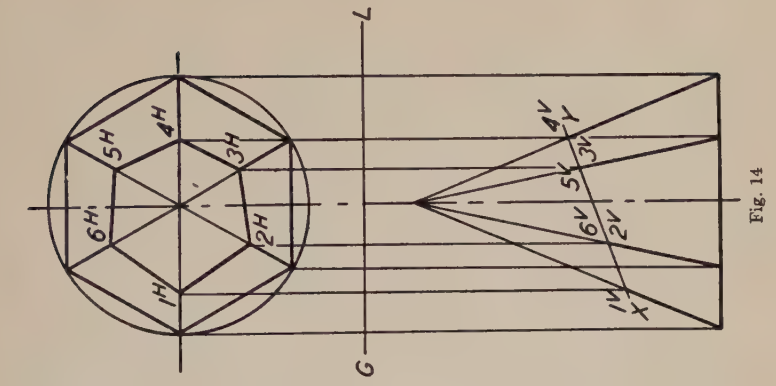


Fig. 14

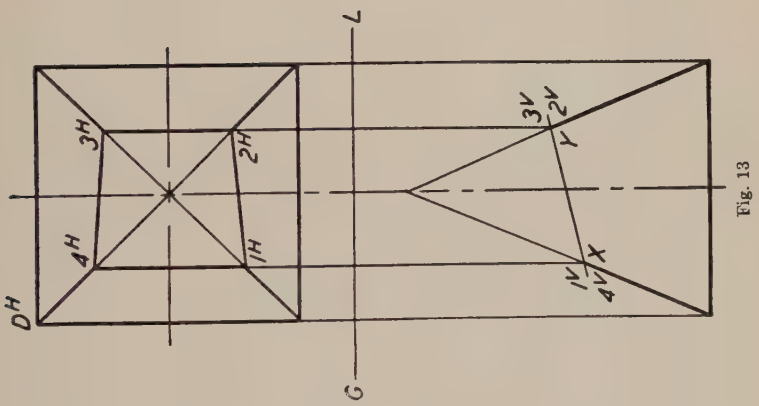


Fig. 13

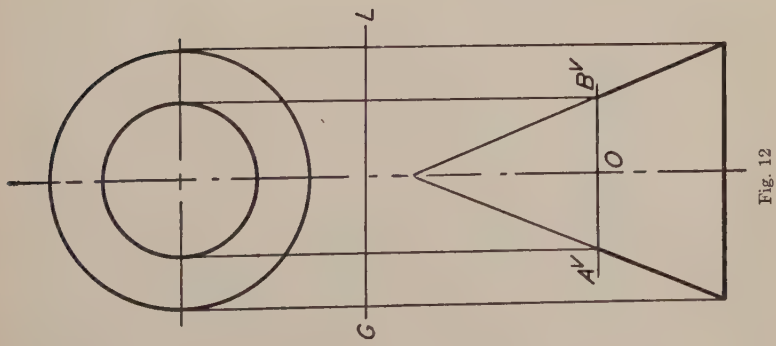


Fig. 12

must be a circle the center of which in plan coincides with the apex of the cone. The radius must equal OB as shown in elevation.

In Figs. 13 and 14 a square pyramid and a hexagonal pyramid are cut by a plane or surface $X\ Y$ which is not parallel to the base, and therefore it is not parallel to H. The surface forming the intersection is perpendicular to the vertical. We know that the surface is perpendicular to the vertical, because the line $X\ Y$ in elevation is the vertical projection of the edge of the intersecting plane. See Rules of Projection.

The horizontal projections are found by projecting the intersection in elevation up to the plan view as shown at points 1^H, 2^H, 3^H and 4^H in Fig. 13 and points 1^H, 2^H, 3^H, 4^H, 5^H, and 6^H in Fig. 14. Owing to the intersecting plane being at an angle to the base, the resulting section will not be similar in shape to the base. This holds good for all figures that are cut or intersected by a plane or surface which is not parallel to the base of the object.

Example 1. Find the horizontal projection of a cross section of the pyramid shown in Fig. 15 made by a plane perpendicular to the vertical plane but inclined at an angle to the horizontal plane of projection, and let all edges of the base be at an angle with the vertical plane as represented by the line $M\ N$.

Having drawn the vertical line SS' as the center line of the figures, its point of intersection with the line $M\ N$ is the center of the plan. Since not one of the sides of the base is to be parallel with $M\ N$, draw a diameter $A'D'$, making the required angle with $M\ N$; and from the points A' and D', proceed to set out the angular points of the hexagon $A'B'C'D'E'F'$, as in Fig. 15. Then join the angular points which are diametrically opposite and project the figure thus obtained upon the vertical plane, as shown by points A, F, B, E, C, and D.

Now if the cutting plane be represented by the line $X\ Y$ in the elevation, it is obvious that it will expose, as the section of the pyramid, a polygon whose angular points are the intersections of the various edges with the cutting plane and will be projected in perpendiculars drawn from the points where the plane meets these edges, respectively. (All points projected must be in line.) From the points 1, 2, 3, etc., draw the perpendiculars $1'$—$1'$, $2'$—$2'$, $3'$—$3'$, etc., to meet the lines $A'D'$, $F'C'$, and $B'E'$. When the adjoining

points of intersection of these lines are joined, a six-sided figure will be formed which will represent the section required. The edges *FS* and *ES* are concealed in the elevation but are necessary for the construction of the plan and have been expressed in dotted lines. The portion of the pyramid situated above the cutting plane, which

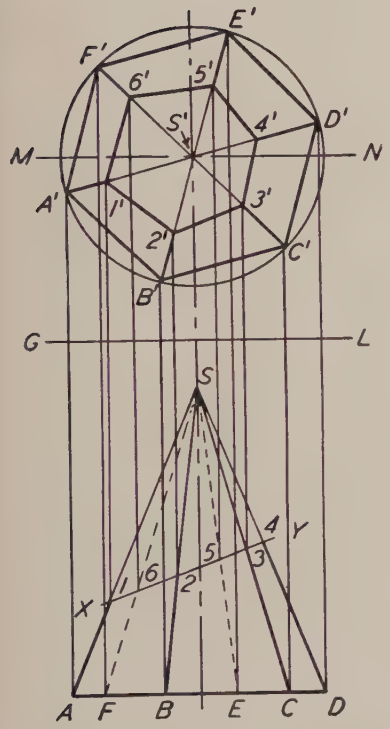


Fig. 15

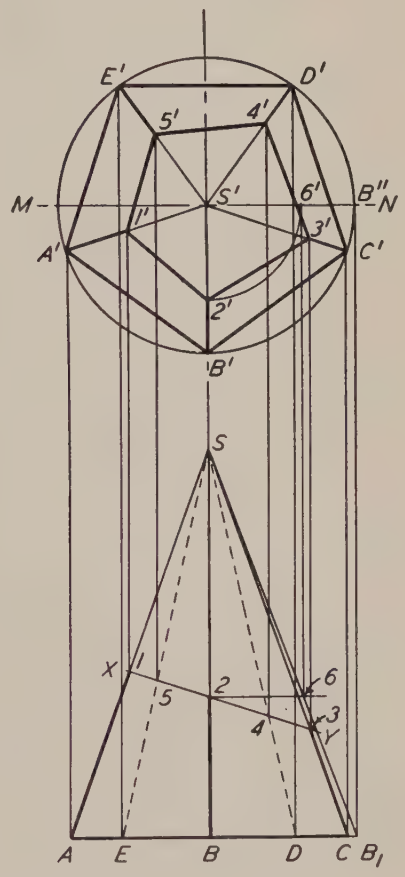


Fig. 16

is shown in light lines, though supposed to be removed, is necessary in order to draw the lines representing the edges in the plan view.

Example 2. Find the horizontal projection of the cross section of a regular five-sided pyramid, cut by a plane perpendicular to the vertical but inclined at an angle to the horizontal plane of projection;

and let one edge, BS, of the pyramid be in an imaginary plane which is perpendicular to both the horizontal and the vertical planes of projection, as shown in Fig. 16.

The plan of the pyramid is constructed by describing from the center S' a circle circumscribing the base and from B' dividing the circumference into five equal parts and joining the adjoining points of division by straight lines. These form the polygon $A'B'C'D'E'$, whose angles when joined to the center S' show the projections of the edges of the pyramid. Then following the method just explained, the elevation and the horizontal projection of the section made by the plane XY are obtained. But this method will not suffice for the determination of the point $2'$ because the perpendicular drawn from the corresponding point 2 in the elevation coincides with the projection of the edge BS.

Let the pyramid supposedly be turned a quarter of a revolution around its axis, then the line $B'S'$ will assume the position $S'B''$. Project the point B'' in plan to B_1 in elevation and join SB_1. Then since the required point must also have described a quarter of a circle in a plane parallel to the horizontal plane and its new position must be in the lines SB_1, it is obvious that its vertical projection is the point 6, the intersection of a horizontal line drawn through 2 with the line SB_1. The distance from 2 to 6 may then be used to determine the distance from S' to $2'$, which determines the position of point $2'$ in the plan; or, following a more methodical process, by projecting the point 6 to $6'$ and describing a circle from the center S' passing through $6'$, its intersection with $B'S'$ is the point sought.

Planes with Cones or Cylinders. Sections cut by a plane from a cone have been defined as **conic sections.** These sections may be any of the following: two straight lines, a circle, an ellipse, a parabola, or a hyperbola. All except the parabola and hyperbola may be cut from a cylinder.

Methods have been given for constructing the ellipse, parabola, and hyperbola without projections; it will now be shown that they may be obtained as actual intersections.

In Fig. 17 the plane cuts the cone obliquely. To find points on the curve in plan, take a series of horizontal planes as represented by line XYZ, etc., between points C and D. One of these planes W should be taken through the center of line CD. The points C and D

must be points on the curve, since the plane cuts the two contour elements at these points. Contour elements are those forming the

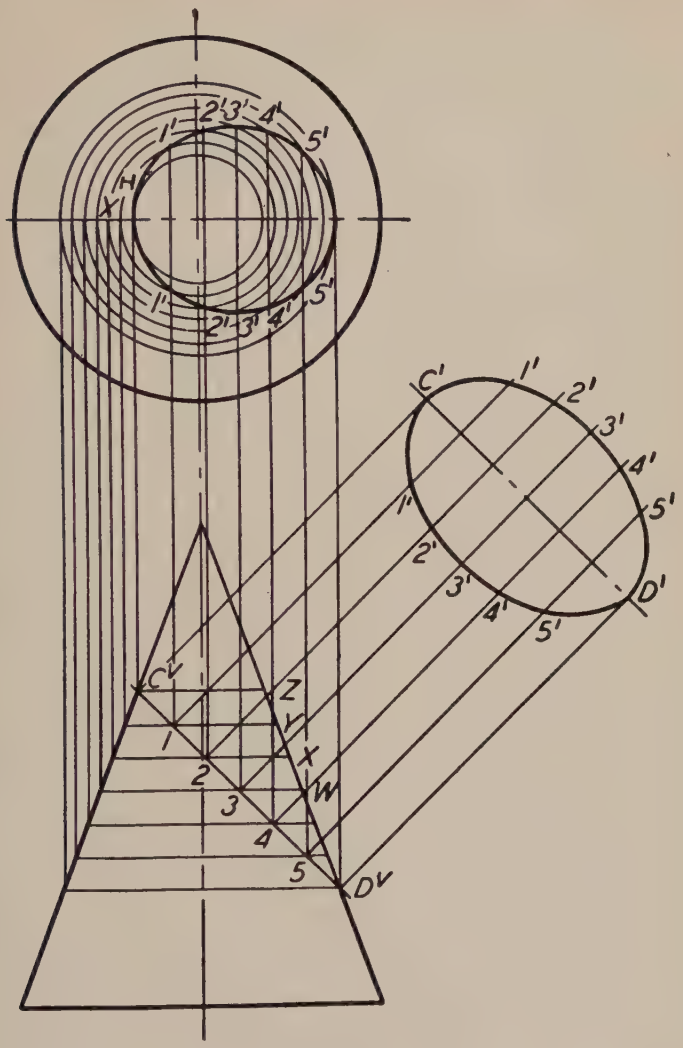


Fig. 17

outline of the cone or figure. The horizontal projections of the contour elements will be found in a horizontal line passing through the center of the base; hence, the horizontal projection of C and D

will be found on this center line and will be the extreme ends of the curve.

The plane X cuts the surface of the cone in a circle, as it is parallel to the base and the diameter of the circle is the distance between the points where X crosses the two contour elements. This circle X^H on the plan has its center at the horizontal projection of the apex. The circle X and the curve cut by the plane are both on the surface of the cone, and their vertical projections intersect at the point *2*. Point *2* on the elevation then represents two points *2'*, *2'* in plan on the circle X^H and are points on the required intersection. Planes Y and Z, and as many more as may be necessary to determine the curve accurately, are used in the same way. The curve found is an **ellipse.** It is readily seen that the true size of this ellipse is not shown in the plan, for the cutting plane forming the curve or intersection is not parallel to the horizontal plane.

In order to find the actual size of the ellipse, it is necessary to place its plane in a position parallel either to the vertical or to the horizontal. The actual length of the long diameter $C^V D^V$ of the ellipse must be shown in elevation, because the line is parallel to the vertical plane. The plane of the ellipse then may be revolved about $C^V D^V$ as an axis until it becomes parallel to V, when its true size will be shown. For the sake of clearness of construction, $C^V D^V$ is imagined moved over to the position $C'D'$, parallel to $C^V D^V$. The lines *1'—1'*, *2'—2'*, *3'—3'*, etc., on the plan show the true width of the ellipse, as these lines are parallel to H, but are projected closer together than their actual distances. In elevation these lines are shown as the points *1*, *2*, *3*, etc., at their true distance apart. Hence, if the ellipse is revolved around its axis $C^V D^V$ and the distances *1'—1'*, *2'—2'*, *3'—3'*, etc., as taken from the plan view, are laid off on lines perpendicular to $C^V D^V$ drawn from points *1*, *2*, *3*, *4* on the cutting plane in elevation, the true shape and size of the ellipse will be shown by the curve drawn through the points *1'—1'*, *2'—2'*, *3'—3'*, etc., on the lines perpendicular to $C^V D^V$.

In Fig. 18 a plane cuts a cylinder obliquely. This is a simple case, as the horizontal projection of the curve coincides with the base of the cylinder. To obtain the true size of the section, which is an ellipse, any number of points are assumed on the plan and projected down on the cutting plane at *1*, *2*, *3*, etc. The lines drawn

through these points perpendicular to line *1—7* are made equal in length to the corresponding distances *2′—2′*, *3′—3′*, etc., on the plan, because *2′—2′* is the true width of the curve at *2*.

When finding the true shape of the intersection of an oblique plane and a cylinder, it is very often more convenient to project the true shape to one side of the elevation in a manner similar to that shown for the ellipse in Fig. 17. The steps shown in Fig. 19 are a simple application of the principle taught in Fig. 17.

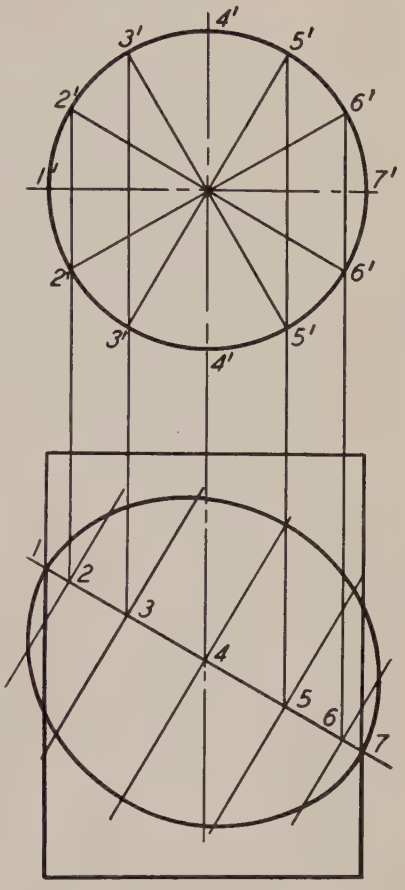


Fig. 18

Step 1, Fig. 19. After completing the plan and elevation views, divide the plan view into equal parts and project these points down until they intersect the cutting plane in the elevation.

Step 2, Fig. 19. Project lines at right angles to the cutting plane as shown and draw center line *AB* parallel to the cutting plane and at any convenient distance from it.

Step 3, Fig. 19. Take distances *1—1*, *2—2*, *3—3*, etc., from the plan and lay these off equally on each side of the center line *AB*, as shown at *1′—1′*, *2′—2′*, *3′—3′*, etc., and draw the curve through the points found. The lines *1—1*, *2—2*, *3—3*, etc., on the plan view are true length because they are parallel to *H*.

If a cone is intersected by a plane which is parallel to only one of the elements, as in Fig. 20, the resulting curve is a **parabola.** While the parabola is different in shape to the ellipse, the method of finding its true shape is exactly the same as that used in finding the ellipse.

After completing the plan and elevation views and locating the cutting plane, it is necessary to find first the horizontal projection of the cutting plane. To do this, divide the cutting plane on the elevation into any number of equal parts and draw horizontal lines through the divisions *A*, *B*, *C*, *D*, etc., until they intersect the edges of the cone and then project these intersections up into the plan view, as shown in Step 1, Fig. 20.

Step 2, Fig. 20. Construct the circles in the plan view. The center of the circles are found at the center of the plan view. Project the intersections on the cutting plane up into the plan view until they intersect the circles.

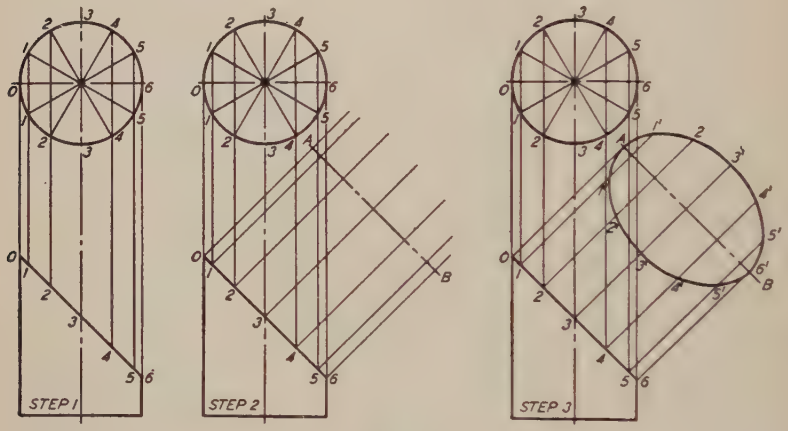


Fig. 19

Step 3, Fig. 20. Draw the curve in the plan view and the horizontal projection of the cutting plane is complete. The lines drawn between the points on the curve in the plan view are parallel to the horizontal plane and are therefore true length. Locate the center line for the true shape of the intersection at any convenient distance from the cutting plane, but parallel to it, and also project lines from the intersections on the cutting plane and perpendicular to it.

Step 4, Fig. 20. Take the true distance from the plan view on lines *1—1*, *2—2*, *3—3*, etc., and lay them off equally each side of the center line *X Y* on lines *1′—1′*, *2′—2′*, *3′—3′*, etc., and draw the curve through these points.

If the intersecting plane is parallel to more than one element

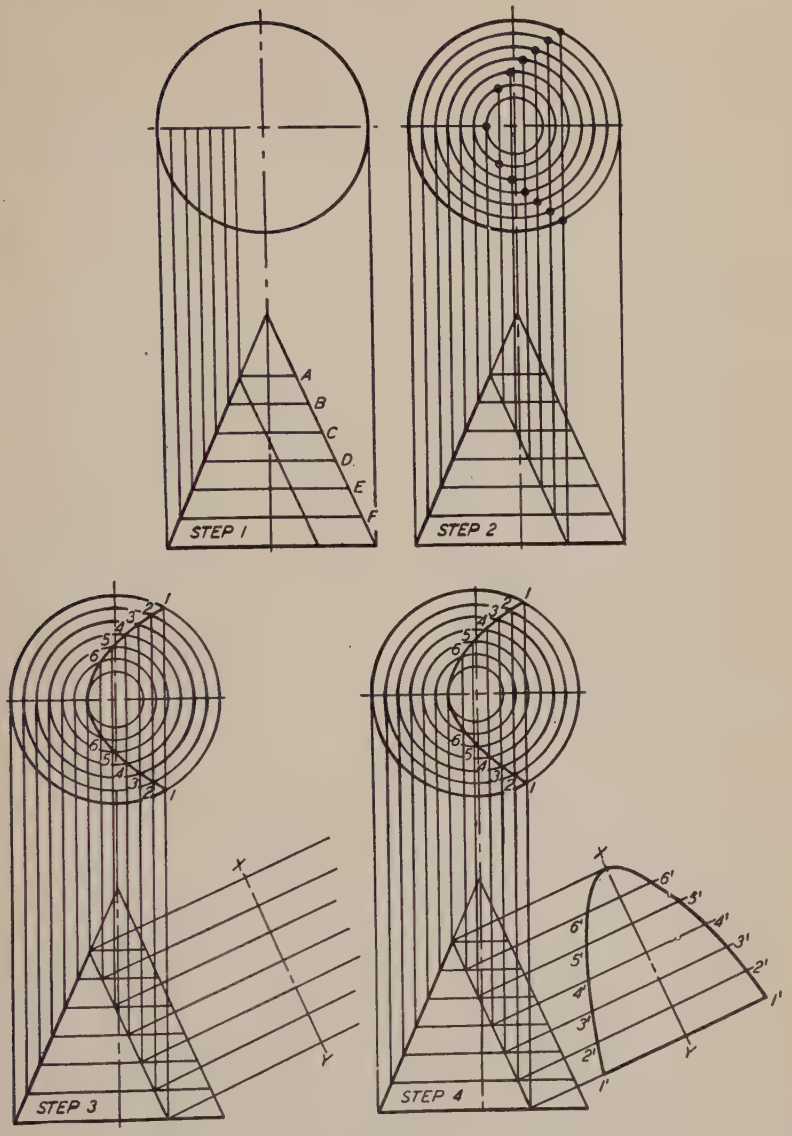


Fig. 20

or is parallel to the axis of the cone, a **hyperbola** is produced. In Fig. 21, the plane A is parallel to the vertical plane. Because the cutting plane is parallel to the vertical plane, it must be first located in the top or plan view and then projected down to the elevation view.

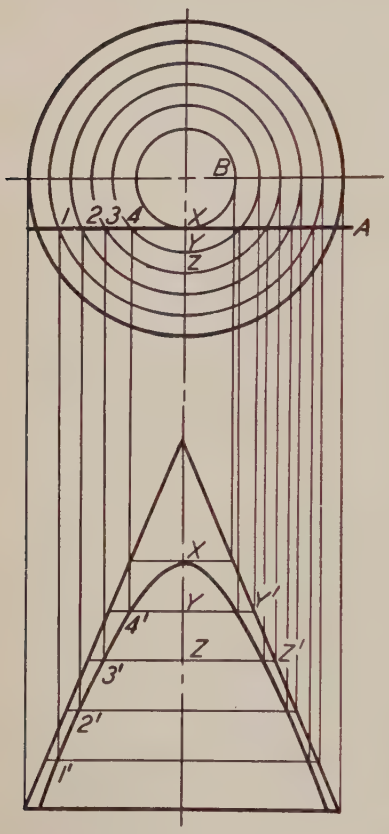


Fig. 21

In this instance the curve found will appear in its true size, because plane A is parallel to the vertical. Observe that the highest point on the curve is found by drawing the circle X on the plan tangent to the given plane. One of the points B where this circle crosses the diameter is projected down to the contour element of the cone and the horizontal plane X drawn. This gives the height of the intersection on the front view or elevation.

Intermediate planes, as Y, Z, etc., are chosen in the elevation and corresponding circles drawn in plan. To do this, project the intersecting points on the contour elements Y', Z', etc., up to the plan view until they intersect the diameter, and then with a center at the center of the plan view, draw circles through the intersections on the diameter line.

The points 1, 2, 3, etc., where these circles are crossed by the plane A are also points on the curve. These points are projected down until they intersect the corresponding planes in the elevation, as at $1'$, $2'$, $3'$, etc., and the curve is drawn through these intersections.

It will be seen that the true lengths which give the distance between the points on the curve, as found on lines Y, Z, etc., can be

found on the horizontal projection of the cutting plane or line *A* in the plan view by measuring from the center line to the points of intersection *1*, *2*, *3*, etc., of the circles and the cutting plane.

Thus it is seen that to find the true shape of an intersection, the distances or measurements must be taken from projections which are parallel to one of the planes of projection.

MECHANICAL DRAWING

DEVELOPMENTS

In the lesson on intersections it was shown that when the shape and position of an object is such that its surfaces or edges are not parallel to the planes of projection, the projections of that object do not show its true size or shape. Therefore, there must be some way to find the true size and shape of an object before a correct pattern of it can be drawn.

True Length of Lines

In order to find the true size and shape of an object, revolve the projections of the object until they are parallel to the different plans of projection. This is termed **finding the true length of a line.**

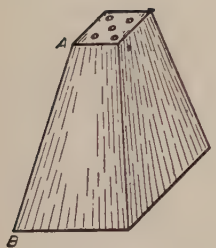


Fig. 1

Fig. 2

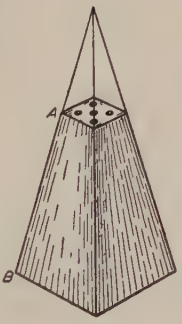


Fig. 3

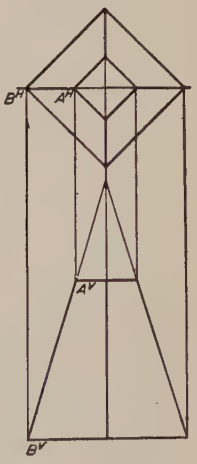


Fig. 4

In Fig. 1 is shown the picture of a square salt shaker, and in Fig. 2 are shown the top and front views of this object, or its projections. Notice that the top of the salt shaker, Fig. 1, is smaller than the bottom, giving a tapering form from bottom to top. A careful study of the shape of the salt shaker shows that all of the surfaces, or sides, and all of the edges, or lines, on this object slope upward and back-

ward as they get nearer to the top of the object. Now if projections are made of this object as shown in Fig. 2, we see from a careful study of the projections that none of the edges are parallel to the ground line (G L), therefore the true size and shape cannot be found from its projections because none of the surfaces or edges are parallel to either plane or projection.

Let us then assume that all the edges of the salt shaker have been continued until they meet at the top and form a square pyramid, Figs. 3 and 5. Now let us imagine that the object in Fig. 1 has been revolved until line AB is parallel to the vertical plane, Fig. 3. If we

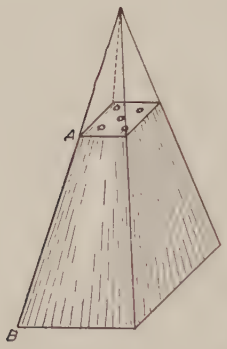


Fig. 5

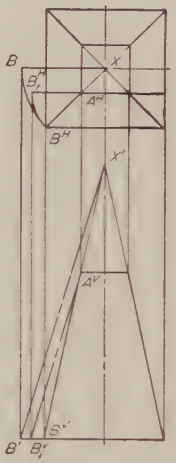


Fig. 6

were to make projections of the shaker in this position, they would appear as in Fig. 4. The projection of line $A^H B^H$ is now parallel to the ground line.

As we do not have the object to revolve, we must revolve every line in its projections to get the same position as shown in Fig. 4; but in order to find the true length of the pyramid edge, it is only necessary to revolve one of the edge lines in the projections. If the edge AB is revolved and a horizontal projection is made of this line, the line A^H, B^H in Fig. 2 in the plan view would assume the position as shown by A^H, B_1^H in Fig. 6 in the plan view. This projection being parallel to the GL shows that this edge is now parallel to the vertical plane; and if we can find a vertical projection of this edge, we

can then find its true length from it, because we have the projection of a line which is parallel to one of the planes of projection, namely the vertical plane. To find the vertical projection of the revolved line, draw a line from the point B_1^H down to the elevation until it intersects the base line of the object extended, as shown at B_1^V. A line drawn from A^V to B_1^V is the true length of the edge of the salt shaker, and $X'\ B'$ is the true length of the pyramid of which the salt shaker is a part. Fig. 6.

Rule. *To find the true length of a line revolve one of its projections until it is parallel to the ground line, then find the projection of the revolved line on the other plane of projection.*

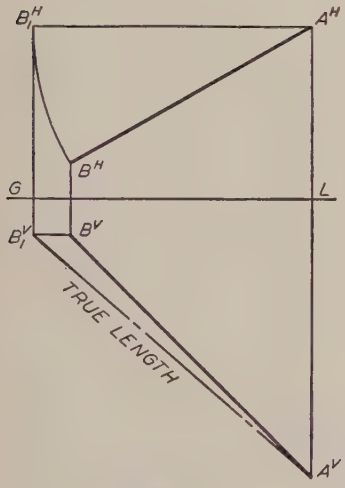


Fig. 7

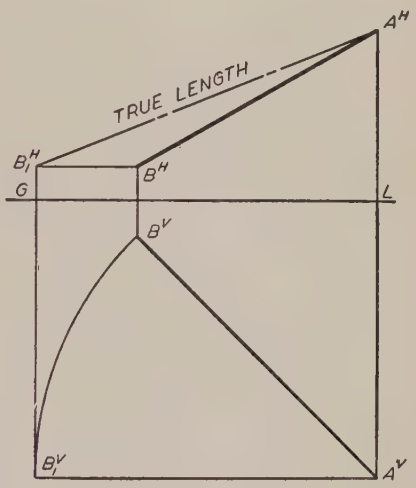


Fig. 8

No matter which projection is revolved first, the result is the same. In Fig. 7 we have the horizontal and vertical projections of a line which is not parallel to either plane of projection. The horizontal projection has been revolved with center at A^H, until the end B^H has reached B_1^H. The line is now parallel to V. A line is now drawn from point B_1^H downward until it intersects a horizontal line drawn from B^V as shown at B_1^V. The line drawn between points A^V and B_1^V is the true length of the actual line.

In Fig. 8 we again have the same projections as shown in Fig. 7, but in this case the vertical projection is revolved first with its

center at A^V until B^V reaches B_1^V when the line becomes parallel to H. Now draw a line from B_1^V up into the plan until it intersects a horizontal line drawn from point B^H as shown at B_1^H. A line drawn from A^H to B_1^H is the true length line.

If the true length lines in Figs. 7 and 8 are measured, they will be found to be the same length.

The student must realize that when working with drawings he is working with the **projections** of the object and not with the object

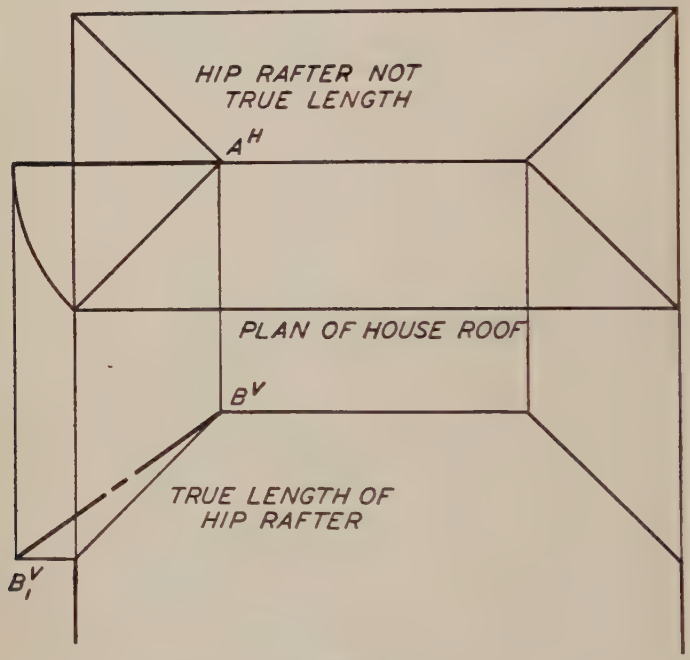


Fig. 9

itself; that the object is made from the drawings and he does not have the object to measure to find the distances that cannot be found from the projections. Therefore he will see that some method must be applied in actual practice to find these unknown dimensions.

In Fig. 9 is shown a practical application of the method of finding the true length of a line or edge on the object. The projections of the roof, as shown by the plan and elevation views of the roof in Fig. 9, show that the edges of the roof or hip rafters are not

parallel to either plane of projection. Therefore we can not find the true length of these rafters from their projections.

In the plan view the left front rafter has been revolved around with the center at A^H at the top end, or peak of the roof, until it is parallel to the vertical plane. A line has been then drawn from the end of this line down into the elevation view until it intersects the horizontal line B_1^V, drawn from the lower end of the vertical projection. A line drawn from B^V to B_1^V, is the true length of the hip rafter.

Development of Surfaces

When we speak of the development of an object, we mean the laying out or unfolding of the object on a flat surface.

In sheet metal work a pattern must be made such that when a sheet is cut according to the pattern drawn, it can be so formed that it will be of the required size and shape.

The drawing of this pattern on the flat sheet of metal is the developing of the surface or surfaces required to form the object. The draftsman therefore develops the object on the drawing paper from which the mechanic takes his measurements for the layout of the pattern on the sheet metal.

Any surface which can be smoothly wrapped about by a sheet of paper can be developed. Figures made up of planes and single curved surfaces only would be of this nature. Double curved surfaces and warped surfaces cannot be developed; and when patterns of such surfaces are desired, they must be made by an approximate method which requires two or more pieces to make the complete pattern.

If you find the true size of all the faces of an object made up of planes and join them in proper order at their common edges, the developed surface will be formed. The best way to do this is to find the **true length of the edges of the object.** The projections of any object must be drawn before the development can be made, but it is necessary only to draw such views as are required for finding the true length of the elements and true sizes of cut surfaces.

Right Cylinder. In Fig. 10 is represented a right cylinder rolling on a plane or flat surface. The development is formed by one complete revolution of the cylinder and is a rectangle, the width

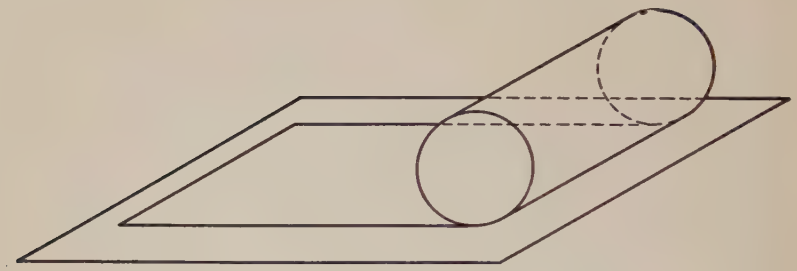


Fig. 10

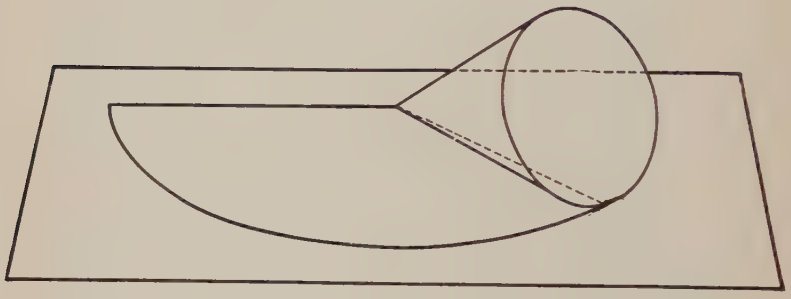


Fig. 11

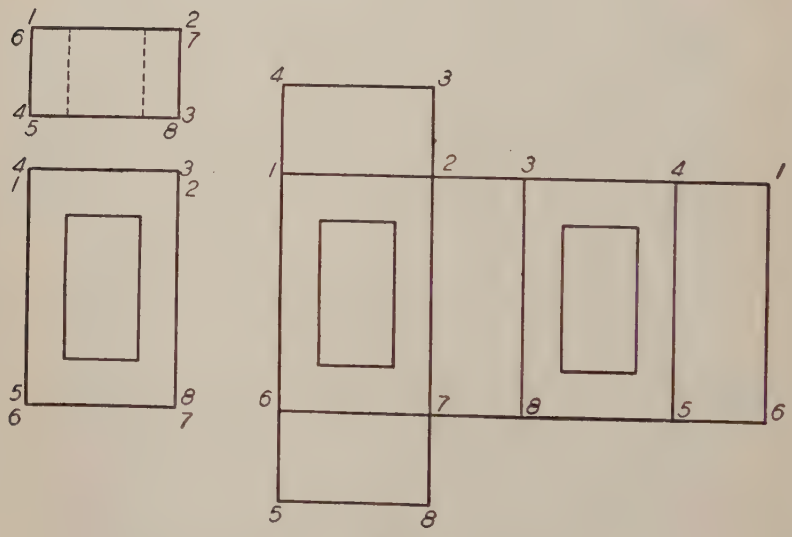


Fig. 12

being equal to the height of the cylinder and the length to the circumference.

Right Cone. In Fig. 11 is represented a right cone rolling out its development, which is a sector of a circle. The arc equals the circumference of the circle forming the base of the cone, and the radius equals the slant height.

Rectangular Prism. In order to find the development of the rectangular prism in Fig. 12, the back face *1-2-7-6* is supposed to be placed in contact with a plane and then the prism turned on the edge *2-7* until the side *2-3-8-7* is in contact with the same plane, and this process continued until all four faces have been placed on the same plane. The rectangles *1-2-3-4* and *6-7-8-5* are for the top and bottom, respectively. The development then is the exact size and shape of a covering for the prism.

If a rectangular hole is cut through the prism, the openings in the front and back faces will be shown in the development in the centers of the two broad faces.

The development of a right prism, then, consists of as many rectangles joined together as the prism has sides, these rectangles being the exact size of the faces of the prism, and two polygons the exact size of the bases. It will be found helpful in developing a solid to number or letter all corners on the projections, then designate each face when developed in the same way as in the figure.

Cone. If a cone is placed on its side on a plane surface, one element will rest on the surface. Now if the cone is rolled on the plane, the vertex remaining stationary, until the same element is in contact again, the space rolled over will represent the development of the convex surface of the cone. Fig. 13 shows a cone cut by a plane parallel to the base. In Fig. 14 let the vertex of the cone be placed at *V* and one element of the cone coincide with *V-F-1*. The length of this element is taken from the elevation, Fig. 13, of either contour element. It is not necessary to find the true length of the edge of the cone in this case because the contour element is parallel to the vertical plane and therefore is true length. If one element is imagined as being on the surface of the cone and its projections drawn as at F^H1^H and F^V1^V, it is seen that F^H1^H is parallel to the ground line and therefore is parallel to the vertical plane, and its true length can be obtained from its vertical projection.

All of the elements of the cone are of the same length, so that when the cone is rolled, each point of the base as it touches the plane will be at the same distance from the vertex. Therefore, it follows that in the development of the base, the circumference will become the arc of a circle of a radius equal to the length of the element and of a length equal to the distance around the base. To find this length divide the circumference of the base in the plan into any number of equal parts, say twelve, and lay off twelve such

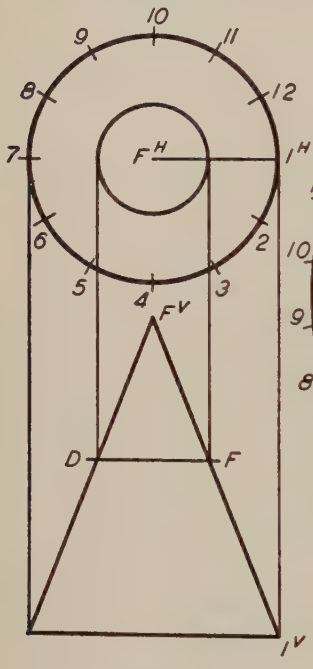


Fig. 13

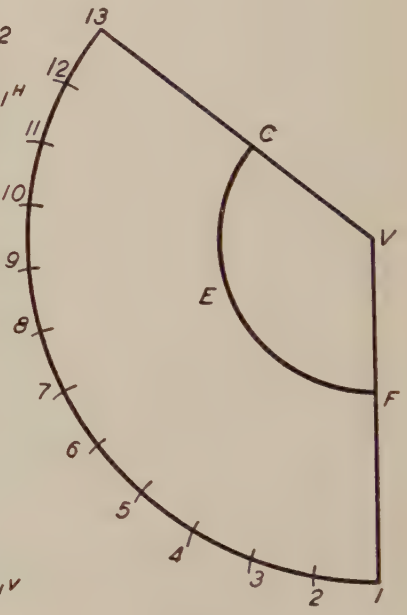


Fig. 14

spaces, *1* to *13*, along an arc drawn with a radius equal to $F^V 1^V$. Join *1* and *13* with *V*, and the resulting sector is the development of the cone from vertex to base.

In order to represent on the development the circle cut by the section plane *DF*, Fig. 13, draw from the vertex *V*, Fig. 14, as a center and with $F^V F$ as a radius, Fig. 13, the arc *FC*, Fig. 14. The development of the frustum of the cone will be a portion of a circular ring. This of course does not include the development of

the bases, which would be simply two circles the same size as shown in the plan.

Fig. 15 shows the elevation of a truncated circular cone for which a development is required. The method for finding the main development is the same as that shown in Fig. 14. The points laid out on the arc should be connected to the center for the arc, as shown in Fig. 16. To find the development of the truncation or intersect-

Fig. 15

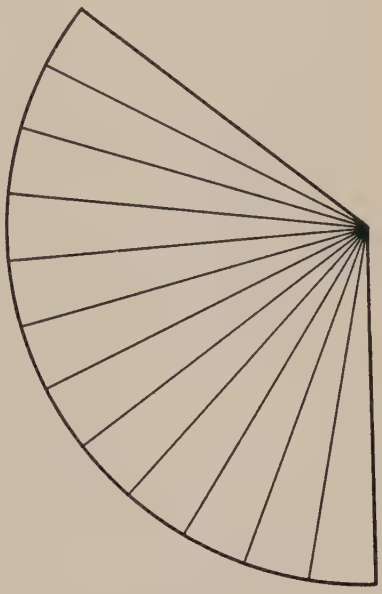


Fig. 16

ing plane we must first find points of intersection on the cutting plane as shown in Figs. 17 and 18.

Draw the elevation view, locate the cutting plane, and below this draw half of the base circle, Fig. 17. Divide this semi-circle up into any number of equal parts and project these points up until they intersect the base line on the elevation. It is a good plan to number these lines for easy reference. Connect these points with the apex of the cone and these lines will give the intersections on the cutting plane.

Project these intersections across horizontally until they intersect the contour element 6 as at *A*, *B*, *C*, etc., Fig. 18. Set the com-

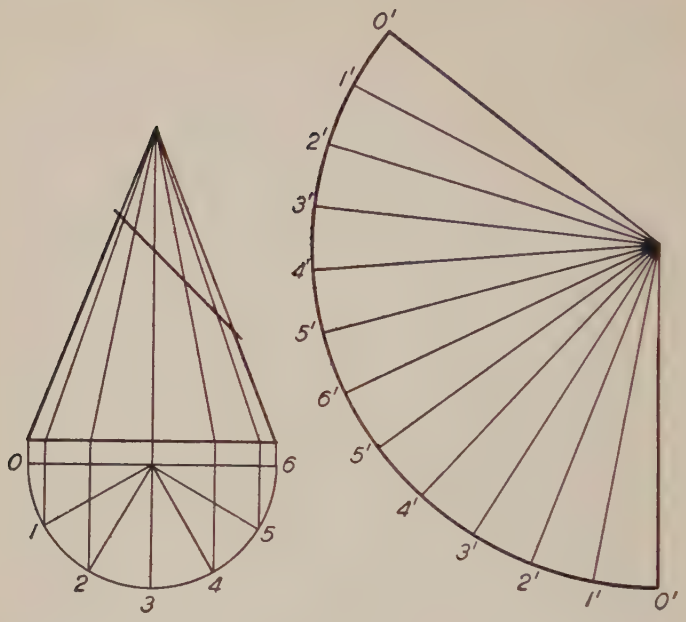


Fig. 17

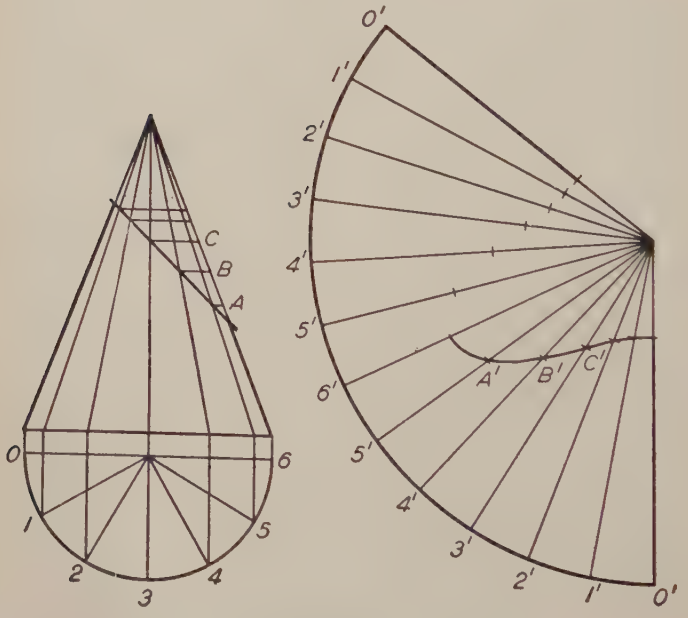


Fig. 18

pass to the distances found between the apex of the cone and the intersections on the contour element *6* and lay these distances off on the corresponding lines in the development as at *0'*, *1'*, *2'*, *3'*, etc., and as at *A'*, *B'*, *C'*, etc. Draw the curve through these intersections on the main development and the true development is completed.

A great deal of time and work can be saved by projecting the points of intersection on the cutting plane across to one element on the cone and then revolving these points directly into the main development, as shown in Fig. 19, which shows one-half of the development.

It will be seen from the two foregoing examples that when the object being developed is part of a cone, the cone must be completed

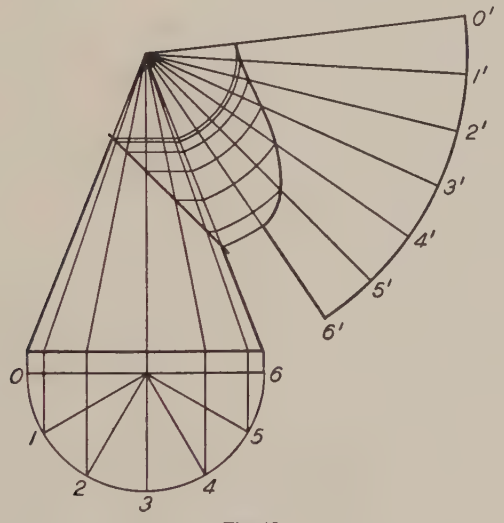


Fig. 19

first by extending the elements until they meet at a common point, which of course is the apex of the cone, and then all measurements are taken from this common point.

Fig. 20 shows the plan and elevation of a regular triangular pyramid and Fig. 21 its development. If face *C* is placed on the plane, its true size will be shown in the development. The true length of the base of the triangular pyramid is shown in the plan. However,

as the slanting edges are not parallel to the vertical, their true length is not shown in the elevation but must be obtained by the method indicated in Fig. 20.

Having found the true length, draw an arc as shown in Fig. 21, with a radius equal in length to the length of the true length line, Fig. 20, and on this arc lay off three spaces equal in length to one of the base edges of the cone as taken from the plan view, and connect these points with the center for the arc. Connect the points on the arc with straight lines and the three faces of the pyramid are completed. These, together with the base, constitute the complete development.

In Fig. 22 are shown the projections of a truncated square pyramid. The edges of this pyramid are not parallel to any plane of projection, so before a development can be made the true length of the edge must be found. Having the true length, the procedure of laying out the development is the same as that in Fig. 21, excepting that we have one more surface to lay out on the development, Fig. 23. However, the development is not yet complete because the location of the intersecting plane or truncation and also the true shape of the intersections are to be found and attached to the development.

To find the true shape of the intersections, project lines upward perpendicular to the cutting plane, and draw a center line parallel to the cutting plane and at any convenient distance from it, as at *X Y*. Next, take the distances *A A* and *B B* from the plan view, because they are parallel to *H* and are therefore true length, and lay them off equally each side of *X Y*, as at *A'A'* and *B'B'*.

To find the location of the cutting plane on the development, first project points *1* and *2* across until they intersect the true length line. Next, set the compass to distances *C-1'* and *C-2'* and scribe the arcs as shown on the development at *C'-1'* and *C'-2'*. Connect the points of intersection and the location of the cutting plane is finished.

The next step is to attach the true shape of the intersections to the development by taking the dimensions from the true shape and laying them off as shown by points *3*, *3*, and *4*, *4*, on the development.

If you take a piece of transparent paper and lay it over the development in Fig. 23 and make a tracing of it and then cut out a

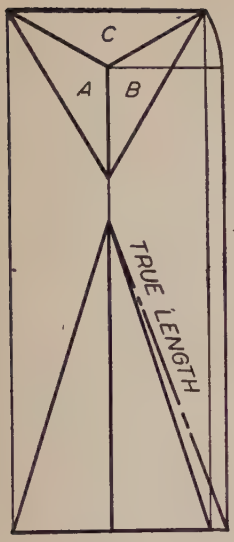


Fig. 20

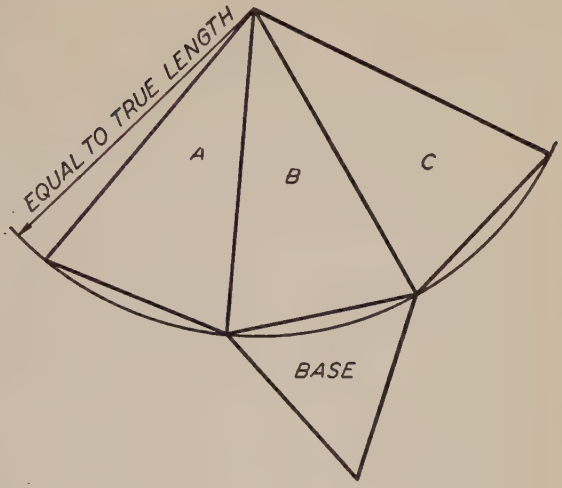


Fig. 21

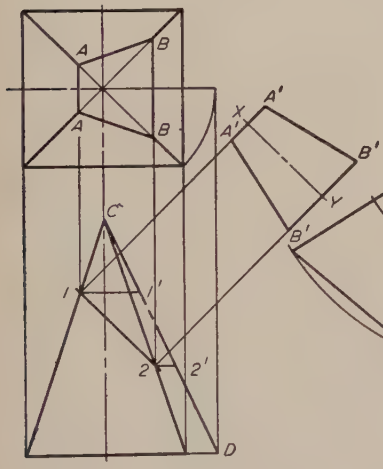


Fig. 22

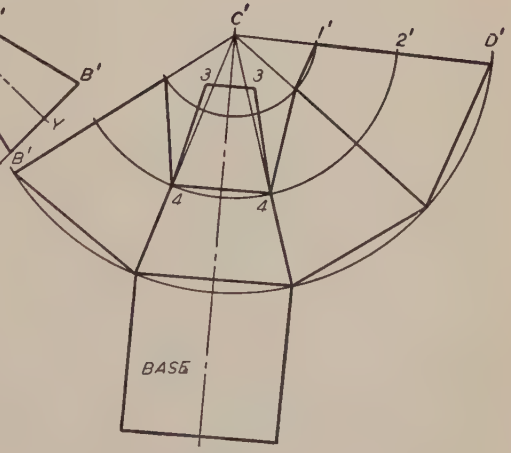


Fig. 23

pattern, you will find that you can actually form the object by folding the pattern on the heavy lines.

Truncated Circular Cylinder. If a truncated circular cylinder is to be developed or rolled upon a plane, the elements being parallel will appear as parallel lines, and the base line being perpendicular to the elements will appear as a straight line of length equal to the circumference of the base. The base of the cylinder in Fig. 24 is divided into twelve equal parts, *1*, *2*, *3*, etc., and commencing at point *1* on the development, these twelve parts are laid off along the straight line, giving the total width.

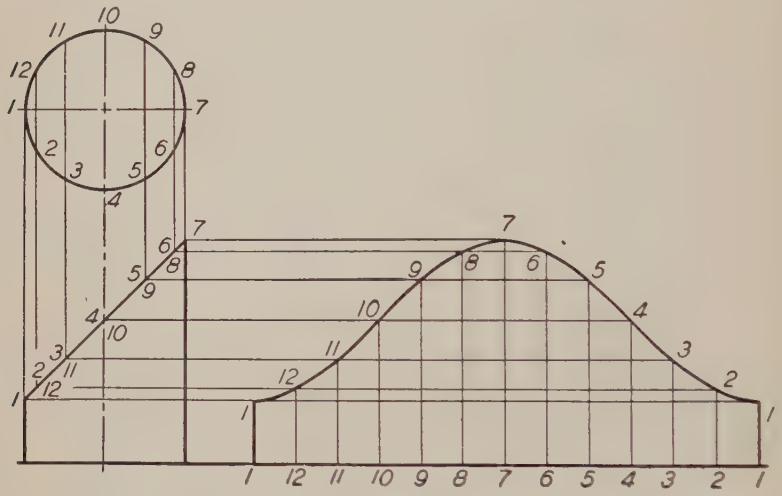


Fig. 24

Draw in elevation the elements corresponding to the various divisions of the base and note the points where they intersect the oblique plane. As the cylinder is rolled, beginning at point *1*, the successive elements, *1*, *12*, *11*, etc., will appear at equal distances apart and equal in length to the lengths of the same elements in elevation. Thus point *10* on the development is found by projecting horizontally across from point *10* in elevation. It will be seen that the curve formed is symmetrical, the half on the left of point *7* being similar to that on the right. The development of any similar surface may be found in the same manner.

The principle of cylinder development is used in laying out elbow joints, pipe ends cut off obliquely, etc. In Fig. 25 is shown the plan

and elevation of a three-piece elbow and collar, and developments of the four pieces are shown in Fig. 26. In order to construct the various parts making up the joint, it is necessary to know what shape and size must be marked out on the flat sheet metal, so that when cut out and rolled up the three pieces will form cylinders with the ends fitting together as required. Knowing the kind of elbow desired, first draw the plan and elevation, and from these make the developments. Let the lengths of the three pieces *A*, *B*, and *C* be

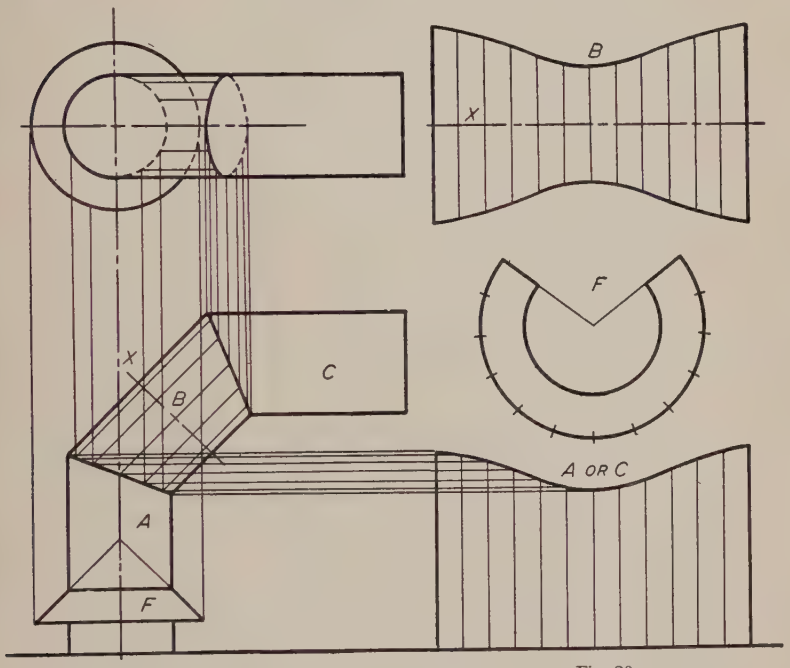


Fig. 25

Fig. 26

the same on the upper outside contour of the elbow; the piece *B* at an angle of 45 degrees; the joint between *A* and *B* bisects the angle between the two lengths, and in the same way the joint between *B* and *C*. The lengths *A* and *C* will then be the same and one pattern will answer for both. The development of *A* is made exactly the same as explained in Fig. 24 and this is also the development of *C*.

It is very necessary to remember that in developing a cylinder the base must always be at right angles to the elements, and if the cylinder as given does not have such a base, it becomes necessary to

cut the cylinder by a plane perpendicular to the elements and use the intersection as a base. This point must be clearly understood in order to proceed intelligently. A section at right angles to the elements is the only section which will unroll in a straight line and is, therefore, the section from which the other sections must be developed. As *B*, Fig. 25, has neither end at right angles to its length, the plane *X* is drawn at the middle and perpendicular to the length. *B* has the same diameter as *C* and *A*, so the section cut by *X* will be a circle of the same diameter as the base of *A*, and is shown in the development at *X*.

The elements on *B* are drawn from the points where the elements on the elevation of *A* meets the joint between *A* and *B* and are equally spaced as shown on the plan of *A*. Commencing with the left-hand element in *B*, the length of the upper element between *X* and the top corner of the elbow is laid off above *X*, Fig. 26, giving the first point in the development of the end of *B* fitting with *C*. The lengths of the other elements in the elevation of *B* are measured in the same way and laid off from *X*. The development of the other end of the piece *B* is laid off below *X*, using the same distances, since *X* is halfway between the ends. The development of the collar is simply the development of the frustum of a cone, explained in Fig. 14. The joint between *B* and *C* is shown in plan as an ellipse, the construction of which is easily understood from a study of the figure.

MECHANICAL DRAWING

ISOMETRIC PROJECTION

There are a great many mechanics who are unable to read working drawings and it is therefore necessary to make a drawing which will show in one view practically everything that is shown in the three views as used in orthographic projection. Isometric projection is one of the methods by which this presentation of an object in one view is accomplished, but it has certain disadvantages which prevent its use in manufacturing or engineering work.

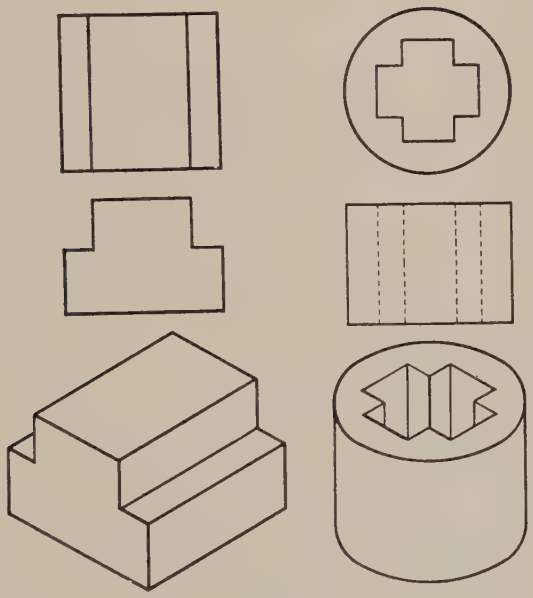


Fig. 1

Isometrical drawings always appear larger than the object being drawn. If the object is of complicated shape and construction, it is very difficult to show all details in the single view because of the mass of dotted lines and other lines it is necessary to use in order to show the construction of the object but which makes the drawing hard to read. In isometric the drawing is always made

so that one corner of the object is nearest the observer and which is considered to be parallel to the vertical plane. In Fig. 1 are shown the working and isometric drawings of a T-shaped block and a cylinder with a rectangular shaped hole through it. Notice how much larger the isometrical drawing appears to be than the working drawings.

Isometric of a Cube. To obtain an isometrical drawing of a cube or a view which shows the three dimensions in such a way that measurements may be taken from them, draw the cube in its simplest position, that is, with all its faces parallel to the planes of projection, as shown at the left of Fig. 2. Two faces are then

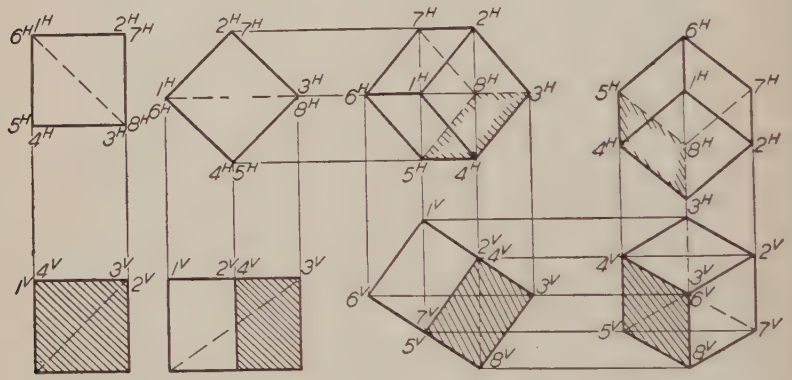


Fig. 2

parallel to the vertical plane V. The two views show that a diagonal is passed through the cube from the front upper right-hand corner to the back lower left-hand corner and is represented by a dotted line. The shaded face is the same face throughout the different positions in this figure. Next, swing the cube around so that the diagonal line is parallel to V. This brings one corner of the object directly facing the observer with the shaded face to the right of the corner. Next, tilt the cube up on the extreme lower right-hand corner until the diagonal is horizontal, and in this position the diagonal is parallel to both planes of projection. The shaded portion, or face, is now the lower of the two faces but is still to the right of the corner. The plan view is found by projecting from this front view and the previous plan view.

In the next position the cube is rotated toward the right on the corner 8^V, and in this position the diagonal line is perpendicular to V and parallel to H. The plan is the same in shape as in the previous plan but in the elevation a corner has again come into view with the shaded portion to the left.

Distinction between Isometric Projection and Isometric Drawing. In the last position in Fig. 2 the cube is so placed that all the faces make the same angle with V and, if projected on that plane, the projections would be parallelograms of equal size and shape. If the edges of the cube were projected on to the vertical plane, the projections would be all the same length but equally foreshortened because of their angle to this plane of projection. Since all these edges are equally foreshortened and since a drawing may be made to any scale, it is customary to make all isometrical lines of a drawing full length. This gives the same proportions and is a much simpler method than trying to revolve an object until it is in such a position in which all faces and edges are equal in shape and length. This would be a very difficult thing to do with objects of complicated construction. In the last position, Fig. 2, the edges are 30 degrees to the horizontal; therefore, in isometrical drawing, lines showing length and breadth are drawn as 30-degree lines while those showing height are vertical lines.

True Length Lines. Edges or lines that are either parallel to a T-square line or perpendicular to V in orthographic projection are drawn in isometric as 30-degree lines and true length; and those that are actually vertical are made vertical in isometric and also true length. Isometric axes are, therefore, 30-degree lines and vertical lines. Front vertical edges are vertical lines and base edges are 30-degree lines.

In Fig. 3 is shown the orthographic views of a square block or cube in its simplest position and in Fig. 4 is the isometric of this cube. All the edges are shown in their true length, hence, all the surfaces appear of the same size. In Fig. 4 the edges of the base are at 30 degrees, because the base edges of the cone in the orthographic views are parallel to a T-square line. Note that the height is shown by the vertical lines.

For rectangular objects, such as prisms, cubes, etc., the base edges are at 30 degrees only when the prism or cube is in its simplest

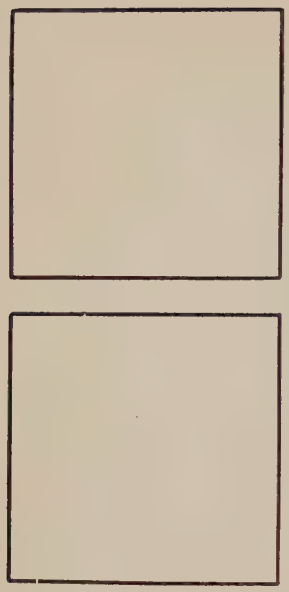


Fig 3

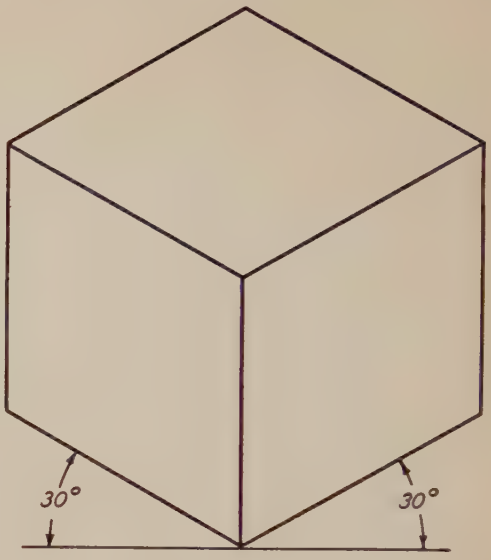


Fig. 4

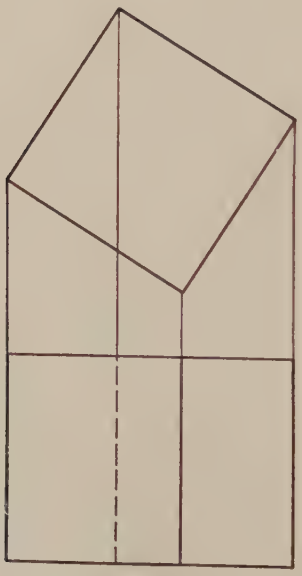


Fig. 5

possible position, that is, with two faces parallel to the vertical plane.

If the isometric of the cube in the position shown in Fig. 5 were required, it could not be drawn with the base edges at 30 degrees nor would these appear in their true length. It follows then that in isometrical drawing true lengths appear only as 30-degree lines or as vertical lines.

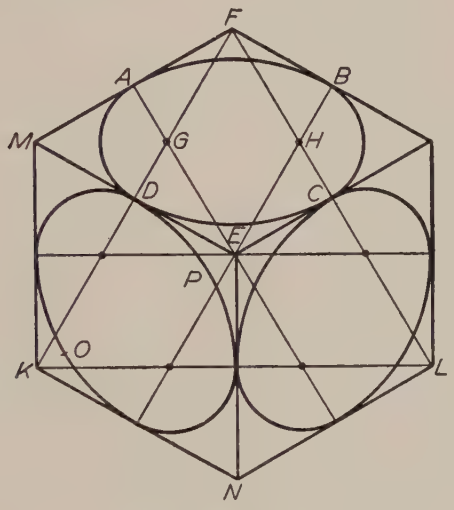


Fig. 6

Characteristics of Various Isometrics. The isometrical drawing of a circle is an ellipse and is drawn in an isometric square, Fig. 6, which has its edges equal in length to the diameter of the circle to be drawn in isometric. In the method shown, four centers are used. Considering the upper face of the cube, lines are drawn from the obtuse angles F and E to the centers of the opposite sides. The intersections of these lines give points G and H, which serve as centers for the ends of the ellipse. With G as center and a radius GA, the arc AD is drawn; and with F as center and a radius FD, the arc DC is drawn; the ellipse is finished by using centers H and E. This construction is applied to all three faces.

The true dimensions of the ellipse are taken between the points where the ellipse touches the isometric square. The measurement is never taken across the major axis. It will be seen that if lines

are drawn through these points, they will be either 30-degree lines or vertical lines; and as these lines are isometric axes, they are always true length. If the isometric view slopes upward from left

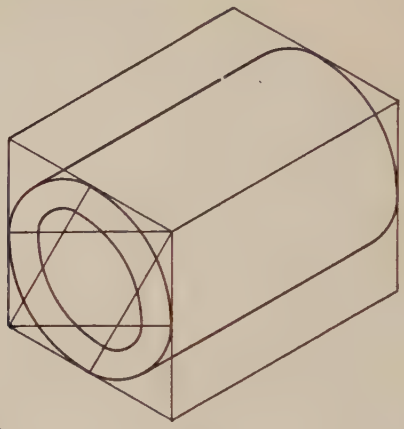


Fig. 7

to right, the method shown in the left-hand face of the cube would be used, as shown in Fig. 7; if it slopes from right to left, the method

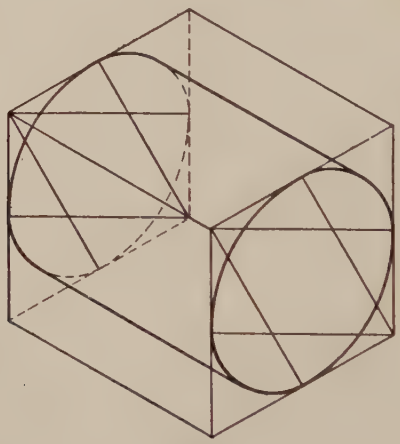


Fig. 8

shown in the right-hand face of the cube would be used, as shown in Fig. 8; and if the view is to be vertical, use the top face of the cube, as shown in Fig. 9.

In Fig. 9 are shown the working drawings of a hollow cylinder and the necessary steps to convert this into isometric drawing.

It is very often necessary to draw a radius or arc in isometric.

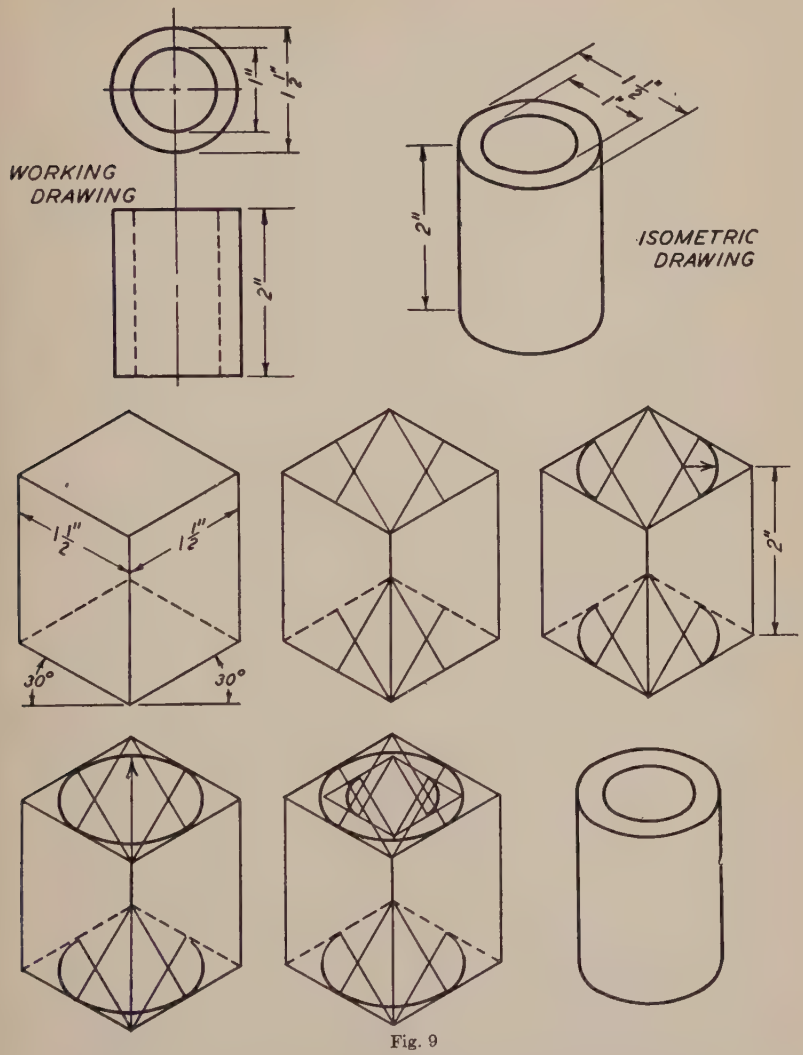


Fig. 9

It has been stated that the isometric of a circle is an ellipse and, as an arc is part of a circle, the isometric of an arc must therefore be part of an ellipse.

To correctly draw the arc in isometric, the diameter of the circle of which the arc is a part must first be determined. The diameter of a circle of which an arc is a part is always twice the radius of the arc. Having found the diameter of the circle, construct an isometric

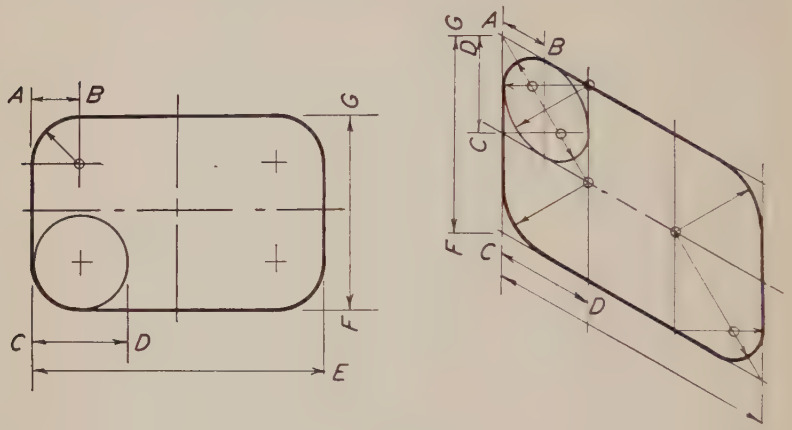


Fig. 10

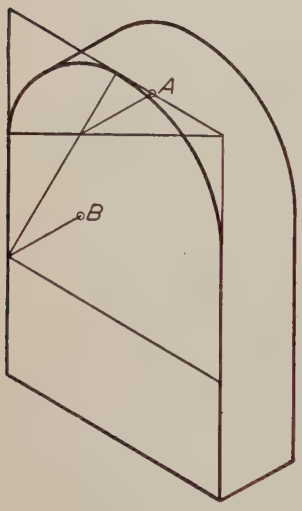


Fig. 11

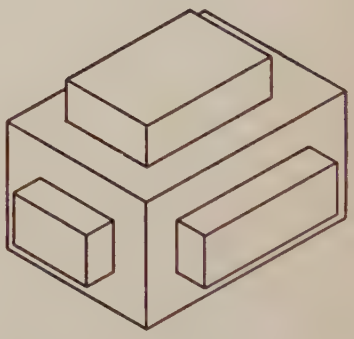


Fig. 12

square with edges that are equal in length to the diameter of the circle and then proceed to construct the ellipse as shown in Fig. 10 or according to the position of the object as explained in Figs. 7, 8 and 9.

In Fig. 11 is shown a block or prism with a semicircular top. First find the isometric of the square in which the ellipse is drawn as shown in Fig. 10, then draw the curve by the method shown in Fig. 7.

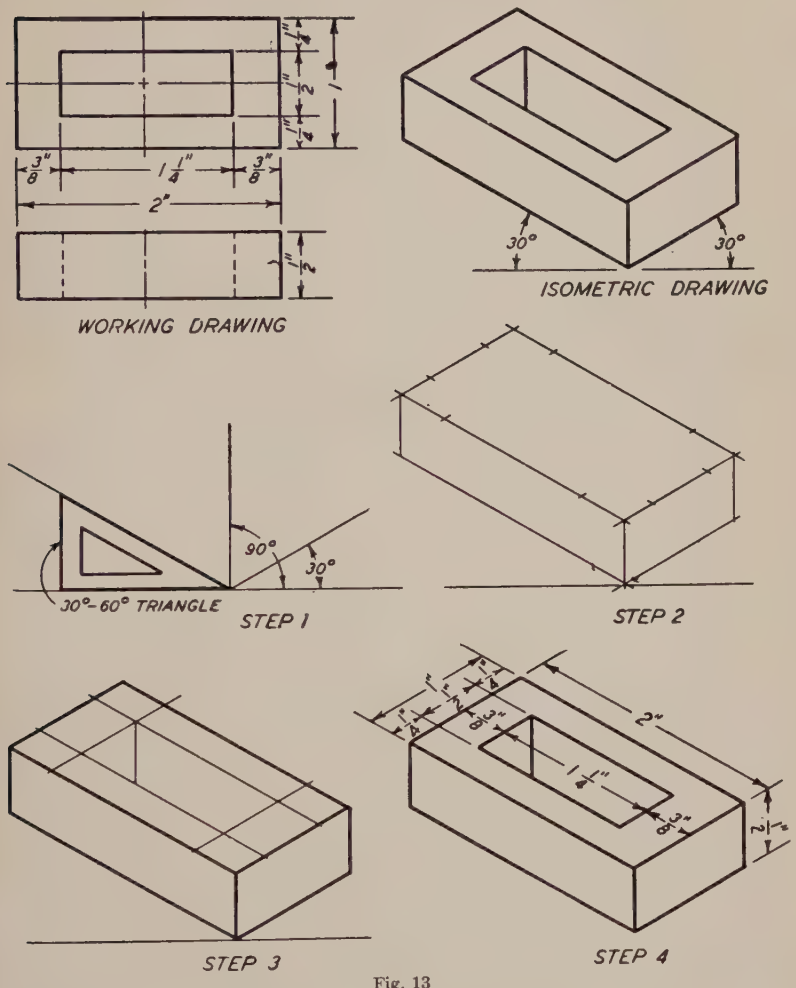


Fig. 13

Blocks. Fig. 12 represents a block with smaller blocks projecting from three faces. Note how the edges of the small projecting blocks are parallel to the edge of the large or main block. In Fig. 13 is shown the working drawings of a rectangular-shaped block with a

rectangular hole through it, and the steps for constructing it in isometric.

Fig. 14 shows a framework of three pieces, two at right angles and a slanting brace. The horizontal piece is mortised into the upright, as indicated by the dotted lines.

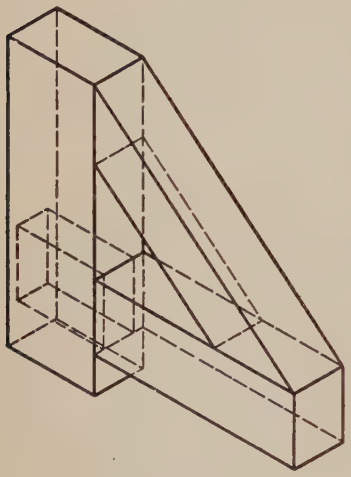


Fig. 14

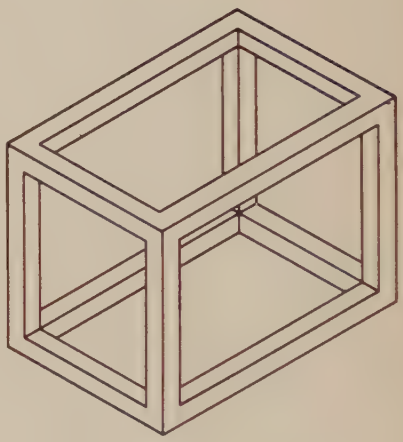


Fig. 15

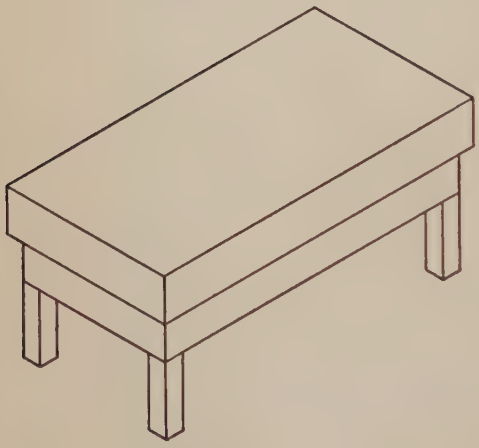


Fig. 16

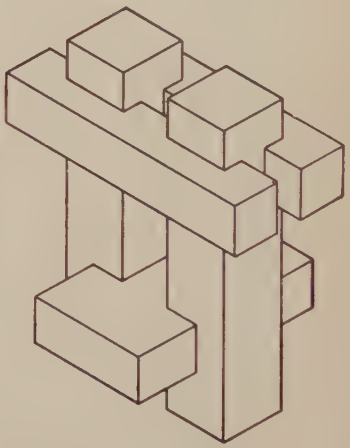


Fig. 17

Fig. 15 shows a hollow block or just a box skeleton. Notice how all lines and edges are parallel. These parallel isometric lines are also shown in Fig. 16. True lengths can be found on these ver-

tical and isometric lines or axes. In Figs. 17 and 18 are shown two more examples of parallel isometric line drawings.

Co-ordinate Isometric Lines or Axes. The length of lines or edges which in orthographic projection do not conform to the state-

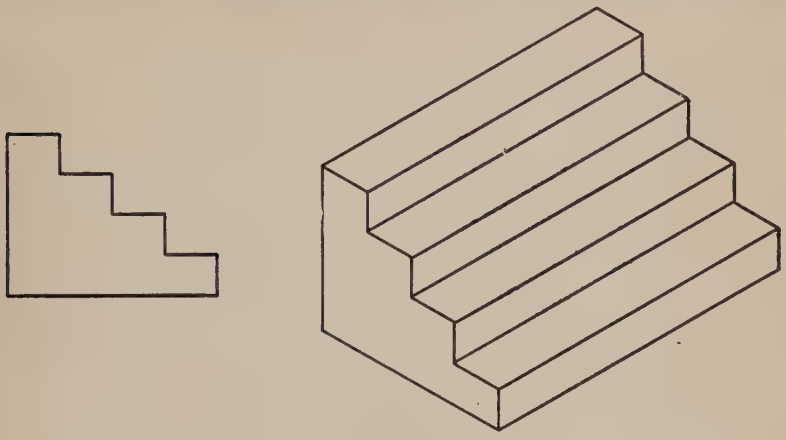


Fig. 18

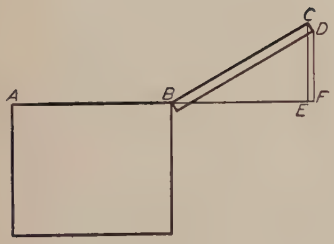


Fig. 19

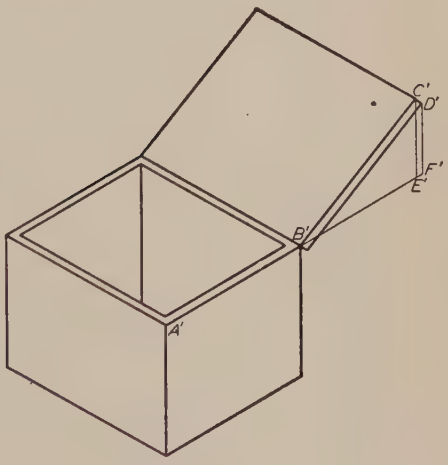


Fig. 20

ment given on Page 3 cannot be laid off directly on isometric lines, therefore co-ordinate isometric lines or axes must be used; but before this can be done, the true lengths of these lines must be found from the orthographic projections.

In Fig. 19 is shown the elevation view or orthographic projection

of a box, and in Fig. 20 is shown the isometric of this box. The line $B'F'$, which is line $A'B'$ extended, is a 30-degree co-ordinate; while the lines $E'C'$ and $D'F'$ are vertical co-ordinates. The distances taken from the corresponding points in Fig. 19, as indicated by the letters, are laid off on these co-ordinate axes and then the different points connected to form the box lid. It will be seen that line BC in Fig. 19 is neither parallel to a T-square line or perpendicular to V, therefore, this distance in isometric cannot be laid off directly on $B'C'$ in Fig. 20.

In Fig. 21 the orthographic view of a hexagon is shown, which had to be drawn first to find the distance across the corners. Notice how the isometric rectangle in Fig. 22 is first constructed with the

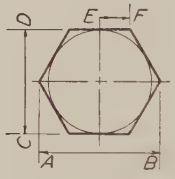


Fig. 21

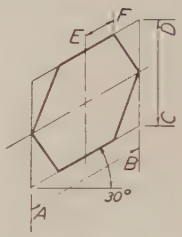


Fig. 22

30-degree lines equal in length to the distance across the corners of the hexagon, while the vertical height is equal to the distance across the flats of the hexagon. In this figure the sides of the rectangle are the isometric co-ordinates.

In Fig. 23 the plan and elevation or orthographic views of a carpenter's sawhorse are shown. To draw this in isometric the co-ordinates, as shown in Fig. 24, must first be drawn and then the distances as taken from the orthographic are laid off on these co-ordinates. Although the lines that are parallel to EA in the plan view are perpendicular to the GL, the legs of the horse are not actually perpendicular to V, hence the use of the co-ordinates. Vertical distances are laid off on the vertical co-ordinate and horizontal distances are laid off on the 30-degree co-ordinate.

In Fig. 25 the isometric of a house is represented, showing a dormer window and a partial hip roof. AB is a hip rafter and CD is a valley. Let the pitch of the main roof be shown at 1 and let M

be the middle point of the top of the end wall of the house. Then by measuring vertically up a distance ML equal to the vertical height AN shown in *1*, a point on the line of the ridge will be found at L.

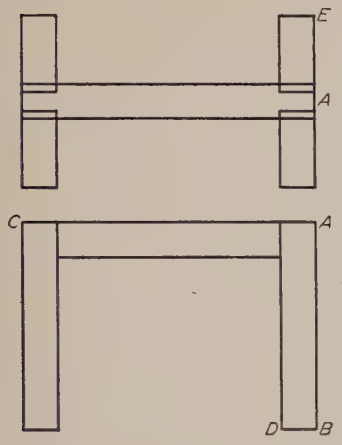


Fig. 23

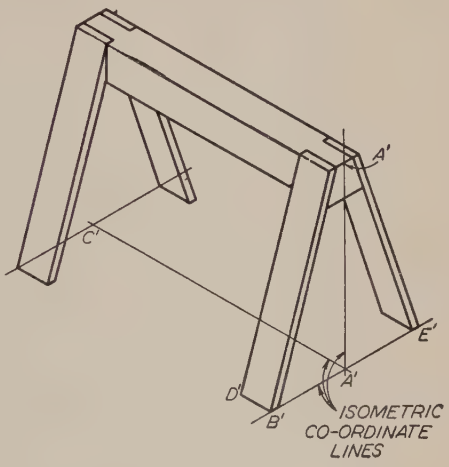


Fig. 24

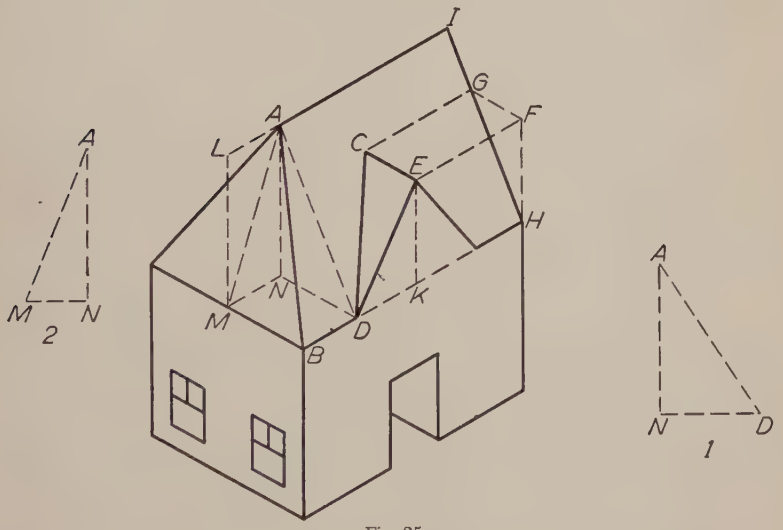


Fig. 25

Line LI is equal to BH, and IH is then drawn. Let the pitch of the end of the roof be given at *2*. It is shown that the peak of the roof, or the end A of the ridge, is back from the end wall a

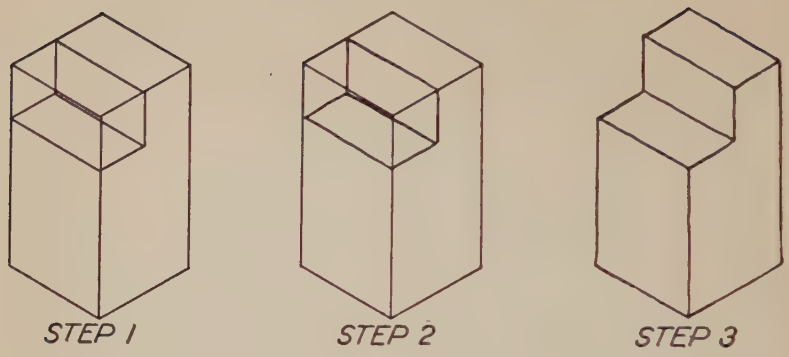


Fig. 26

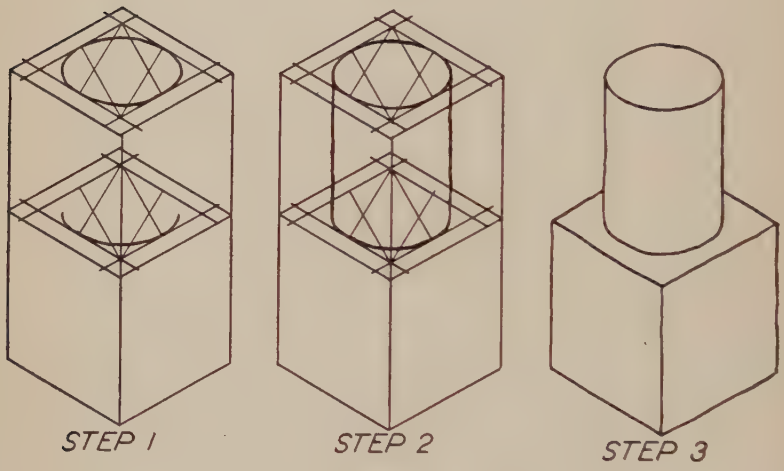


Fig. 27

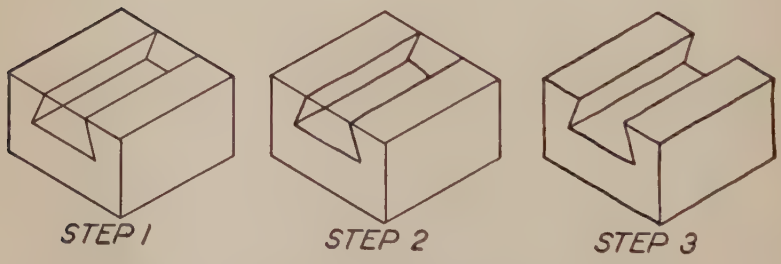


Fig. 28

distance equal to the base of the triangle at *2*. Hence, lay off from *L* this distance on *LI*, giving point *A*, and join *A* with *B* and also with the other corner as shown.

The height of *KE* of the ridge of the dormer we will assume is known, but where this ridge will meet the main roof must be found. The ridge must be a 30-degree line, as it is parallel to the end wall of the house and the ground. Draw from *E* a line parallel to *BH* to meet a vertical line drawn through *HF*. This point is on the vertical plane of the end wall of the house, hence in the plane *IH*. If now a 30-degree line be drawn from *F* parallel to *BM*, it will meet the roof of the house at *G*. The dormer ridge *G* and *F* are in the same horizontal plane, hence will meet the roof at the same distance below the ridge *AI*. Therefore, draw the 30-degree line *GC* and connect *C* with *D*.

Lines *MN*, *NA*, *ND*, *EK*, *EF*, *FG*, *CG* are all co-ordinate lines or isometric axes, while the triangles at *1* and *2* are the orthographic views of the portions of the ends of the roof, and as these lines do not conform to the statement given on Page 3, co-ordinates must be used to draw the isometric of the roof.

Freehand Isometric Sketching. It may take an artist to make a good sketch of an object, but the skill required to make an isometric sketch is within the reach of everyone. In making an isometric sketch of an object always start with a solid prism; then outline on this prism the lines necessary to cut the figure you want to draw out of the solid block. To finish the drawing, all that is necessary is to erase the parts which would be cut away if you were working on a solid block.

The various sketches given in Figs. 26, 27, and 28 will illustrate the method of working out the lines and erasing the construction lines after the figure is completed.

MECHANICAL DRAWING

OBLIQUE PROJECTION

Oblique Compared with Isometric Projection. In oblique projection as in isometric the end sought for is the same, namely, a more or less complete representation in one view of any object. Oblique projection differs from isometric in that one face of the object is parallel to the vertical plane of projection, Fig. 1, while in

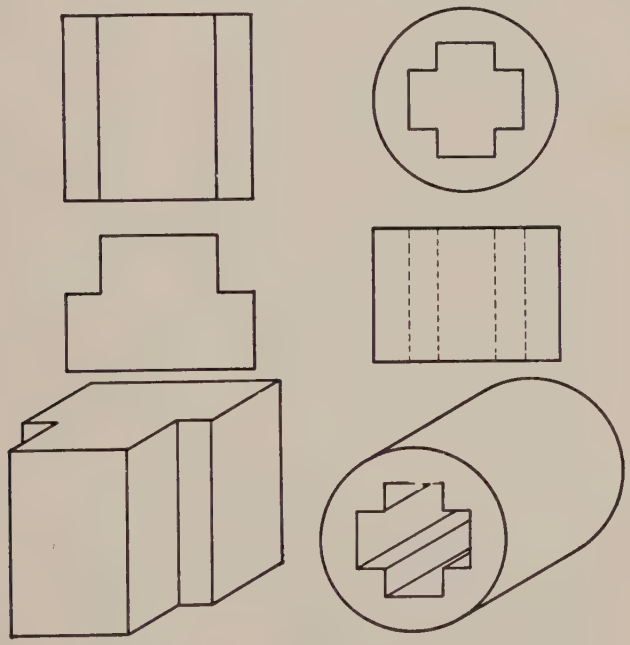


Fig. 1

isometric one corner is presented as being parallel to the vertical plane.

All faces that are parallel to the front face in orthographic projection are shown parallel to the front face in oblique.

The base edges in isometric are 30-degree lines, while in oblique they may be placed at any convenient angle.

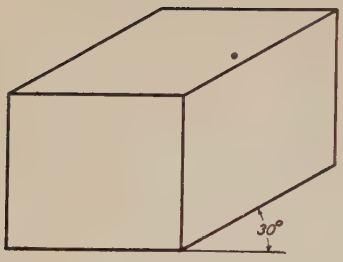


Fig. 2

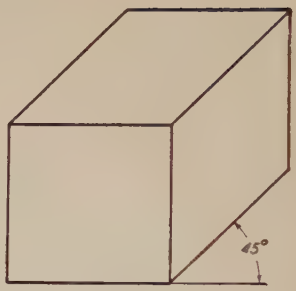


Fig. 3

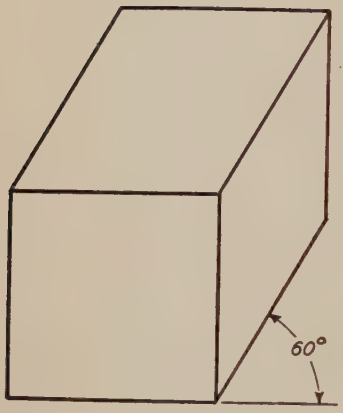


Fig. 4

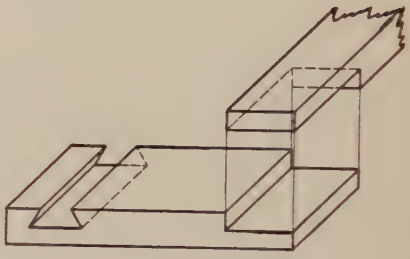


Fig. 5

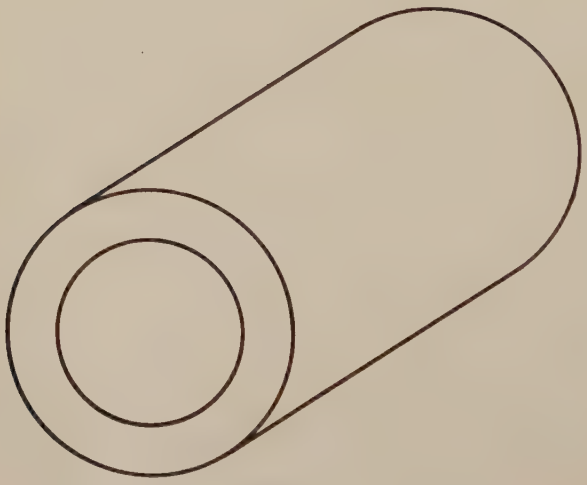


Fig. 6

Another point of difference is that oblique projection cannot be deduced from orthographic projection by the revolution of a cube as in isometric.

Characteristics of Method. In oblique projection all lines in the front face are shown in their true lengths and in their true rela-

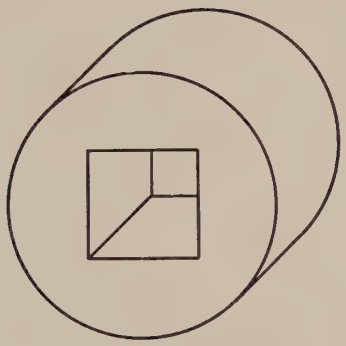


Fig. 7

tion to one another; and in orthographic projection lines which are perpendicular to this front face are shown in their true lengths at any angle that may be desired for any particular case. Lines not in the plane of the front face nor perpendicular to it must be determined by co-ordinates as in isometric, but these co-ordinates will not be at 30 degrees but at the same angle at which the drawing is

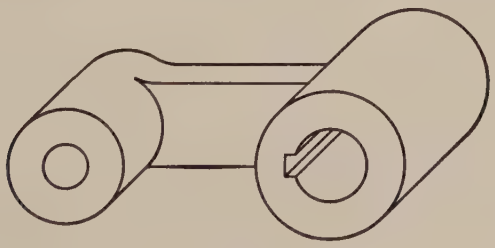


Fig. 8

being made. It will be seen that in this system objects that are circular on the front face may be drawn as true circles in oblique and true size. In Figs. 2, 3, and 4 are shown a cube in oblique with the edges at 30-degree, 45-degree, and 60-degree slant, respectively. In choosing the angle care should be taken not to make it too sharp,

as it will be found difficult to show the construction clearly in some cases and it also gives a very distorted view of the object.

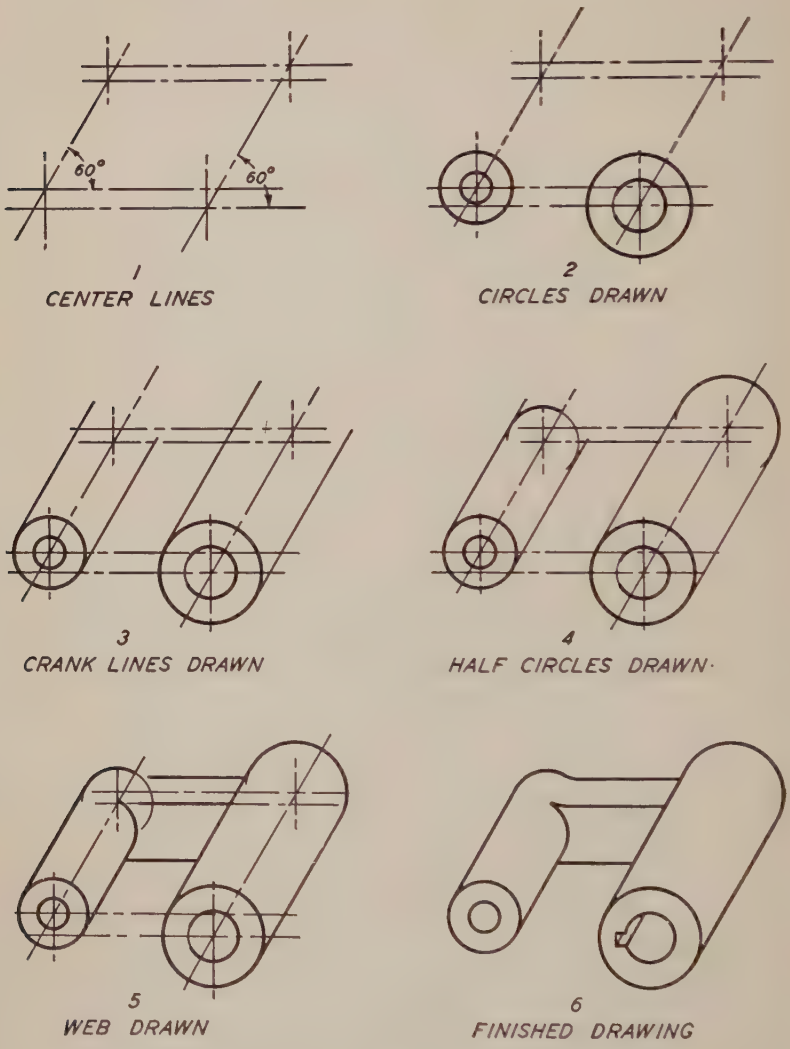


Fig. 9

In Fig. 5 is shown the oblique view of a miter joint. Notice how the height and length of the piece is shown on the front faces.

In Fig. 6 the oblique view of a cylinder is shown. It will be seen that although the drawing is placed at a different angle to the cubes in Figs. 2, 3, and 4, the front face is shown as a true circle and is true size.

In Fig. 7 is shown an oblique view of a cylinder with a rectangular hole through it. Note that the hole is shown in the center of the cylinder front face and is true size.

The oblique view of a crank is shown in Fig. 8 and the steps for the complete oblique drawing are shown in Fig. 9.

It has been shown that when the front face of an object is circular and parallel to *V*, it is represented in oblique as a true circle. This is true whether the object be drawn toward the left or the right, but sometimes a hole or circular portion must be shown on the top or side of the object, and these must be represented as ellipses.

It does not matter at what angle the drawing is placed, the method used is exactly the same in every case.

Step 1—Fig. 10.

First locate the center lines of the ellipse, then draw an oblique rectangle with edges equal in length to the diameter of the circle to be drawn. These edges should be parallel to the edges of the surface on which the ellipse is to be drawn.

Step 2—Fig. 10.

Draw a line through the diagonal corners of the rectangle.

Step 3—Fig. 10.

Draw perpendicular lines from the intersection of the center line and sides of the rectangles until they intersect the diagonal lines drawn through the corners. The intersections on the diagonal lines will be the centers for the long arcs of the ellipse.

Step 4—Fig. 10.

Draw lines from the intersection of the center line and top and bottom edges of the rectangle to the centers for the long arcs. The centers for the short arcs are found at the intersection of these two lines as shown.

If the angle at which the drawing is being made is 30 degrees, it will be found that the isometric method can be applied to the drawing of the ellipse.

Where arcs are to be drawn in oblique, the diameter of the circle of which the arc is a part must first be determined and then the oblique rectangle constructed and the ellipse drawn. The position of the rectangle for the ellipse will depend upon the position of the ellipse on the object. If it is on the top or vertical, then the rectangle sides will consist of horizontal lines parallel to the front face at the top and bottom, and the lines for the sides are at the same angle as the base line of the object. If the ellipse is to be on the side, then the top and bottom lines of the rectangle will be the same angle as the base lines of the object and parallel to them, while the side lines

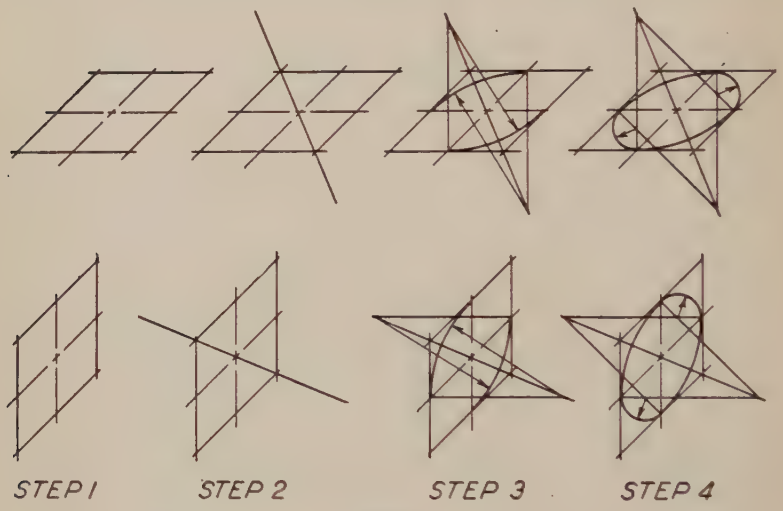


Fig. 10

of the rectangle will be vertical lines and parallel to the vertical lines on the front face.

Co-ordinate Lines. Co-ordinate lines in isometric were found to be 30-degree lines and vertical lines. Co-ordinate lines in oblique will be vertical lines and lines of the same angle as that at which the drawing is being made. The plan and elevation of a wooden brace are shown in Fig. 11, and the oblique drawing of this object is shown in Fig. 12. The lines showing the portion projecting upward, Fig. 11, are not perpendicular to the front face, therefore, the position of this portion must be determined by co-ordinates, as shown in Fig. 12. Note how the horizontal distances *A* and *B* are laid off on a

45-degree co-ordinate A' and B', which is parallel to the edges of the object. The vertical heights of the projecting portion C and D are laid off on the vertical co-ordinates at C' and D'. These lines are parallel to the vertical lines on the front face.

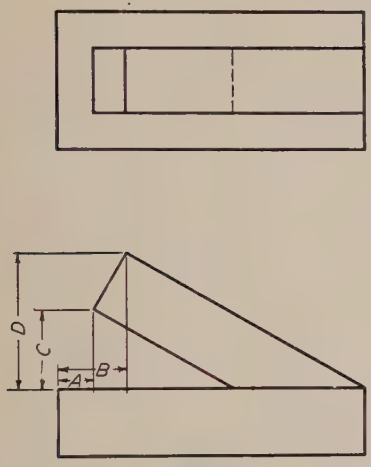


Fig. 11

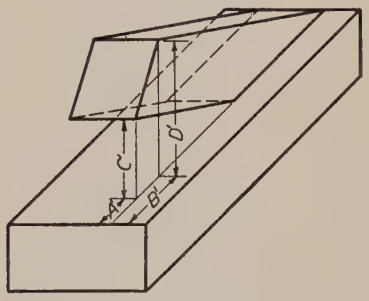


Fig. 12

LINE SHADING

Object of Line Shading. In finely finished drawings it is frequently desirable to make the various parts stand out by showing the graduations of light and shade on the curved surfaces. This is specially true of such surfaces as cylinders, cones, and spheres. The effect is obtained by drawing a series of parallel or converging lines on the surface at varying distances from one another, the width of the lines also varying according to the idea of the draftsman. These lines are farther apart on the lighter portion of the surface and closer together and heavier on the darker part. It takes constant practice to do good work in shading.

Fig. 13 shows a cylinder with elements drawn on the surface equally spaced on the plan. On account of the curvature of the surface, however, the elements are not equally spaced on the elevation in order to give the effect of the graduations of light on the curved surface. The result is that in drawing the elevation of the cylinder the distances between the elements are made gradually less

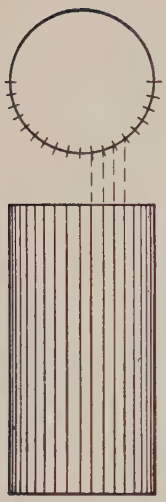


Fig. 13

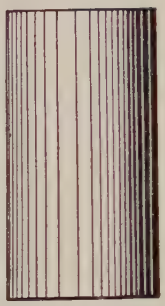


Fig. 14

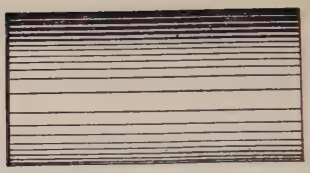


Fig. 15

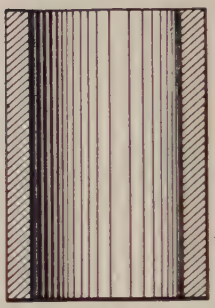


Fig. 16

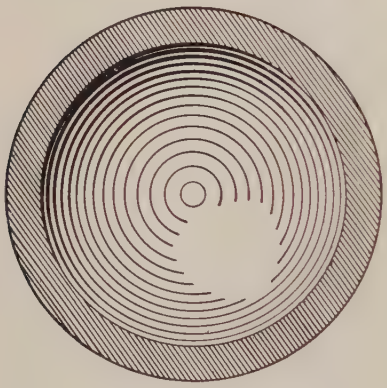


Fig. 17

Fig. 18

Fig. 19

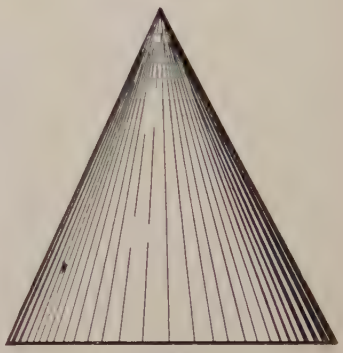


Fig. 20

from the center toward each side, thus giving a correct representation of the convexity of the cylinder. This effect is intensified by making the outside elements heavier as well as closer together, as shown in Figs. 14 to 20. Concavity is shown in the same manner, the heavy shading always appearing on the left to indicate the deeper shadow, Figs. 16 and 17.

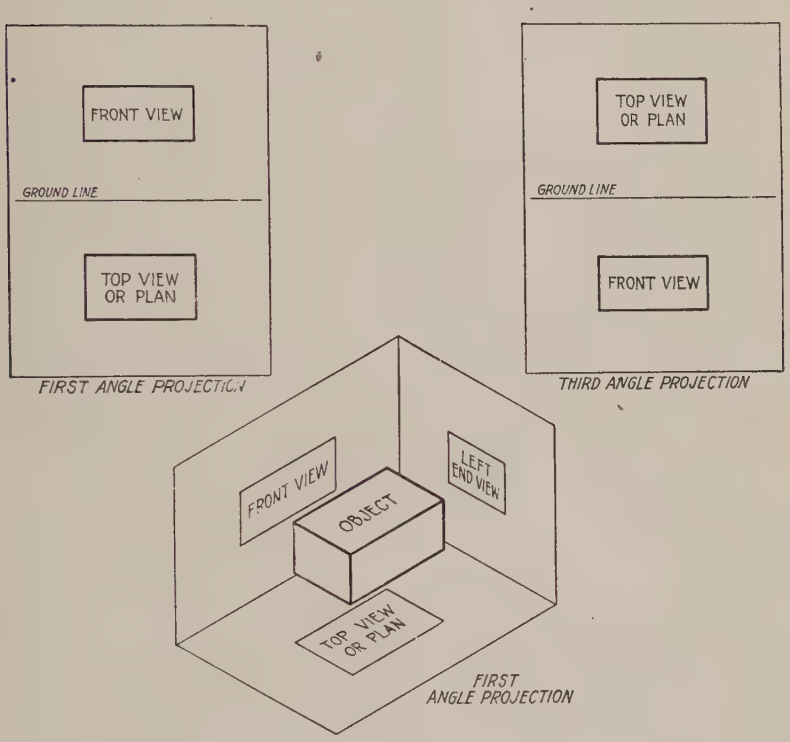


Fig. 21

Fig. 14 is a cylinder showing the heaviest shade at the right, a method often used. Fig. 15 represents a cylinder in a horizontal position and Fig. 16 represents a section of a hollow vertical cylinder. Figs. 18 to 20 show other examples of familiar objects.

In the elevation of the cone shown in Fig. 20, the shaded lines should diminish in weight as they approach the apex. Unless this is done, it will be difficult to avoid the formation of a blot at that point.

FIRST ANGLE PROJECTION

Thus far we have discussed and illustrated drawings made in the third angle projection. There is, however, a method called first angle projection, which is used but little now in this country although at one time it was in almost general use. Because draftsmen and engineers may have to work occasionally with old drawings or drawings made in foreign countries, it is well for them to understand first angle projection.

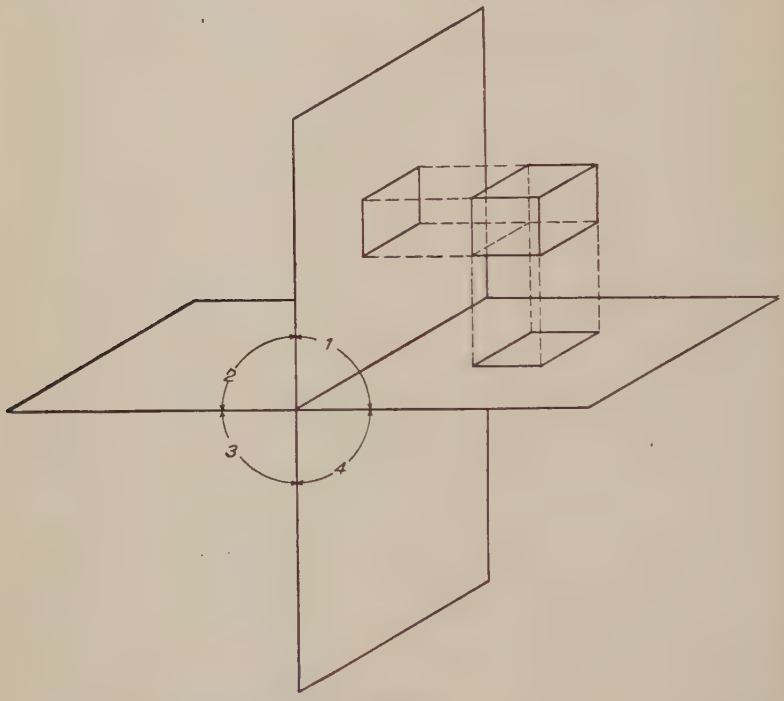


Fig. 22

This method of projection places the front view or elevation above the top view or plan; the left-end view at the right of the front view; and the right-end view at the left of the front view. Fig. 21 shows this method of first angle projection. Fig. 22 illustrates the position the object would take if a screen were used to represent the vertical and horizontal planes. It will be noted in first angle projection that the lines of the object instead of being observed through the screen and traced thereon are projected away from the eye beyond

the object and therefore the side of the object that is drawn on the screen is not the one actually nearest to it.

When drawings are made in the first angle projection, the position of the top view of the plan appears below the intersecting line of the planes and the front elevation appears above the ground line. Fig. 21 shows the same drawing made in both first angle and third angle projection.

Fig. 23 shows an architect's sketch of a stair in first angle projection. Architects' drawings are usually made showing only one

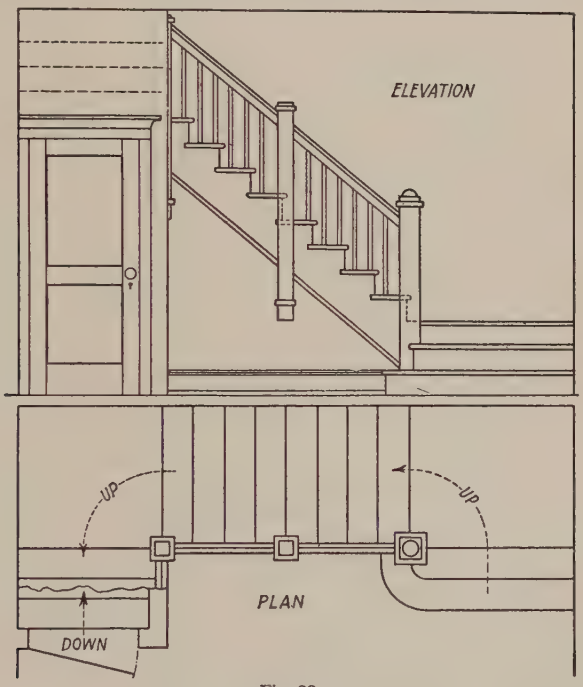


Fig. 23

view on a sheet. It will be seen that so far as the making of the drawing is concerned, the method of projection is the same for both angles of projection, only the various views of the object appear in different positions on the drawing paper. One can always tell whether a drawing has been made in third angle projection by noting whether the top view or plan is above or below the ground line. In first angle projection the top view is below the ground line. In the third angle projection the top view is above the ground line.

ARCHITECT'S DRAWING OF THE JOHN G. SHEDD AQUARIUM
Located on the Lake Front in Grant Park, Chicago
Courtesy of Underwood & Underwood

FREEHAND DRAWING.

1. The Value of Freehand Drawing to an Architect. Outside of its general educational value freehand drawing is as absolutely essential to the trained architect as it is to the professional painter. It is obviously necessary for the representation of all except the most geometric forms of ornament, and it is equally important in making any kind of a rapid sketch, either of a whole building or a detail, whether from nature or in the study of plans and elevations. It is perhaps not so generally understood that the training it gives in seeing and recording forms accurately, cultivates not only the feeling for relative proportions and shapes, but, also, that very important architectural faculty—the sense of the third dimension. The essential problem of most drawing is to express length, breadth, and thickness on a surface which has only length and breadth. As the architect works out on paper, which has only length and breadth, his designs for buildings which are to have length, breadth, and thickness, he is obliged to visualize; to see with the mind's eye the thickness of his forms. He must always keep in mind what the actual appearance will be. The study of freehand drawing from solid forms in teaching the representation on paper of their appearance, stimulates in the draughtsman his power of creating a mental vision of any solid. That is, drawing from solids educates that faculty by means of which an architect is able to imagine, before it is erected, the appearance of his building.

2. Definition of Drawing. A drawing is a statement of certain facts or truths by means of lines and tones. It is nothing more or less than an explanation. The best drawings are those in which the statement is most direct and simple; those in which the explanation is the clearest and the least confused by the introduction of irrelevant details.

A drawing never attempts to tell all the facts about the form depicted, and each person who makes a drawing selects not only the leading truths, but also includes those characteristics which

appeal to him as an individual. The result is that no two people make drawings of the same subject exactly alike.

3. The Eye and the Camera. The question immediately arises: Why should we not draw all that we see; tell all that we know about our subject? Since the photograph does represent, with the exception of color, all that we see and even more, another question is raised: What is the essential difference between a photograph of an object and a drawing of an object? These are questions which bring us dangerously near the endless region of the philosophy of fine arts. Stated simply and broadly, art is a refuge invented by man as an escape from the innumerable and bewildering details of nature which weary the eye and mind when we attempt to grasp and comprehend them.

Without going into an explanation of the differences in structure between the lens of a camera and the lens of the eye, it may be accepted as a general statement that in spite of apparent errors of distortion the photograph gives us an exact reproduction of nature. Every minutest detail, every shadow of a shade, is presented as being of equal importance and interest, and it is easy to demonstrate that the camera sees much more detail than the human eye. In any good photograph of an interior the patterns on the walls and hangings, the carving and even the grain and texture of woods are all presented with equal clearness. In order to perceive any one of those details as clearly with the eye it would be necessary to focus the eye on that particular point, and while so focused all the other details of the room would appear blurred. The camera, on the contrary, while focused at one point sees all the others with almost equal clearness. This fact alone is enough to demonstrate the danger of assuming that the photograph is true to the facts of vision. Again, a photograph of an antique statue will exaggerate the importance of the weather stains and disfigurements at the expense of the subtle modelling of the muscular parts which the eye would instinctively perceive first.

Nature, then, and the photograph from nature, is a bewildering mass of detail. The artist is the man of trained perceptions who, by eliminating superfluous detail and grasping and presenting only the essential characteristics, produces a drawing in which we see the object in a simplified but nevertheless beautiful form.

In looking at the drawing we become conscious of the subject and its principal attributes; we comprehend and realize these with far less effort of the mind and eye than we should expend in taking in and comprehending the real object or a photograph of it. Compared to nature it is more restful and more easily understood, and the ease with which it is comprehended constitutes, the psychologists say, a large part of the pleasure we take in art; it certainly explains why we enjoy a drawing of an object when we may take no pleasure in the object itself, or a photograph of it.

4. Restraint in Drawing. The practical application of the preceding broad definition is neither difficult nor abstruse. The beginner in drawing usually finds his work swamped in a mass of detail, because his desire is to be absolutely truthful and accurate, and the more he has read Ruskin* and writers of his school the more does he feel that art and nature are one, and that the best drawing is that which most successfully reproduces nature with photographic fidelity. It may be taken for granted that a drawing must be true; true to nature. But truth is at best a relative term, and while it may be said that every normal eye sees practically the same, yet, after all, the eye sees only what it is trained to see. It is the purpose of all teaching of drawing to train the eye to see and the hand to put down the biggest and most important truths and to sacrifice small and unimportant details for the sake of giving greater emphasis or accent to the statement of the larger ones. "Art lives by sacrifices" is the expression of the French, the most artistic nation of modern times. The experience of the beginner is very practical testimony to the truth of the expression, for he very soon realizes that he has not the ability, even if it were best, to draw all he sees, and he has to face the question of what to leave out, what to sacrifice. Sense will tell him that he must at all costs retain those elements which have the most meaning or significance, or else his drawing will not be intelligible. So he is gradually taught to select the vital facts and make sure of them at least. It is true that the more accomplished the draughtsman becomes the greater will be his ability to successfully represent the lesser truths, the smaller details he sees,

*Note.—Ample corroboration for all that is stated above may be found in Ruskin, but it is embedded in a mass of confusing and contradictory assertions. Ruskin is a very dangerous author for the beginner.

because having trained his perception to the importance of grasping the big truths he has also attained the knowledge and ability to express the smaller facts without obscuring the greater ones. Nevertheless the question of what to sacrifice remains one of the most important in all forms of representation. One of the commonest criticisms pronounced by artists on the work of their colleagues is that "he has not known when to stop"; the picture is overloaded and obscured with distracting detail.

5. Learning to See. It is very important that the student of drawing shall understand in the beginning that a very large part of his education consists in learning to see correctly. The power to see correctly and the manual skill to put down with accuracy what he sees—these he must acquire simultaneously. It is usually difficult at first to convince people that they do not naturally and without training see correctly. It is true that there is formed in every normal eye the same image of an object if it is seen from the same position, but as minds differ in capacity and training, so will they perceive differently whatever is thrown upon the retina or mirror of the eye.

It is a matter of common observation that no two people agree in their description of an object, and where events are taking place rapidly in front of the eyes, as in a football game, one person with what we call quick perceptions, will see much more than another whose mind works more slowly; yet the same images were formed in the eyes of each. The person who understands the game sees infinitely more of its workings than one who does not, because he **knows what to look for;** and to draw with skill one must also know what to look for. Many people who have not studied drawing say they see the top of a circular table as a perfect circle in whatever position the eye may be in regard to the table. Others see a white water lily as pure white in color, whether it is in the subdued light of an interior or in full sunlight out of doors. In questions of color it is a matter of much study, even with persons of artistic gifts and training, to see that objects of one color appear under certain conditions to be quite a different color.

6. Outline. The untrained eye usually sees objects in outline filled in with their local color, that is, the color they appear to be when examined near the eye without strong light or shade

thrown upon them. One of the first things the student has to learn is that there are no outlines in nature. Objects are distinguished from each other not by outlines but by planes of light and dark and color. Occasionally a plane of dark will be so narrow that it can only be represented by a line, but that does not refute the statement that outlines do not exist in nature. Very often only one part of an object will be detached from its surroundings. Some of its masses of light may fuse with the light parts of other forms or its shadows with surrounding shadows. If enough of the form is revealed to identify it, the eye unconsciously supplies the shapes which are not seen, and is satisfied. The beginner in drawing is usually not satisfied to represent it so, but draws definitely forms which he does not see simply because he knows they are there. Obviously then it is necessary to learn what we do not see as well as what we do.

7. Although there are no outlines in nature, most planes of light and shade have definite shapes which serve to explain the form of objects and these shapes all have contours, edges or boundaries where one tone stops and another begins. As the history of drawing shows, it has always been a convention of early and primitive races to represent these contours of objects by lines, omitting effects of light and shade. To most people to-day the outline of an object is its most important element—that by which it is most easily identified—and for a large class of explanatory drawings outlines without light and shade are sufficient. By varying the width and the tone of the outline it is even possible to suggest the solidity of forms and something of the play of light and shade and of texture.

8. Since, in order to represent light and shade, it is necessary to set off definite boundaries or areas and give them their proper size and contour, it follows that the study of outline may very well be considered a simple way of learning to draw, and a drawing in outline as one step in the production of the fully developed work in light and shade. An outline drawing is the simplest one which can be made, and by eliminating all questions of light and shade the student can concentrate all his effort on representing contours and proportions correctly. But he should always bear in mind that his drawing is a convention, that it is not as he actually

sees nature, and that it can but imperfectly convey impressions of the surfaces, quality and textures of objects.

9. It is often asserted that whoever can learn to write can learn to draw, but one may go further and assert that writing is drawing. Every letter in a written word is a drawing from memory of that letter. So that it may be assumed that every one who can write already knows something of drawing in outline, which is one reason why instruction in drawing may logically begin with the study of outline.

Some good teachers advocate the immediate study of light and shade, arguing that since objects in nature are not bounded by lines to represent them so it is not only false but teaches the student to see in lines instead of thinking of the solidity of objects. But these arguments are not sufficient to overbalance those in favor of beginning with outline, especially in a course planned for architectural students to whom expression in outline is of the first importance.

MATERIALS.

10. Pencils. Drawings may be made in "black and white" or in color. A black and white drawing is one in which there is no color and is made by using pencil, charcoal, crayon or paint which produces different tones of gray ranging from black to white.

The pencil is the natural medium of the architect and the materials for pencil drawing are very inexpensive and require little time for their preparation and care. Drawings in pencil are very easily changed and corrected if necessary. All the required plates for this course are to be executed in pencil.

The pencil will make a drawing with any degree of finish ranging from a rough outline sketch to the representation of all the light and shade of a complicated subject. In addition it is the easiest of all mediums to handle. Students are sometimes led to think that it is more artistic to draw in charcoal crayon or pen and ink. It may be that an additional interest is aroused in some students by working in these materials, but the beginner must assure himself at once that artistic merit lies wholly in the result and not at all in the material in which the work is executed.

Pencils are made in varying degrees of hardness. The softest

is marked BBBBBB or 6B; 5B is slightly less soft and they increase in hardness through the following grades: 4B, 3B, 2B, B, HB, F, H, 2H, 3H, 4H, 5H, 6H. A pencil should mark smoothly and be entirely free from grit. The presence of grit is easily recognized by the scratching of the pencil on the paper and by the unevenness in the width and tone of the line. The leads of the softer pencils are the weaker and are more easily broken. They give off their color the most freely and produce blackest lines. What hardness of pencils one should use depends upon a number of considerations, one of the most important being the quality of paper upon which the drawing is made.

Quick effects of light and shade can be best produced by the use of soft pencils because they give off the color so freely and the strokes blend so easily into flat tones.

A medium or hard pencil is necessary when a drawing is to be small in size and is intended to express details of form and construction rather than masses of light and shade. This is because the lines made by hard pencils are finer, and more clean and crisp than can be obtained by using soft grades. The smaller the drawing, the more expression of detail desired, the harder the pencil should be; a good general rule for all quick studies of effects of light and shade is to use as soft a pencil as is consistent with the size of the drawing and the surface of the paper. Beginners, however, are obliged to make many trial lines to obtain correct proportions, and in that way produce construction lines so heavy that the eraser required to remove them leaves the paper in a damaged condition. Until the student can draw fairly well he should begin every piece of work with a medium pencil and take care to make very light lines and especially to avoid indenting the paper.

It should be understood that pencil drawings ought never to be very large. There should always be a proportional relation between the size of a drawing and the medium which produces it. The point of a pencil is so small that to make a large drawing with it consumes a disproportionate amount of time. For large drawings, especially such showing light and shade, crayon or charcoal are the proper materials for they can be made to cover a large surface in a very short time. The larger the area to be covered the larger should be the point and the line producing it.

Special pencils with large leads can be obtained for making large pencil drawings.

11. Paper. In general the firmer the surface of the paper the harder the pencil one can use on it. For a medium or hard pencil the paper should be tough and rather smooth but never glazed. Many very cheap grades of paper, for example that on which newspapers are printed, take the pencil very well but have not a sufficiently tough surface to allow the use of the eraser. They are excellent for rapid sketches made very directly without alterations.

Paper for effects of light and shade should be soft and smooth. For this work the cheaper grades of paper are often more suitable than the expensive sorts. Paper with a rough surface should always be avoided in pencil drawings, as it gives a disagreeable "wooly" texture to the lines.

12. Holding the Pencil. Any hard and fast rules for the proper use of the pencil would be out of place, but until the student has worked out for himself the ways which are the easiest and best for him he cannot do better than adopt the following suggestions, which will certainly aid him in using the pencil with effect and dexterity.

The most important points in drawing are to be accurate and at the same time direct and free. Of course, accuracy—the ability to set down things in their right proportions—is indispensable; but the abilty to do this in the most straightforward way without constraint, fumbling, and erasures is also necessary. Art has been defined as the doing of any one thing supremely well.

The pencil should be held lightly between the thumb and forefinger three or four inches from the point, supported by the middle finger, with hand turned somewhat on its side.

There are three ways in which it is possible to move the pencil; with the fingers, the wrist, or the arm. Most people find it convenient to use the finger movement for drawing short, vertical lines. In order to produce a long line by this movement it is only necessary to make a succession of short lines with the ends touching each other but not overlapping, or by leaving the smallest possible space between the end of one line and the beginning of the next. The wrist movement produces a longer line and is used

naturally to make horizontal lines. For a very long sweep of line the movement of the arm from the shoulder is necessary. This is, perhaps, the most difficult way of drawing for the beginner, but it affords the greatest freedom and sweep, and many teachers consider it the only proper method.

13. Position. The draughtsman should sit upright and not bend over his drawing, as that cramps the work and leads him to look, while working, at only a small portion of his drawing instead of comprehending the whole at a glance.

The surface to receive the drawing must be held at right angles to the direction in which it is seen, otherwise the drawing will be distorted by the foreshortening of the surface. A rectangular surface such as a sheet of paper is at right angles to the direction in which it is seen when all four corners are equally distant from the eye. A fairly accurate test may be made in the following manner: Locate the center of the paper by drawing the diagonals. Flat against this point place the unsharpened end of a pencil. Tip the surface until the length of the pencil disappears and only the point and sharpened end are visible, then the surface will be at right angles to a line drawn from the eye to its center. The pencil represents this line for a part of the distance because if properly held it is at right angles to the surface.

FIRST EXERCISES.

Before trying to draw any definite forms the student should practice diligently drawing straight lines in horizontal, vertical, and oblique positions, and also circles and ellipses.

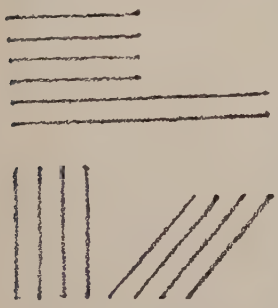


Fig. 1. Lines, Connecting Points.

14. Straight Lines. In drawing the straight line exercises points should first be placed lightly and the line drawn to connect them as in Fig. 1. Draw a series of ten or fifteen lines in each position, placing the points to be connected by the lines one inch apart and leaving a space of one quarter of an inch between each line. Next draw a series placing the points two inches apart, then a group with the points four inches apart, and finally a set which

will give lines eight inches long. Start to draw vertical lines from the top, horizontal lines from the left to right, oblique lines which slant upward toward the right, from the lower point, and those slanting upward toward the left, from the upper point. Use all three pencils, 3H, F and a solid ink pencil for these exercises, and take the greatest care not to press too strongly on the paper with the harder grades. They are intended to make rather light gray lines. Where dark lines are desired always use the solid ink pencil. Try also making the exercises with different widths of line regulated by the bluntness of the point, and do at least one set using the solid ink pencil and making very wide lines as near together as is possible without fusing one line with another. In all of these exercises the lines should each be drawn with one pencil stroke without lifting the pencil from the paper and absolutely no corrections of the line should be made.

15. Circles and Ellipses. In practicing drawing circles start from a point at the left and move around toward the right as in Fig. 2. Draw a series of ten circles half an inch in diameter, forming each with a single pencil stroke. Next draw a group of ten with a one-inch diameter, still keeping to the single pencil stroke. Follow these with a set, each being two inches in diameter and another set with a three-inch diameter. In drawing these larger circles the free arm movement will be found necessary and the lines may be swept about a number of times for the purpose of correcting the first outline and giving practice in the arm movement. As the circles increase in diameter the difficulty of drawing them with accuracy by a single stroke increases also, but instead of erasing the faulty positions and laboriously patching the line, it is better to make the corrections as directed, by sweeping other lines about until a mass of lines is formed which gives the shape correctly. The single outline desired will be found somewhere within the mass of lines and may be accented with a darker line and the other trial lines erased.

Fig. 2. Circles.

Draw a series of ten ellipses, Fig. 3, with a long diameter of

half an inch, forming each with a single pencil stroke. Follow with a group of ten, having the long diameter one inch in length, joining each outline with a single pencil stroke. Proceed with a set having a long diameter of two inches and a set with a long diameter of three inches. Follow the same instructions for these last two groups as were laid down for drawing the larger circles, that is, sweep the lines about several times with the free arm movement.

Fig. 3. Ellipses.

In drawing horizontal straight lines the elbow should be held close to the body. For vertical lines and for all curved lines the elbow should be held as far from the body as possible.

These exercises and similar ones of his own invention should be practiced by the student for a long period, even after he is studying more advanced work. Any piece of waste paper and any spare moments may be utilized for them. As in acquiring any form of manual skill, to learn to draw requires incessant practice, and these exercises correspond to the five-finger exercises which are such an important part of the training in instrumental music. While they are not very interesting in themselves the training they give to the muscles of the hand and arm is what enables the draughtsman to execute his work with rapidity, ease, and assurance.

The student should bear in mind that a straight freehand line ought not to look like a ruled line. A part of the attraction of freehand drawing, even of the simplest description, is the sensitive, live quality of the line. A straight line is defined in geometry as one whose direction is the same throughout, but slight deviations in a freehand straight line, which recover themselves and do not interfere with the general direction are legitimate, as the hand, even when highly trained, is not a machine, and logically should not attempt to do what can be performed with more mechanical perfection by instruments. Where freehand straight lines are used to indicate the boundaries of forms, the slight inevitable variations in the line are really more true to the facts of vision than a ruled line would be, inasmuch as the edges even of geometric solids appear softened and less rigid because they are affected by the play of light and by the intervening atmosphere.

This the beginner will not be able to see at first, for in this case as in so many others, his sight is biased by his knowledge of what the object is and how it feels.

16. Freehand Perspective. One of the chief difficulties in learning to draw is, as before stated, in learning to see correctly, because the appearance of objects so often contradicts what we know to be true of them. More than one beginner has drawn a handle on a mug because he knew it was there, regardless of the fact that the mug was turned in such a way that the handle was not visible. The changes which take place in the appearance of forms through changes in the position from which they are seen, are governed by the principles of perspective. Although students of this course are supposed to be familiar with the science of perspective, it is necessary to restate certain general principles of perspective with which the freehand draughtsman must be so familiar that he can apply them almost unconsciously as he draws. The most important of these are demonstrated in the following paragraphs, and their application should be so thoroughly understood that they become a part of the student's mental equipment. In theory the draughtsman draws what he sees, but practically he is guided by his knowledge as to how he sees.

The principles can be most clearly demonstrated through the study of certain typical geometric forms which are purposely stripped of all intellectual or sentimental interest, so that nothing shall divert the attention from the principles involved in their representation. The student will readily recognize the great variety of subjects to which the principles apply and the importance of working out the exercises and mastering them for the sake of the knowledge they impart. These principles can be explained very clearly by the use of the glass slate, which is a part of the required outfit for this course. All drawings should be made from the models in outline and in freehand on the glass, using the Cross pencil. The drawing should be tested and corrected according to the instructions for testing.

17. Tracing on the Slate. In beginning to study model drawing the model may be traced upon the slate held between the model and the eye and at right angles to the direction in which the object is seen. (See section 13.) In order to do this with

accuracy it is absolutely necessary that the slate shall not move and it is equally necessary that the position of the eye shall not change. As neither of these conditions can be fulfilled exactly without mechanical contrivances for holding both the slate and the head fixed, it follows that the best tracing one can make will be only approximately correct and even that only if the object is of a very simple character. The more complicated the object the less satisfactory will be the tracing from it. Perhaps the best method is to mark the important angles and changes of direction in the contour with points and then rapidly connect the points with lines following the contours. Although the result may not be very correct, if carefully made the tracing will at least demonstrate the principal points wherein the appearance of an object differs from and contradicts the facts, and that is the sole object of the tracing. It awakens in the student the power of seeing accurately as it teaches the mind to accept the image in the eye as the true appearance of an object even if that image differs from the actual shape and proportion of the object as we know it by the sense of touch. *Except as it helps us to learn to see, the tracing gives no training in freehand drawing other than the slight manual exercise involved in drawing the line.*

18. Testing with the Slate. The great value of the slate for the beginner in freehand drawing is the ease with which the accuracy of a drawing may be tested. To obtain satisfactory results the models should be placed about a foot and a half in front of the spectator and the drawings made rather large. The drawing should be made freehand, in outline, and the greatest care taken to make it as accurate as possible before testing it because the object in making the drawing is to exercise the hand and eye. Drawing exercises should not be confounded with the preliminary exercises in tracing whose only object is to emphasize the fact that forms appear different as the position of the eye changes.

In order to test a drawing place the slate at right angles to a line from the eye to the model according to the directions in section 13. Holding the slate at this angle and keeping one eye closed move it backward and forward until the lines of the drawing cover the lines of the model. Any difference in the general direction of the lines or proportions can be readily observed. Cor-

rections should not be made by tracing, but errors should be carefully noted and the alterations made freehand from a re-study of the models. If the drawing is too large to cover the lines of the model, errors may be discovered by testing the different angles of the drawing with those of the model. If all the angles coincide the drawing must be correct.

In making the tests the slate should be held firmly with both hands, and it cannot be emphasized too strongly that the test is of no value unless the slate is at right angles to the direction in which the model is seen. When groups of models or other complicated subjects are being tested only the directions of important lines and proportions of leading masses can be compared. It must be clearly understood that it takes some practice and much care to test the drawing of a simple form, and that the slate is not to be used as a means of tracing. The student will soon discover that it is impossible to trace any form or group having much detail or multiplication of parts owing to the impossibility of holding the slate and the eye for long in the same position at the same time.

Do not expect too much of the slate. Even the first exercises in tracing simple forms will show the student that unless he has acquired some facility in making lines freehand he cannot trace lines. Indeed it has often been observed that no one can trace who cannot draw. Another difficulty in using the slate at first is the resistance which the pencil encounters on the glass. It calls for a different pressure and touch from that used with a pencil on paper, so that the beginner is often discouraged unnecessarily and becomes impatient with the slate, partly because he expects too much from it and partly because he has not learned how to use it. Do not try to make perfect lines on the slate. Be satisfied at first to indicate the general direction of lines. Understand also that the slate is only to be used in beginning to draw. The student should as soon as possible emancipate himself from the use of the tests and depend upon the eye alone for judging the relations of proportions and lines. From the beginning a drawing should be corrected by the eye as far as possible before applying any tests.

FREEHAND PERSPECTIVE.*

19. The Horizon Line or Eye Level. This, as the name implies, is an imaginary horizontal line on a level with the eye. It is of great importance in representation, as all objects appear to change their shape as they are seen above or below the horizon line.

The following experiments should be made before beginning to draw any of the exercises in freehand perspective. Fasten two square tablets together at right angles to each other so that the adjacent corners exactly coincide, giving two sides of a cube. Hold it at arm's length with the edge where the two planes touch, parallel to the eyes and the upper plane level. Lower it as far as the arms allow, then raise it gradually to the height of the eyes, and above as far as possible, holding it as far out as possible. Observe that the level tablet appears to become narrower as it approaches the eye level, and when it is opposite the eye it becomes only a line showing the thickness of the cardboard. Observe that this line or front edge of the tablet always appears its actual length while the side edges have been gradually appearing to become shorter. As the tablet is lifted above the horizon the lower side begins to appear very narrow at first, but widening gradually the higher the tablet is lifted. It will be seen also that when the tablet is below the horizon line the side edges appear to run upward, and when the tablet is above the eye its side edges appear to run downward, toward the horizon.

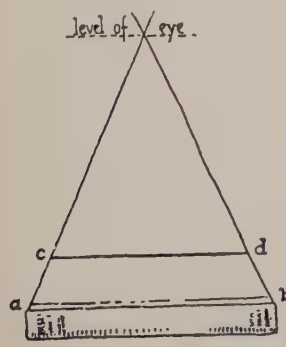


Fig. 4. Book with Strings.

That they and similar lines appear to converge and vanish in the horizon line is proved by the following experiment:

Place a book on a table about two feet away with its bound edge toward the spectator and exactly horizontal to the eye, that is, with either end equally distant from the eye. Between the cover and the first page and as near the back as possible place a string, leaving about two feet of it on either side. Hold the left end of the

*Note.—Through the courtesy of its author and publishers, these exercises in freehand perspective have been adopted from the text-book on "Freehand Drawing," by Anson K. Cross. Ginn & Co., Boston.

string in the right hand and move it until it coincides with or covers the left edge of the book. Hold the right end of the string in the left hand and move it until it covers the right edge of the book. The two strings will be seen to form two converging or vanishing lines which meet at a point on the level of the eye, that is, in the horizon line. This and the preceding experiment illustrate the following rule:

Rule 1. Horizontal retreating lines above the eye appear to descend or vanish downward, and horizontal retreating lines below the eye appear to ascend or vanish upward. The vanishing point of any set of parallel, retreating, horizontal lines is at the level of the eye.

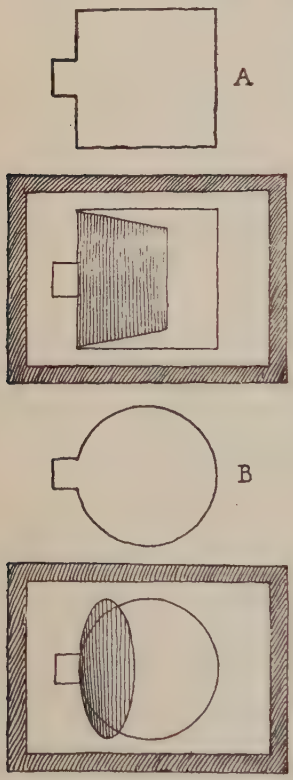


Fig. 5. Slates with Square and Circle Flaps.

It is necessary to remember that the horizon line is changed when the spectator's position is changed. This is very noticeable when one stands on a high hill and observes that the roof lines of houses which one is accustomed to see vanishing downward to the level of the eye, now vanish upward, since the eyes have been raised above the roofs.

Retreating lines are those which have one end nearer the eye than the other.

Exercise 1. Foreshortened Planes and Lines. Cut from paper a tracing of the square tablet, which is a part of the set of drawing models, and leave a projecting flap as at A, Fig. 5. Paste the flap on the under side of the slate, with the edges of the square parallel to the edges of the slate, and trace the actual shape of the square.

Holding the slate vertical and so that half the square is above and half below the level of the eye, turn the square somewhat away from the slate and trace the appearance. Turn it still farther and trace. Turn it so that the surface disappears and becomes a line.

Trace a circular tablet and cut it out of paper, leaving a flap as at B, Fig. 5. Paste the flap on the back of the slate, as with the square, and trace its real appearance. Turn the circle away at a moderate angle and trace its appearance. Trace it as it appears at a greater angle and finally place it so that it appears as a line.

Try similar experiments with the triangle, the pentagon, and the hexagon and observe that these exercises all show that lines and surfaces under certain conditions appear less than their true dimensions, and that this diminution takes place as soon as the surfaces are turned away from the glass slate.

When the square rests against the slate, with the centers of the square and slate coinciding, and the slate held so that half is above and half below the horizon line, all four corners of the square will be at equal distances from the eye so that a line from the eye to the center of the slate and of the square is at right angles to the surface of the slate, the latter represents in these experiments what in scientific perspective is called the picture plane. Thus a surface or plane appears its true relative dimensions only when it is at right angles to the direction in which it is seen.

It is for this reason that it is always necessary to arrange the surface on which a drawing is made, at right angles to the eye, otherwise the surface and drawing upon it become foreshortened; that is, they appear less than their true dimensions.

It is easy to see from the drawing of the foreshortened square in Fig. 4, that of the two equal and parallel lines *a b* and *c d* the nearer appears the longer, although neither of the lines are foreshortened as the respective ends of each are equally distant from the eye. This illustrates the following rule :

Rule 2. Of two equal and parallel lines, the nearer appears the longer.

Exercise 2. The Horizontal Circle. Hold the circular tablet horizontally and at the level of the eye. Observe that it appears a straight line.

Place the tablet horizontally on a pile of books about half way between the level of the eye and the level of the table. Trace the appearance upon the slate.

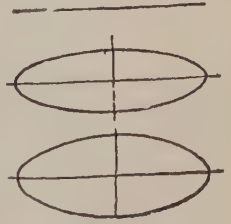


Fig. 6. Horizontal Circles.

Place the tablet on the table and trace its appearance.

While making both tracings the distance between the eye and the object, and the eye and the slate should be the same.

Hold the tablet at different heights above the level of the eye and observe that the ellipse widens as the height above the eye increases. These exercises illustrate the following rules:

Rule 3. A horizontal circle appears a horizontal straight line when it is at a level of the eye. When below or above this level the horizontal circle always appears an ellipse whose long axis is a horizontal line.

Rule 4. As the distance above or below the level of the eye increases the ellipse appears to widen. The short axis of any ellipse which represents a horizontal circle changes its length as the circle is raised or lowered. The long axis is always represented by practically the same length at whatever level the circle is seen.

Place the tablet on the table almost directly below the eye and trace its appearance.

Move it back to the farther edge of the table and trace it. It will be seen that where the level of the circle remains the same. its apparent width changes with the distance from the eye to the circle.

Exercise 3. Parallel Lines. Place the square tablet on the table $1\frac{1}{2}$ feet from the front, so that its nearest edge appears horizontal; that is, so that it is at right angles to the direction in which it is seen. By tracing the appearance the following rules are illustrated:

Fig. 7. Parallel Lines.

Rule 5. Parallel retreating edges appear to vanish, that is, to converge toward a point.

Rule 6. Parallel edges which are parallel to the slate, that is, at right angles to the direction at which they are seen, do not appear to converge, and any parallel edges whose ends are equally distant from the eye appear actually parallel.

Exercise 4. The Square. Place the square tablet as in Exercise 3, and it will be seen that two of the edges are not foreshortened but are represented by parallel horizontal lines. The others vanish at a point over the tablet on a level with the eye.

Now place the tablet so that its edges are not parallel to those

of the desk and trace its appearance on the slate. None of its edges appear horizontal, and when the lines of the tracing are continued as far as the slate will allow, the fact that they all converge will be readily seen; the drawing illustrates the following rule:

Rule 7. When one line of a right angle vanishes toward the right, the other line vanishes toward the left.

The drawing also shows that the edges appear of unequal length and make unequal angles with a horizontal line and illustrates the following rule:

Rule 8. When two sides of a square retreat at unequal angles, the one which is more nearly parallel to the picture plane (the slate) appears the longer and more nearly horizontal.

Exercise 5. The Appearance of Equal Spaces on Any Line. Cut from paper a square of three inches and draw its diagonals. Place this square horizontally in the middle of the back of the table, with its edges parallel to those of the table, and then trace its appearance and its diagonals upon the slate. (Fig. 8.)

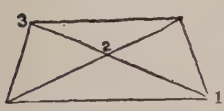


Fig. 8. Equal Space on any Line.

Note.—The diagonals of a square bisect each other and give the center of the square.

Compare the distance from the nearer end, 1, of either diagonal to the center of the square, 2, with that from the center of the square to the farther end of the diagonal, 3, for an illustration of the following rule:

Rule 9. Equal distances on any retreating line appear unequal, the nearer of any two appearing the longer.

Exercise 6. The Triangle. Draw upon an equilateral triangular tablet a line from an angle to the center of the opposite side. (This line is called an altitude.)

Connect the triangular tablet with the square tablet, and place them on the table so that the base of the triangle is foreshortened, and its altitude is vertical. Trace the triangle and its altitude upon the slate. The tracing illustrates the fact that the nearer half of a receding line appears longer than the farther half (see Rule 9), and also the following rule:

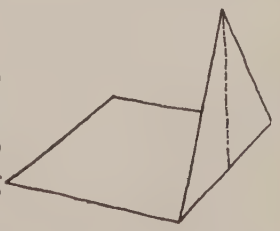


Fig. 9. The Triangle.

Rule 10. The upper angle of a vertical isosceles or equilateral triangle, whose base is horizontal, appears

in a vertical line erected at the perspective center of the base.

Exercise 7. The Prism. Connect two square tablets by a rod to represent a cube, and hold the object so that one tablet only is visible, and discover that it must appear its real shape, A, Fig. 10. This illustrates the following rule:

Rule 11. When one face only of a prism is visible, it appears its real shape.

Place the cube represented by tablets (Fig. 10) in the middle of the back of the desk, and trace its appearance. First, when two

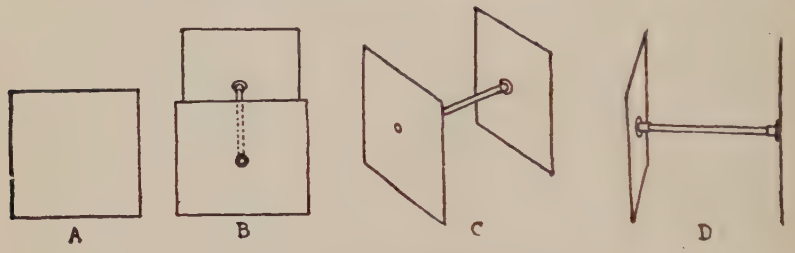


Fig. 10. The Prism.

faces only of the solid would be visible (B); and, second, when three faces would be seen (C). These tracings illustrate the following rule:

Rule 12. When two or more faces of a cube are seen, none of them can appear their real shapes.

Place the cubical form on the desk, with the tablets vertical, and one of them seen edgewise (D) and discover that the other tablet does not appear a straight line. This illustrates the following rule:

Rule 13. Only one end of a prism can appear a straight line at any one time.

Exercise 8. The Cylinder. Connect two circular tablets by a $2\frac{1}{2}$-inch stick, to represent the cylinder. Hold the object so that one end only is visible, and see that it appears a circle (Fig. 11).

Place the object on the table, so that its axis is horizontal but appears a vertical line, and trace its appearance. The tracing illustrates the following rule:

Rule 14. When an end and the curved surface of a cylinder are seen at the same time, the end must appear an ellipse (Fig. 12).

Place the object horizontally, and so that one end appears a vertical line, and trace to illustrate the following rule:

Fig. 11. The Cylinder—Circle.

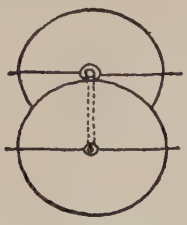


Fig. 12. The Cylinder—Axis Horizontal.

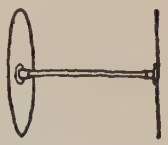


Fig. 13. The Cylinder—One End Straight Line.

Rule 15. When one end of a cylinder appears a straight line, the other appears an ellipse. (*Fig. 13.*)

Place the object upright on the table, and trace its ends and axis. Draw the long diameters of the ellipse, and discover that they are at right angles to the axis of the cylinder. This illustrates the following rule:

Rule 16. The bases of a vertical cylinder appear horizontal ellipses. The nearer base always appears the narrower ellipse. (*Fig. 14.*)

Place the object with its axis horizontal and at an angle, so that the surfaces of both tablets are visible. Trace the tablets and the rod, and then draw the long diameters of the ellipses, and discover that they are at right angles to the axis of the cylindrical form. The axes of the ellipses are inclined, and the drawing illustrates the following rules:

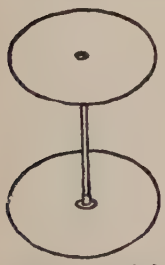


Fig. 14. The Cylinder—Upright.

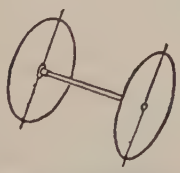


Fig. 15. The Cylinder—Axis Horizontal and at an Angle.

Rule 17. The bases of a cylinder appear ellipses, whose long diameters are at right angles to the axis of the cylinder, the nearer base appearing the narrower ellipse.

Note.—The farther end may appear narrower than the nearer, but must always appear proportionally a wider ellipse than the nearer end.

Rule 18. Vertical foreshortened circles below or above the level of the eye appear ellipses whose axes are not vertical lines.

Rule 19. The long axis of an ellipse representing a vertical circle below or above the level of the eye is at right angles to the axis of a cylinder of which the circle is an end.

Rule 20. The elements of the cylinder appear to converge in the direction of the invisible end. This convergence is not represented when the cylinder is vertical.

Note 1.—Less than half the curved surface of the cylinder is visible at any one time.

Note 2.—The elements of the cylinder appear tangent to the bases and must always be represented by straight lines tangent to the ellipses which represent the bases. When the elements converge, the tangent points are not in the long axes of the ellipses. See Fig. 12, in which if a straight line tangent to the ellipse be drawn, the tangent points will be found above the long axes of the ellipses.

Fig. 16. The Cone.

Exercise 9. The Cone. Hold the cone so that its axis is directed toward the eye, and the cone appears a circle. Hold the cone so that its base appears a straight line, and it appears a triangle. (Fig. 16.)

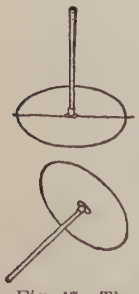


Fig. 17. The Cone—Tablet with Rod.

Place a circular tablet, Fig. 17, having a rod attached, to represent the axis of the cone, so that the axis is first vertical and second inclined. Trace both positions of the object, and discover that the appearance of the circle is the same as in the case of the cylinder. The tracings illustrate the following rule:

Rule 21. When the base of the cone appears an ellipse, the long axis of the ellipse is perpendicular to the axis of the cone.

Note 1.—More than half the curved surface of the cone will be seen when the vertex is nearer the eye than the base, and less than half will be seen when the base is nearer the eye than the vertex. The visible curved surface of the cone may range from all to none.

Note 2.—The contour elements of the cone are represented by straight lines tangent to the ellipse which represents the base, and the points of tangency are not in the long axis of this ellipse.

Exercise 10. The Regular Hexagon. In Fig. 18 the opposite sides are parallel and equal. The long diagonal A D is parallel to the sides B C and E F, and it is divided into four equal parts by the short diagonals B F and C E, and by the long diagonals B E or C F.

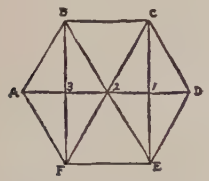


Fig. 18. Hexagon.

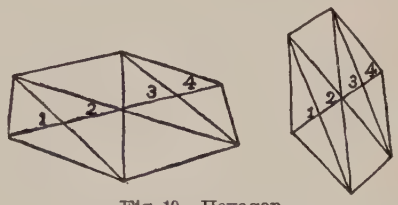


Fig. 19. Hexagon.

The perspective drawing of this figure will be corrected by giving the proper vanishing to the different sets of parallel lines, and by making the divisions on the diagonal A D perspectively equal.

Draw the long and short diagonals upon a large hexagonal tablet. Place this tablet in a horizontal or vertical position, Fig. 19, and then trace upon the slate its appearance and the lines upon it. The tracing illustrates the following rule:

Rule 22. In a correct drawing of the regular hexagon, any long diagonal when intersected by a long diagonal and two short diagonals, will be divided into four equal parts.

Exercise 11. The Center of the Ellipse Does Not Represent the Center of the Circle. Cut from paper a square of three inches, after having inscribed a circle in the square. Draw the diameters of the square and then place the square horizontally at the middle

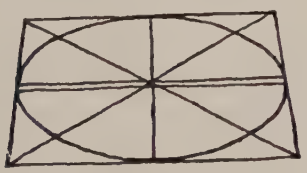


Fig. 20. Center of Circle not Center of Ellipse.

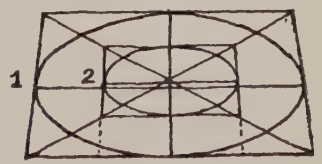


Fig. 21. Concentric Circles.

of the back of the table, with its edges parallel to those of the table. Trace the square, its diameters, and the inscribed circle, upon the slate. The circle appears an ellipse, and as the long axis of an ellipse bisects the short, it is evident that it must come below the

center of the square, and we discover that the center of the ellipse does not represent the center of the circle, and that the diameter of the circle appears shorter than a chord of the circle.

Exercise 12. Concentric Circles. Cut a 4-inch square from practice paper, and draw the diagonals. With the center of the square as center draw two concentric circles, 4 inches and 2 inches in diameter.

Place the card horizontally upon the table, as illustrated, and trace its appearance upon the slate, together with all the lines drawn upon it.

Draw the vertical line which is the short axis of both ellipses. Bisect the short axis of the outer ellipse, and draw the long axis of this ellipse. Bisect the short axis of the inner ellipse, and draw its long axis. It will be seen that the long axes are parallel but do not coincide, and that both are in front of the point which represents the center of the circles.

Each diameter of the larger circle is divided into four equal parts. The four equal spaces on the diameter which forms the short axis appear unequal, according to Rule 9. The diameter which is parallel to the long axes of the ellipses has four equal spaces upon it, and they appear equal. This diameter is behind the long axes, but generally a very short distance; and in practice, if the distance 1 2 between the ellipses measured on the long axis is one-fourth of the entire long axis, then the distance between the ellipses measured on the short axis must be a perspective fourth of the entire short axis. This illustrates the following rule:

Rule 23. Foreshortened concentric circles appear ellipses whose short axes coincide. The distance between the ellipses on the short axis is perspectively the same proportion of the entire short axis, as the distance between the ellipses measured on the long axis, is geometrically the same proportion of the entire long axis.

Exercise 13. Frames. In the frames are found regular concentric polygons with parallel sides, the angles of the inner polygons being in straight lines connecting the angles of the outer polygon with its center. In polygons having an even number of sides, the lines containing the angles of the polygons form diagonals of the figure, as in the square.

In polygons having an odd number of sides, the lines containing the angles of the polygon are perpendicular to the sides opposite the angles, as in the triangle.

Draw upon large triangular and square tablets the lines shown in Fig. 22. Place the tablets horizontally on the table, or support them vertically, and trace upon the slate the appearance

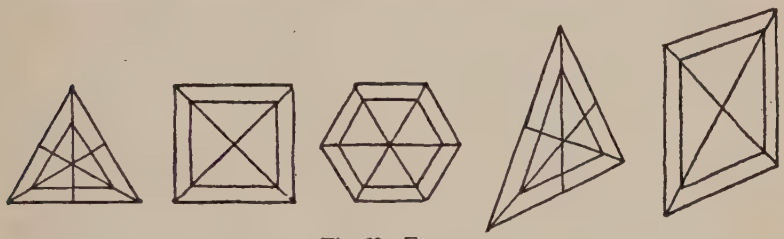


Fig. 22. Frames.

of the edges and all the lines drawn upon them. The tracings illustrate the following rule:

Rule 24. In representing the regular frames, the angles of the inner figure must be in straight lines passing from the angles of the outer figure to the center. These lines are altitudes or diagonals of the polygons.

20. After making the tracings described in the foregoing exercises, draw (not trace) freehand on the slate the various tablets, arranged to illustrate each one of the exercises. This is really drawing from objects, and where the rods are used to connect the tablets the figures are equivalent to geometric solids. After the proportions of the surfaces are correctly indicated, lines connecting the corresponding corners of the tablets should be drawn to complete the representation of solid figures. The lines indicating the rods and those lines which in a solid form would naturally be invisible, may be erased. By the use of the three rods of different lengths, three figures of similar character but different proportions may be obtained. These should each be drawn, but each in a different position.

The following directions, which are based on general principles, apply to all drawing whether from objects or from the flat, for work in pencil or in any other medium; drawing from another drawing, a photograph or a print, whether at the same size or larger, is called working from the flat.

21. General Directions for Drawing Objects. First observe carefully the whole mass of the object, its general proportions and the direction of lines as well as the width of the angles. Then sketch the outlines rapidly with very light lines, and take care that all corrections are made, not by erasing but by lightly drawing new lines as in Fig. 23. By working in this manner much time is saved and the drawing gains in freedom. Where the drawing is kept down to only one line which is corrected by erasure, the line becomes hard and wiry, and there is a tendency to be satisfied with something inaccurate rather than erase a line which has taken much time to produce. There is always a difficulty at first in drawing lines light enough, and it is well for the beginner to make the first trial lines with a rather hard pencil. Practice until the habit of sketching lines lightly is fixed. The ideal is to be able to set down exact proportions at the first touch. This, however, is attained by comparatively few artists, and only after long study, but the student will soon find himself able to obtain correct proportions with only a few corrections.

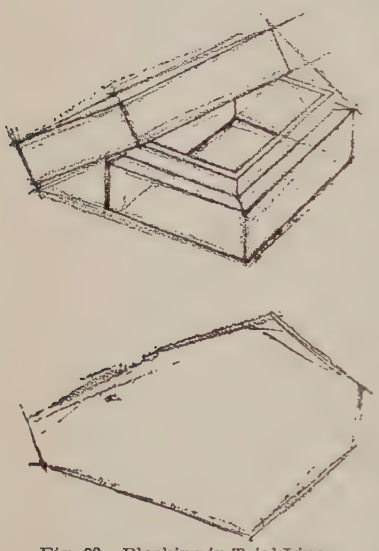


Fig. 23. Blocking in Trial Lines.

22. It cannot be too strongly emphasized that the student must teach himself to regard the subject he is depicting, as a whole, and to put down at once lines that suggest the outline of the whole. This he will find contrary to his inclination, which with the beginner is always to work out carefully one part of the drawing before suggesting the whole.

There are two objections to this; in the first place, much time having been spent on one part, it is almost inevitable that the addition of other portions reveals faults in the completed part, and unnecessary time is consumed in correcting. The second objection is that a drawing made piecemeal is sure to have a disjointed look, even if the details are fairly accurate in their relative proportions.

The idea of unity is lost and some one detail is apt to assume undue importance, instead of all details being subordinated to the general effect of the whole. It is always most important to state the general truths about the subject rather than small particular truths, which impair the general statement. This applies particularly to small variations in the outline which should be omitted until the big general direction or shape has been established.

23. Where an outline drawing is desired, after the correct lines have been found, they should be made stronger than the others and then all trial lines erased. In doing this the eraser will usually remove much of the sharpness of the correct lines so that only a faint indication of the desired result remains. These should be strengthened again with a softer pencil and each line produced, as far as possible, directly with one touch ; in the case of curves and very long lines, breaking the line and beginning a new one as near as possible to the end of the previous line, but taking care that the lines do not lap.

As soon as the student has acquired some proficiency in drawing the single figures made from the tablets, groups of two or three objects should be attempted. Combinations of books or boxes with simple shapes, or vases, tumblers, bowls and bottles will illustrate most of the principles involved in freehand perspective. Outline sketches may be made on the slate first and tested in the usual way, and afterward the same group may be drawn larger on paper. The chief difficulty in drawing a group is to obtain the relative proportions of the different objects. There is the same objection to completing one object and then another as there is to drawing a single object in parts. The whole group must be suggested at once. This can best be done by what is called blocking in, by lines which pass only through the principal points of the group. The block drawing gives hardly more than the relative height and width of the entire group and the general direction of its most important lines. But if these are correct, the subdivision of the area within into correct proportions is not difficult The longer and more important lines of the parts are indicated and short lines and details lost.

24. Testing Drawings by Measurement. In drawings which are not made on the slate the following method of testing propor-

tions is usual. With the arm stretched forward to its greatest length, hold the pencil upright so that its unsharpened end is at the top. Move it until this end coincides with the uppermost point of the object. Holding it fixed and resting the thumb against the pencil, move the thumb up and down until the thumb nail marks the lowest point of the object. The distance measured off on the pencil represents the upright dimension. Holding the pencil at exactly the same distance from the eye, turn it until it is horizontal and the end of the pencil covers the extreme left point of the object. Should the height and width be equal, the thumb nail would cover the extreme right edge of the object. If the width is greater than the height, use the height as a unit of measurement and discover the number of times it is contained in the width. Always use the shorter dimension as the unit of measurement. The accuracy of the test demands that the pencil should be at exactly the same distance from the eye while comparing the width and height. In order to insure this, the arm must not be bent at the elbow and must be stretched as far as possible without turning the body, which must not move during the operation. The distance from the eye to the object must not change during the test, and the position of the eye and body is first fixed by leaning the shoulders firmly against the back of the chair and keeping them in that position while the test is taking place. It is equally important in both the upright and horizontal measurement that the pencil be held exactly at right angles to the direction in which the object is seen; *i.e.*, at right angles to an imaginary line from the eye to the center of the object. In either position the two ends of the pencil will be equally distant from the eye. The test should be made several times in order to insure accuracy, as there is sure to be some slight variation in the distances each time. Avoid taking measurements of minor dimensions, as the shorter the distances measured the more inaccurate the test becomes. At the best measurements obtained in this way are only approximately correct, and too much care cannot be taken in order to render the test of use. Applied carelessly, the test is not only valueless, but thoroughly misleading. When there is any great conflict between the appearance of the object and the drawing after it has been corrected by the test, it is often safe to assume some mistake in applying the

test and to trust the eye. In such a case the test may be tested by the use of the slate. A few lines and points will be sufficient to indicate the width and height on the slate, and the relative proportions can then be calculated.

The plumb-line affords another method of testing. A thread or a string with any small object for a weight attached to one end, is sufficient. Hold the string so that it hangs vertical and motionless, and at the same time covers some important point in the object. By looking up and down the line the points directly over and under the given point can be determined and the relative distances of other important points to the right and left can be calculated. The plumb-line will also determine all the vertical lines in the object and help to determine divergence of lines from the vertical.

A ruler, a long rod, or pencil held in a perfectly horizontal position is also of assistance in determining the width of angles and divergences of lines from the horizontal.

25. Misuse of Tests. The use of tests may easily be perverted and become mischievous. Since the object of all drawing is to train the hand and eye, it follows naturally that the more the student relies upon tests the less will he depend upon his perceptions to set him right, and the less education will he be giving to his perceptions. There is no greater mistake for a student than to use the measuring test before making a drawing. Spend any amount of time in calculating relative proportions by the eye, but put these down and correct them by the eye, not once but many times before resorting to tests. All the real education in drawing takes place before the tests are made. Let the student remember that the tests may help him to make an accurate drawing, but they will never make him an accurate draftsman in the true sense. Nothing but training the eye to see and the hand to execute what the eye sees, will do that. When the student has reached the end of his knowledge, has corrected by the eye as far as he can, then by applying tests he is enabled to see how far his perceptions have been incorrect. That is the only educational value of the test. Merely to make an accurate drawing with as little mental effort as possible, relying upon test measurements, requires considerable practice and skill in making the tests, but gives very little practice or training in drawing.

26. Light and Shade. Objects in nature, as before explained, detach themselves from each other by their differences in color and in light and shade.

In drawing without color, artists have always allowed themselves a very wide range in the amount of light and shade employed, extending from drawing in pure outline up to the representation of exact light and shade, or of **true values**, as it is called.

Drawings which contain light and shade may be divided into two classes: Form drawing, which is from the point of view of the draftsman, and value drawing, which is from the point of view of the painter.

27. Form Drawing. In form drawing the chief aim, as the name implies, is to express form and not color and texture. In order to do this, shadows and cast shadows are indicated only as far as they help to express the shape. This is the kind of drawing practiced by most of the early Italian masters, and it has been called the Florentine method. It is often a matter of careful outline with just enough shadow included to give a correct general impression of the object. There is usually little variety in the shadow and no subtle graduations of tone, but the shadows are indicated with sufficient exactness of shape to describe the form clearly. Form drawing is a method of recording the principal facts of form with rapidity and ease and of necessity deals only with large general truths. Perhaps its most distinguishing feature is that it does not attempt to suggest the color of the form.

28. Value Drawing. The word value as it is used in drawing is a translation from the French word *valeur*, and as used by artists it refers to the relations of light and dark.

Value drawing represents objects exactly as we see them in nature; that is, not as outline, but as masses of lights and darks. In value drawing the artist reproduces with absolute truth the different degrees of light and shade. While form drawing suggests relief, value drawing represents it, and it also represents by translating them into their corresponding tones of gray, the values of color. In form drawing, a draftsman representing a red object and a yellow one, would be satisfied to give correct proportions and outlines with one or two principal shadows, while a value

drawing of the same objects would show not only the relation of the shadows as they are in nature, but also the further truth that the red object was as a whole darker than the yellow one. The light side of the red object might even be found to be darker in **value** or tone than the shadow side of the yellow form.

29. Values. Drawing has been called the science of art, but artists have rarely approved the introduction of scientific methods in the study of drawing, fearing lest the use of formulas should lead to dull mechanical results. Students are left to discover methods and formulas of their own. It is true that every successful draftsman or artist has a method which he has worked out for himself, but he usually feels it to be so much a matter of his own individuality, that he is reluctant to impose it on students, who are likely to confound what is a vital principle with a personal mannerism, and by imitation of the latter injure the quality of personal expression which is so important in all creative work. So there is an inclination among drawing teachers to distrust anything which tends even to formulate the principles of drawing. Recently there has been, however, a distinct advance in the study of these principles, under the leadership of Dr. Denman W. Ross, of Harvard University, who has made it possible for the first time to speak with exactness of colors and values. As Dr. Ross has permitted the use of his valuable scale in this text book, it will greatly assist in making tangible and clear, what would otherwise be obscure and difficult to explain.

The word **values** as used in the text book refers entirely to relations of light and dark. For instance, the value of a given color, is represented by a tone of gray which has the same density or degree of light and dark that the color has. The value of a spot of red paint on a white ground is expressed by a spot of gray paint which appears as dark on the white ground as does the red paint, but from which the color principle has been omitted. A good photograph of a colored picture gives the values of the picture. A poor photograph, on the contrary, distorts the values and blues are often found too light, while reds and yellows will be too dark to truthfully express the values of the original color.

30. The Value Scale. All possible values which can be represented in drawing, lie between the pure whites of paper or pig-

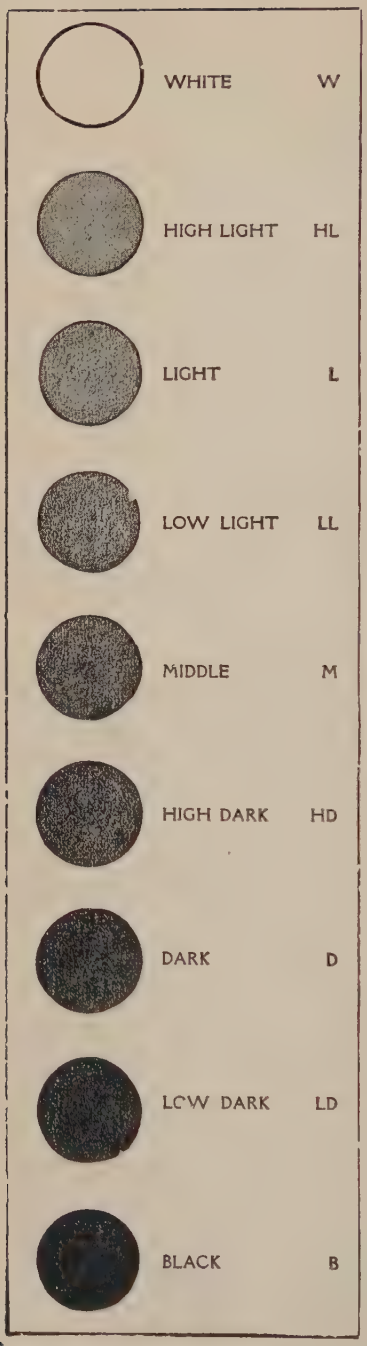


Fig. 24. Value Scale.

ments and the pure black of pencil, ink, or other pigments. In order to think or speak precisely of the great range of values between black and white, it is necessary that they shall be classified in some way. It is not sufficient to say that a given shadow is light, or medium, or dark in value. Dr. Ross has overcome the difficulty by arranging a value scale of nine equal intervals, which covers the whole range from pure white to pure black. Each interval has its appropriate designation and a convenient abbreviation. This scale affords a practical working basis for the study of values. It is evident that while the individual scale does not include all possible values, it can readily be enlarged indefinitely by introducing values between those of the scale as described. As a matter of fact, any differences in value that might come between any two intervals of the scale would rarely be represented, as it is the practice in drawing to simplify values as much as possible; that is to consider the general value of a mass, rather than to cut it up into a number of slightly varying tones which are not necessary for expressing anything of importance in the object.

31. How to Make a Value Scale. Fig. 24 shows a value scale with the names of the intervals and their abbreviations. In making a value scale the student should work in pencil, confining each interval within a circle three-quarters of an inch in diameter. White will be represented by the white paper with a circle penciled about it. Black (B) and white (W) should be established first, then the middle value (M), light (L) and dark (D); afterward the remaining values, low light (LL), high light (HL), low dark (LD) and high dark (HD).

32. How to Use a Value Scale. When the objects to be drawn are neutral in color, that is, are black, white, or gray, the relative values are perceived without special difficulty. When the objects are in color, the draftsman is obliged to translate the color element into terms of light and dark.

In order to determine the value of any surface, it is a help to compare the surface with a piece of white paper held in such a way that it receives the greatest amount of light. It not infrequently happens that two surfaces quite different in color will be of exactly the same value. The student should make a practice of observing the relative values of things about him, even when he is not engaged in drawing.

Place a sheet of white paper in the sunlight as it falls through a window and compare its value with that of white paper further in the room and outside of the sunlight. Try a similar experiment with black. These merely show what everyone may suppose that he knows already—that the less light a surface receives the darker value it appears to have. As a matter of fact, beginners are more ready to accept this truth with regard to color than they are when it relates to black and white.

An instructive way of studying values is to look through a closed window and compare the values of forms outside to the value of the window sash. Even when the sash is painted white, it will often be observed to appear darker than any shadow out of doors.

33. General Directions for Drawing the Examination Plates. The examination plates are planned to give as great a variety to the style of drawing as possible. The architect is called upon to use freehand drawing in two general ways; to make working drawings of ornament, either painted or carved, and to make,

for reference, sketches or notes, more or less elaborate, from ornament already in existence, or from buildings either entire or in part, as well as from their landscape setting. This course will not include drawing of architecture and landscape.

In making a working drawing of ornament every shape and curve should be drawn to perfection, with clean, careful lines, in order that there shall be no opportunity for the craftsman who executes the work to interpret it differently from the designer's intention. Light and shade are used sparingly as the exact amount of relief is indicated by sections.

In making sketches or notes, while proportions must be accurately studied, form may be suggested by a much freer quality of line. In a working drawing light and shade may be merely indicated or may be carried to any degree of elaboration. The natural way of teaching this kind of drawing is to work from the objects themselves or from casts. This is not possible in a correspondence course, but all the principles of sketching may be very well taught by drawing from photographs of ornament, and this method has some decided advantages of its own for a beginner. The light and shade in the photograph are fixed, while in sketching objects out of doors it changes constantly, and even indoors is subject to some fluctuation; and then, in the photograph the object is more isolated from its surroundings and so is less confusing to perceive.

In order to train the sense of proportion as thoroughly as possible, the plates are to be executed on a much larger scale than the examples, but at no fixed scale. Plan each drawing to be as large as possible, where no dimensions are given, but do not allow any point in the drawing to approach nearer than one inch to the border line.

34. Varieties of Shading. In drawing in pen and ink, all effects of shadow are made by lines, and different values are obtained by varying the width of lines, or of the spaces between the lines, or by both. In any case the integrity of each line must be preserved and there can be very little crossing or touching of shade lines, as that causes a black spot in the tone unless lines cross each other systematically and produce **cross hatching.** With the pencil, however, owing to its granular character one may produce a tone without any lines; a tone made up of lines which by touching or overlapping produce a soft, blended effect, in which the general

direction of the strokes is still visible, or a tone made up of pure lines as in pen work. In general it does not matter so much, as in pen drawing, if lines touch or overlap. Indeed, the natural character of the pencil line leads to a treatment which includes both pure lines and more or less blended effects.

35. Directions of Shade Lines. It is always a very important matter to decide what direction shade lines shall take. While it is impossible to give rules for it, a good general principle is to make the direction of the lines follow the contours of the form. The easiest and simplest method is to make all the lines upright. This method is a very popular one with architects. The objections to it are monotony and a lack of expression, but it is certainly a very safe method and far preferable to one where desire for variety has been carried too far and lines lead the eye in a great number of different directions which contradict the general lines of the surface or form. A natural treatment is to adapt the direction of lines to the character of the surface represented; that is, to treat curved surfaces with curving lines and flat planes with straight lines, and in general, lines may very well follow either the contours or the surfaces of the form. In that way variety is obtained and the direction of the shading helps to express the character of the thing represented. This principle must, however, be modified when it leads to the introduction of violently opposing sets of lines. Abrupt transitions must be avoided and the change from one direction to another must be accomplished gradually.

Where a large surface is to receive a tone, the tone can best be made by a series of rather short lines side by side with succeeding series juxtaposed. The lengths of the lines in each of the series must vary considerably in order that the breaks in the lines may not occur in even rows, producing lines of white through the tone. (See Fig. 25.)

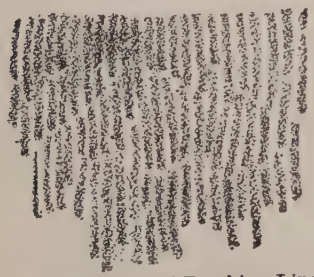


Fig. 25. Method of Breaking Lines Covering a Large Surface.

The crossing of one system of parallel lines by another system is called **cross hatching.** This method probably originated in copperplate engraving, to which it is very well adapted, especially as a means of modifying and deepening

tones. It also changes and breaks up the rather stringy texture produced by a succession of long parallel lines. It has now become somewhat obsolete as a general method for pen or pencil drawing, largely because the result looks labored, for it is always desirable to produce effects more simply and directly, that is, with one set of lines instead of two or more. If the tone made by one set of lines needs darkening, it is now more usual to go over the first tone with another set of lines in the same direction.

A great many drawings have been made with shade lines all in a diagonal direction, but this is open to serious objection and should be avoided. A diagonal line is always opposed to the principle of gravitation, and tends to render objects unstable and give them the appearance of tilting. It is often desirable to begin a tone with diagonal lines which, however, should gradually be made to swing into either an upright or horizontal direction.

FREEHAND DRAWING

PLATE I

The principal dimensions in inches are indicated on the model plate. All dimensions and proportions should however be determined by the eye alone. Measurements may be used as a test after the squares are laid in. The figures on the left should be executed first, in order to avoid rubbing by the hand and sleeve.

Figs. 1, 2, 3, 4, 5 are motives from Egyptian painted decoration. Figs. 1, 2, 4 and 5 are all derived from or suggested by patterns produced by plaiting or weaving. The borders of Fig. 3 are derived from bundles of reeds bound together.

As all the figures are large and simple, they should be executed with a rather wide line drawn with the F pencil. Draw the construction lines on this and on all other plates where they are necessary so lightly that they can be perfectly erased without leaving any indentation in the paper. After the construction lines are drawn out in Figs. 4 and 5, strengthen the lines of the pattern. In erasing, much of the pattern will be removed. This time go over each line with a single stroke of the solid ink pencil. Do not turn the paper in drawing diagonal and vertical lines. They are given especially to train the hand to execute such lines. By turning the paper the exercise becomes one of drawing horizontal lines, which are the least difficult.

Fig. 6 is the skeleton of a very common type of ornament, consisting of curved lines radiating from a point at the base on either side of a central axis.

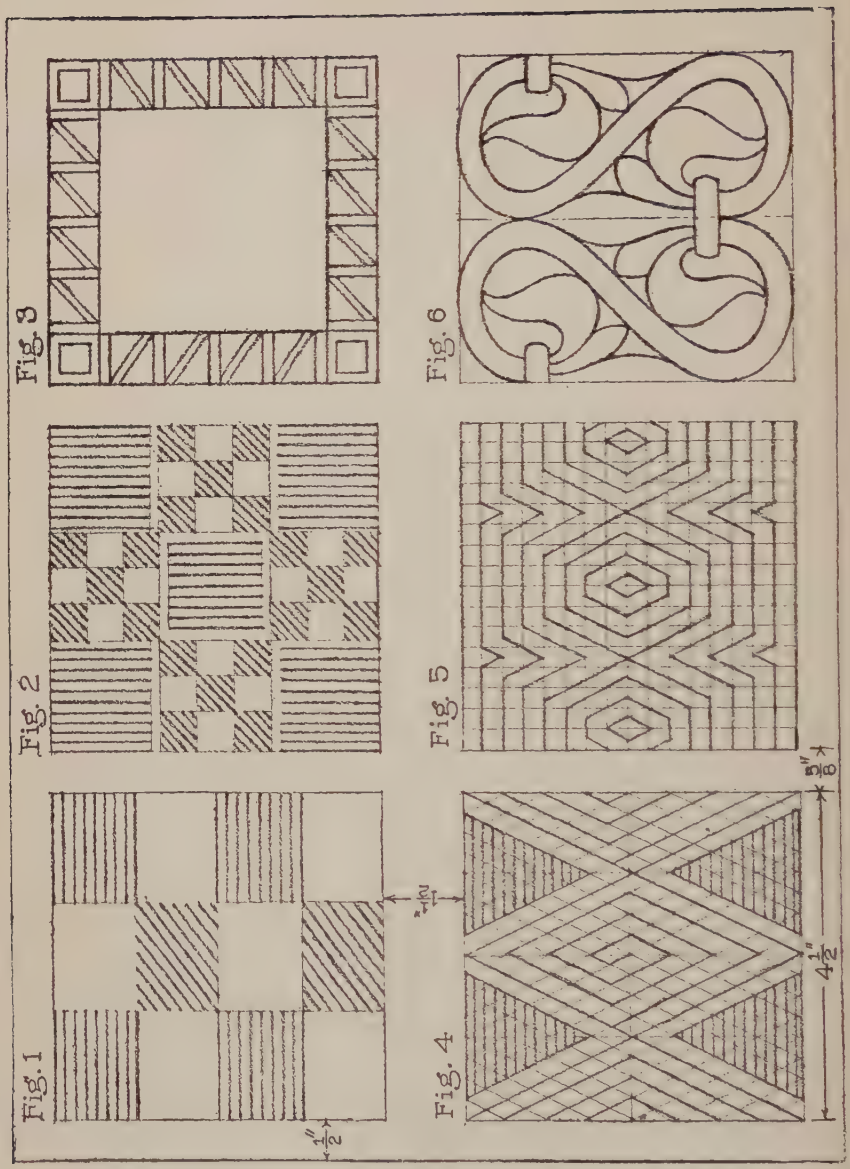


PLATE I.
Motives of Common Types of Ornament.

PLATE II.

Fig. 1 is the basis of a large class of ornament founded on the lines of organic growth, called scrolls or meanders.

Fig. 2 is an Egyptian border consisting of alternate flower and bud forms of the lotus, the most typical and universal of all the Egyptian decorative units. The outline of the flower displays the Egyptian feeling for subtlety and refinement of curve. Observe how the short rounded curve of the base passes into a long subtle curve which becomes almost straight and terminates in a short full turn at the end.

Fig. 3 is a simple form of the guilloche (pronounced geeyōsh), a motive which first becomes common in Assyrian decoration and is afterward incorporated into all the succeeding styles.

Fig. 4 is the skeleton of a border motive where the units are disposed on either side of the long axis of the border.

Figs. 5 and 7 are varieties of the Greek anthemion or honeysuckle pattern, one of the most subtle and perfect of all ornamental forms. Observe in Fig. 5 the quality of the curves—the contrast of full rounded parts with long curves almost straight which characterize the Egyptian lotus. Note in both examples that there is a regular ratio of increase both in the size of the lobes and in the spaces between each, from the lowest one up to the center. It is invariably the rule that each lobe shall be continued to the base without touching its neighbor.

Fig. 6 is an Egyptian "all-over" or repeating pattern painted on wall surfaces. It is made up of continuous circles filled with lotus forms and the intervening spaces with buds.

PLATE III.

This plate is to contain nine outline drawings illustrating *Rules* (not figures) 8, 12, 13, 14, 15, 17, 18, 19, 20. The drawings may be made and corrected on the slate and then copied on to the paper or they may be drawn directly on the paper. They may be from the models or from simple geometric objects such as boxes, blocks, cups, pans, plates, spools, flower pots, bottles, etc.

PLATE IV.*

These are characteristic forms of Greek vases. Fig. 1, the Lechy-

NOTE. This plate and all succeeding ones are to be surrounded by a border line, drawn freehand one inch from the edge of the paper.

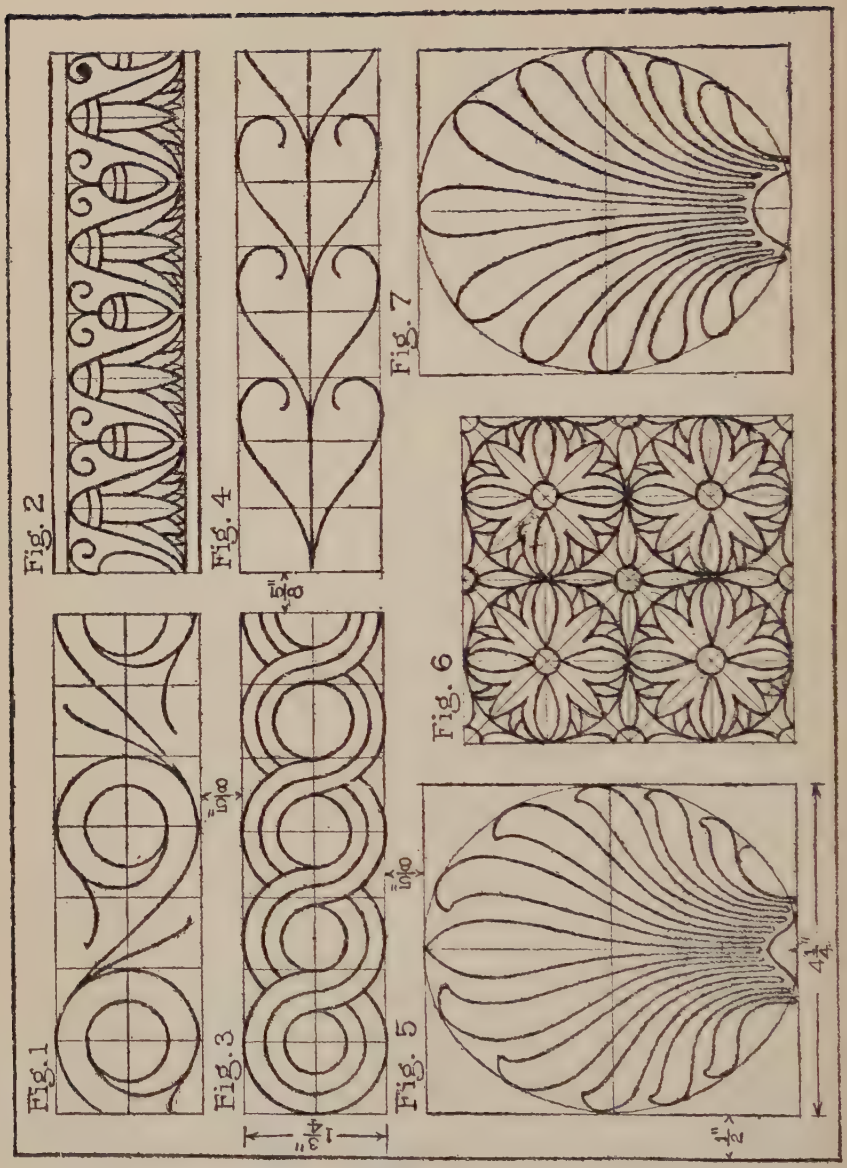


PLATE II.

Typical Egyptian, Assyrian and Greek Motives.

thos, was used to hold oil, Fig. 2, the Kantheros, is one form of the drinking cup, and Fig. 3, the Hydria, for pouring water.

The drawing of these vases includes a great variety of beautiful curves. They are to be executed entirely in outline, and both contours and bands of ornament and the relative sizes of each are to be preserved.

Calculate the heights so that the bases shall each be one inch from above the border line and the upper point of Fig. 3 about one inch below the border line. In sketching them in, first place a construction line to represent the central axis. Across this, sketch the outlines of the horizontal bands and then sketch the contours, following the general directions given in Sections 21 and 25. Remember that lines are to be drawn lightly and corrections made by new lines and not by erasures. Use the arm movement as much as possible in drawing the curves. Before executing the examination paper, practice drawing each vase entirely without corrections of the lines.

PLATE V.

Fig. 1 is from the pavement in the Baptistery at Florence and is in the style called Tuscan Romanesque. The pointed acanthus leaves in the small border at the top, are identical in character with the Byzantine acanthus.

This drawing is to be treated like a sketch made from the object. After sketching in the pattern and correcting in the usual way by drawing new lines, erase superfluous lines and strengthen the outlines by lines made with one stroke. The final outline should, however, be loose and free in character and express the somewhat roughened edges of the pattern in white. This does not mean that the direction of the line must vary enough to distort any shapes. Observe that most of the shapes appear to be perfectly symmetrical only their edges seem slightly softened and broken. Fill in the background with a tone equal to the dark (D) of the value scale. Make this tone by upright lines nearly touching each other and if the value is too light at first, go over them again by lines in the same direction. If a background line occasionally runs over the outline, it will help to produce the effect of the original.

Figs. 2, 3, 4 and 5 comprise typical forms of Greek decorated mouldings. The examples have much the character of a working

PLATE IV. FIG. 1
The Lechythos
Typical Greek Vase Used to Hold Oil

PLATE IV. FIG. 2
The Kantheros
Typical Greek Vase Used for a Drinking Cup

PLATE IV. FIG. 3
The Hydria
Typical Greek Vase Used for Pouring Water

PLATE V. FIG. 1
Pavement from the Baptistery, Florence

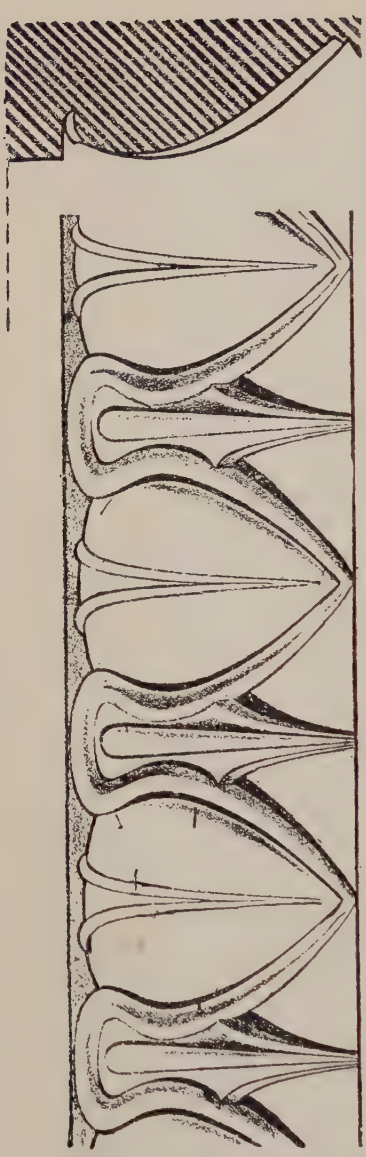


Fig. 2. Plate V Fig. 3.

Typical Forms of Greek Decorated Mouldings

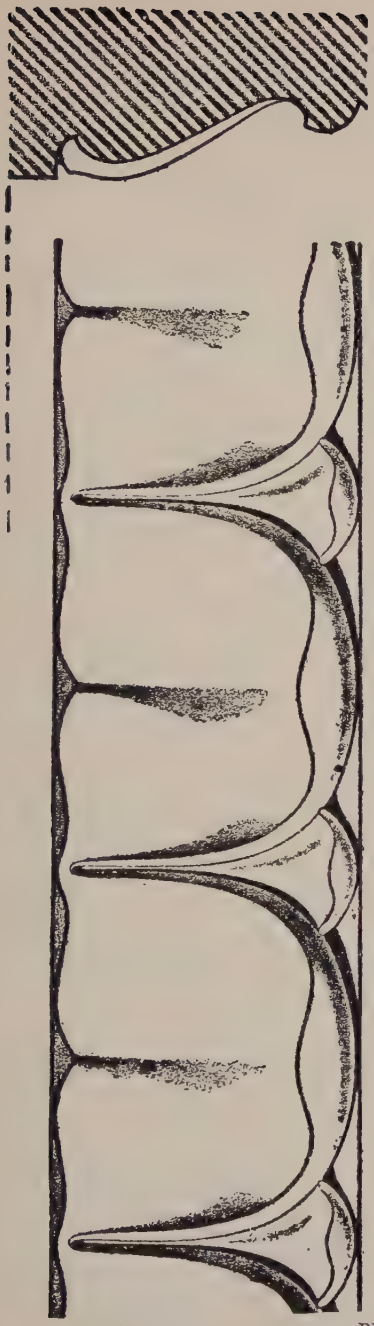


FIG. 4

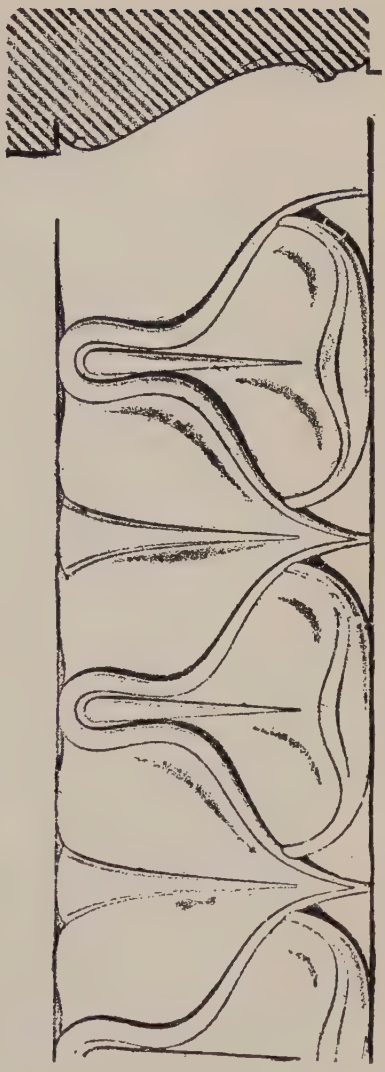


FIG. 5

PLATE V

Typical Forms of Greek Decorated Mouldings

drawing and the plates are to be enlarged copies, but instead of following the character of the light and shade of the original, the shadows are to be executed by upright lines. (See Section 35.) The darker shadows are to be the value of dark (D) of the scale, the lighter ones the value of middle (M).

PLATE VI.

Place these drawings so that there will be at least an inch between them and about half an inch between the border line and the top and bottom.

Fig. 1 is from a drawing of a wrought iron grille in a church in Prague. Some idea of the shape of the pieces of iron is conveyed by the occasional lines of shading. The pattern will be seen to be disposed on radii dividing the circle into sixths. Construct the skeleton of the pattern shown, establishing first an equilateral triangle and the lines which subdivide its angles and sides. About this draw the inner line of the circle and extend the lines which subdivide the angles of the triangles, to form the six radii of the circle. Complete the outlines of the pattern before drawing the shading lines. This drawing with its lines and curves all carefully perfected represents the kind of working drawing which an architect might give to an ironsmith to work with, although in a working drawing, a section of the iron would be given and each motive of the design would propably be drawn out only once and then as it was repeated it would be merely indicated by a line or two sketched in.

Fig. 2 is from a photograph of a wrought iron grille at Lucca in the style of the Italian Renaissance. The drawing to be made from this, the student must consider to be a sketch, the sort of note or memorandum he might make were he before the original.

The accompanying detail gives a suggestion of the proper treatment. The general shape of the whole outline should be indicated and the larger geometric subdivisions; the details of two of the compartments suggested by light lines and those of the remainder either omitted or very slightly suggested. Try to make the drawing suggest the "hammered" quality of the iron. Although the curves are all beautifully felt, there are slight variations in them produced by the hammer, or they are bent out of shape by time, and the thickness of the iron varies sometimes by intention and sometimes by acci-

PLATE VI. FIG. 1.
Wrought Iron Grille, Prague.

PLATE VI. FIG. 2.
Wrought Iron Grille, Lucca.

dent. Take care, however, not to exaggerate the freedom of the lines and do not carry the variation so far that curves are distorted.

Detail of Plate VI, Fig. 2.

Make the drawing in outline first with a line which breaks occasionally, with portions of the line omitted. This helps to indicate the texture of the iron and suggests its free hand-made character.

That part of the background which in the photograph appears black behind the iron, should be filled in with a tone equal to the dark (D) of the value scale. It should only be placed behind the two compartments which are most carefully drawn, with perhaps an irregular patch of it in the adjoining compartment. In making the background use single pencil strokes, side by side, with the solid ink pencil, very near together or occasionally touching. Give a slight curve to each stroke. The direction of the lines may be either upright, or they may keep the leading direction of the general lines of the pattern, but they should not be stiff or mechanical. If the value is not dark enough another set of lines may be made over the first ones, keeping the same direction. The only parts of the ironwork itself which require shading are those twisted pieces which mark the subdivision, the outer edge, and the clasp. For this use a tone equal to the middle (M) of the value scale. Avoid explaining too carefully the twists and use the shading only in the dark side. Use a few broken outlines on the right side, just enough to suggest it and do not darken the flat piece of iron behind the twists except on the shadow side. Do not count the number of twists but indicate them in their proper size and the effect will be near enough for this kind of a drawing. Shade only those twists which are nearest the compartments which are detailed; from them let the detail gradually die away.

PLATE VII.

This figure is a rosette made up of the Roman or soft acanthus,

and the drawing has the general character of a working drawing. Every part is very clearly expressed in outline, slightly shadowed, and a section explains the exact contours. In drawing the outline of the leaflets, observe that one edge, usually the upper is generally expressed by a simple curve and the other edge by a compound curve, the variation in which, however, is slight. Draw a circle first to contain the outer edge of the rosette and sketch in lightly the main rib or central axis of each leaf. Then block in the general form of the leaves, not showing the subdivisions at edges. Next place the *eyes*—the small elliptical spots which separate one lobe from another—and draw the main ribs of each lobe, finally detailing the leaflets in each lobe. In shading use the value dark (D) for the darkest values and the middle value (M) for the others, and instead of producing a perfectly blended tone as in the original, let the tone retain some suggestion of lines, the general direction of which should follow that of the main ribs in the leaves. In the shadow of the rosette on the background, let the lines be upright. Lines naturally show less in very dark values than in lighter tones, for it is difficult to produce the darker values without going over the lines with another set and that has a tendency to blend all the lines into a general tone.

PLATE VIII.

Plate VIII is a sculptured frieze ornament introducing various forms of the Roman or soft acanthus. In this as in all scroll drawing, the skeleton of the pattern should be carefully drawn, then the leaves and rosettes disposed upon it. Always draw the big general form of the acanthus, and proceed gradually to the details as described in the directions for Plate VII. This like Plate VII, has the general character of a working drawing, only in this case there is no section. Use the same values and same suggestions for directions of line as in Plate VII.

PLATE IX.

This plate is an example of the Byzantine acanthus on a fragment in the Capitoline Museum. In drawing this, place the central axis or main rib of the leaf first, then establish the position of the eyes—the egg-shaped cuts which separate the lobes. The general contour of the lobes and their main ribs should next be blocked in before the final disposition of the points or leaflets is determined.

Section through Center

PLATE VII

Acanthus Rosette

PLATE VIII.
Acanthus Frieze.

PLATE IX

This is also to be drawn as Fig. 1, Plate X

Byzantine Acanthus, from a fragment in the Capitoline Museum

The drawing of this plate is to be enlarged to about ten inches in height and well placed on the sheet with the center of the drawing coinciding with the center of the plate. This drawing is to be made by the use of two values only, with white, and the student may select his own values. The object is to select the most important features and to omit as much as possible. It would be well for the student to first try to see how much he can express with one value and white. The values are to be obtained by upright lines. Outlines are to be omitted as far as possible in the finished sketch and forms are to be expressed by the shapes of the masses of shadow. Where only two values and white are to be used, it is desirable to leave as much white as possible and not allow the shadow values to run too near to black as that produces too harsh a contrast with the white. On the other hand, if the shadow values are too high in the scale, that is too near white, the drawing becomes weak and washed out in effect. As this drawing is to be large in scale, it should be made with the solid ink pencil and with wide pencil strokes. After the outline has been sketched in, the shading or "rendering" may be studied, first on tracing paper over the drawing. There should be no attempt at rendering the background in this drawing.

PLATE X.

Figs. 1 and 2 are to be placed on this plate, but Fig. 1 is to be rendered this time as near to the **true values** as it is possible to go by using four values and white in shading The pencil lines should be blended together somewhat, but the general direction of the shading should follow the central axis of the lobes. Only the leaf itself is to be drawn and the background value should be allowed to break in an irregular line about the leaf. It should not be carried out to an edge which would represent the shape of the entire fragment of stone on which the leaf is carved. In studying the shapes of the different shadows it is well at first to exaggerate somewhat and give each value a clean, definite shape even if the edges appear somewhat indefinite in the original. At the last those edges which are blurred may be blended together.

Fig. 2 is a Byzantine capital from the church of San Vitale, at Ravenna. This is to be drawn so that the lines of the column shall fade off gradually into nothing and end in a broken edge instead of

stopping on a horizontal line as in the original. The top of the drawing above the great cushion which rests on the capital proper should also fade off into nothing and with a broken line instead of the horizontal straight line. A small broken area of the background value should be placed either side of the capital. In drawing an object like this which is full of small detail there is danger of losing the larger qualities of solidity and roundness by insisting too much upon the small parts and there is also danger of making the drawing too spotty. It is a good principle to decide at first that the detail is to be expressed either in the shadow or in the light, but not equally in both. This principle is based on one of the facts of vision, for in looking at an object one sees only a comparatively small amount of detail; what falls on either side of the spot on which the eye is focused appears blurred and indistinct. In an object of this kind whose section is circular, one can best express the shape by concentrating the study of detail at the point where the light leaves off and shadow begins, representing less and less detail as the object turns away from the spectator. In this drawing, however, there may be more detail expressed in the shadow than in the light, but remember that outlines of objects in shadow lose their sharpness and become softened. Do not attempt to show all the grooves in the parts in shadow; indicate one or two principal ones and indicate more and more detail as the leaves approach the point where the light begins. There the richness of detail may be fully represented, but as the forms pass into the light, omit more and more detail. Again observe that any small plane of shadow surrounded by intense light, if examined in detail, appears darker by contrast, but if represented as dark as it appears it becomes **spotty** and **out of value.** If observed in relation to the whole object its real value will be seen to be lighter than it appears when examined by itself. Use white and four values to be determined by the student. Guard against too strong contrasts of values within the shadow as it cuts it up and destroys its unity, and in every drawing made, show clearly just which is the shadow side and which is the light. That is, do not place so many shadow values within the light that it destroys it, and do not invade shadows with too many lights and reflected lights. Note that it is characteristic of the Byzantine acanthus to have the points of every tine or lobe touch something; no points are left free, but observe also that the points have some sub-

stance and width at the place they touch and must not be represented by a mere thread of light. It would be a mistake to introduce much variety of direction in the lines in this drawing, especially in the shadows, as it would "break it up" too much. The concave line of the contour of the capital may well determine the dominant direction of the lines which should not be very distinct as lines, but should blend considerably into general tones. Wherever a plane of shadow stops with a clean sharp edge the drawing must correspond, for its interest and expressiveness depend upon its power to suggest differences in surface—those surfaces which flow gradually into one another as well as those in which the transitions are sharp and abrupt.

The student should be very scrupulous about using only the values of the scale, and in the lower left corner of each sheet he should place within half-inch squares examples of each value used on the drawing with its name and symbol indicated.

PLATE XI.

This capital, of the Roman Corinthian order, is in the Museum of the Baths of Diocletian in Rome.

The foliated portions consist of olive acanthus, and the student should carefully study the differences between this and the soft acanthus. It will be noted that the greatest difference is in the subdivision of the edges into leaflets. In the soft acanthus there is always a strong contrast of large and small leaflets and the lobes overlap each other, producing a full rich effect and the general appearance is more like that of a natural leaf. In the olive acanthus the leaflets in one lobe differ slightly from each other in size, are narrower, and bounded by simple curves on either side, where the leaflet of the soft acanthus has the compound curve on one side.

The student may use as many values as he thinks necessary, but he should be conscientious in keeping his values in their scale relations and should place an example of each value used, with its name in one corner of the drawing.

To make a satisfactory drawing of a form so full of intricate detail as this is difficult, as there is a great temptation to put in all one sees. The general instructions for drawing Plate VII are equally applicable here. The student should remember that a drawing is an explanation, but an explanation which can take much for granted.

PLATE X. FIG. 2.
Byzantine Capital, from the Church of San Vitale Ravenna.

PLATE XI
Roman Corinthian Capital, from the Baths of Diocletian

PLATE XII
Italian Renaissance Pilaster

For instance, if the carved ornament on the mouldings or at the top of the capital are expressed where they receive full light, they must become more and more vague suggestions and finally disappear in the strong shadows; so the division line between the two mouldings of the abacus may be omitted in shadow and the mind will fill in what the eye does not see. One could go farther and express the detail only for a short space, letting it gradually die away into light or be merely indicated by a line or two, and still the explanation would be sufficient and far less fatiguing to the eye than literal insistence on every detail for the entire length. It is an excellent plan to look at the original, whether a photograph or the real object, with half closed eyes. This helps decidedly to separate the light masses from the darks and shows how much that is in shadow may be omitted.

The smaller lobes on the olive acanthus have no main ribs and lines are carried from the intersection of each leaf toward the base, the section of the leaflet being concave. The section of the leaflets on the soft acanthus is more V-shaped.

PLATE XII.

This is a portion of a pilaster decoration in the Italian Renaissance style. The acanthus is of the soft Roman type, but much more thin and delicate with the eyes cut back almost to the main ribs and a space cut out between each lobe so there is rarely any overlapping of lobes. Lay out construction lines for the scrolls, block in all forms correctly, detailing little by little, so carrying the whole drawing along to the same degree of finish

PERSPECTIVE DRAWING

PART I

DEFINITIONS AND GENERAL THEORY.

1. When any object in space is being viewed, rays of light are reflected from all points of its visible surface, and enter the eye of the observer. These rays of light are called **visual rays**. They strike upon the sensitive membrane, called the retina, of the eye, and form an image. It is from this image that the observer receives his impression of the appearance of the object at which he is looking.

2. In Fig. 1, let the triangular card *abc* represent any object in space. The image of it on the retina of the observer's eye will be formed by the visual rays reflected from its surface. These rays form a pyramid or cone which has the observer's eye for its apex, and the object in space for its base.

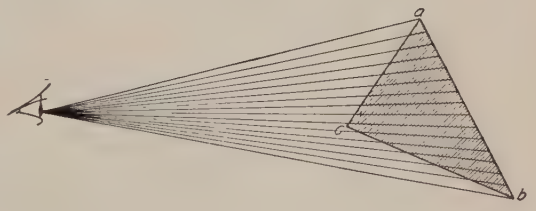


Fig. 1. Pyramid of Rays from Object to Eye

3. If a transparent plane M, Fig. 2, be placed in such a position that it will intersect the cone of visual rays as shown, the intersection will be a **projection** of the object upon the plane M. It will be noticed that the projecting lines, or **projectors**, instead of being perpendicular to the plane, as is the case in orthographic projection,* are the visual rays which all converge to a single point coincident with the observer's eye.

4. Every point or line in the projection on the plane M will appear to the observer exactly to cover the corresponding point or line in the object. Thus the observer sees the point a^P in the projection, apparently just coincident with the point a in the object. This must evidently be so, for both the points a^P and a lie on the same visual ray. In the same way the line $a^P b^P$ in the projection must appear to the observer to exactly cover the line ab in the object; and the projection, as a whole, must present to him exactly the same appearance as the object in space.

5. If the projection is supposed to be permanently fixed upon the plane, the object in space may be removed without affecting the image on the retina of the observer's eye, since the visual rays which were originally reflected from the surface of the object are now reflected from the projection on the plane M. In other words, this projection may be used as a substitute for the object in space, and when placed in proper relation to the eye of the observer, will convey to him an impression exactly similar to that which would be produced were he looking at the real object.

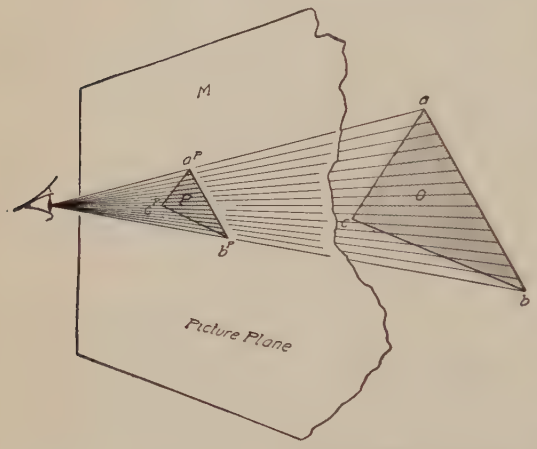


Fig. 2. Diagram Showing Projection of Object on Plane

6. A projection such as that just described is known as a **perspective projection** of the object which it represents. The plane on which the perspective projection is made is called the **Picture Plane.** The position of the observer's eye is called the **Station Point,** or **Point of Sight.**

7. It will be seen that the perspective projection of any point in the object, is where the visual ray, through that point, pierces the picture plane.

8. A perspective projection may be defined as the represen-

tation, upon a plane surface, of the *appearance* of objects as seen from some given point of view.

9. Before beginning the study of the construction of the perspective projection, some consideration should be given to phenomena of perspective. One of the most important of these phenomena, and one which is the keynote to the whole science of perspective, has been noticed by everyone. It is the *apparent* diminution in the size of an object as the distance between the object and the eye increases. A railroad train moving over a long, straight track, furnishes a familiar example of this. As the train moves farther and farther away, its dimensions apparently become smaller and smaller, the details grow more and more indistinct, until the whole train appears like a black line crawling over the ground. It will be noticed also, that the speed of the train seems to diminish as it moves away, for the equal distances over which it will travel in a given time, seem less and less as they are taken farther and farther from the eye.

10. In the same way, if several objects having the same dimensions are situated at different distances from the eye, the nearest one appears to be the largest, and the others appear to be smaller and smaller as they are farther and farther away. Take, for illustration, a long, straight row of street-lamps. As one looks along the row, each succeeding lamp is apparently shorter and smaller than the one before. The reason for this can easily be explained. In estimating the size of any object, one most naturally compares it with some other object as a standard or unit. Now, as the observer compares the lamp-posts, one with another, the result will be something as follows (see Fig. 3). If he is looking at the top of No. 1, along the line *ba*, the top of No. 2 is invisible. It is apparently below the top of No. 1, tor, in order to see No. 2, he has to lower his eye until he is looking in

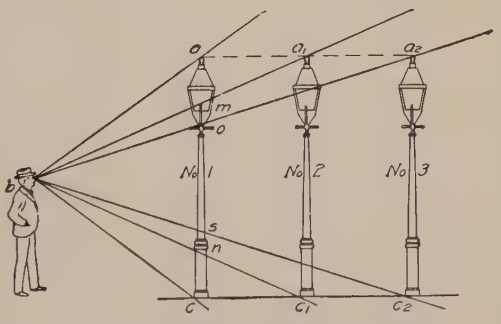


Fig. 3. Diagram Showing Visual Angle

the direction ba_1. He now sees the top of No. 2, but the top of No. 1 seems some distance above, and he naturally concludes that No. 2 appears shorter than No. 1. As the observer looks at the top of No. 2, No. 3 is still invisible, and, in order to see it, he has to lower his eye still farther. Comparing the bottoms of the posts, he finds the same apparent diminution in size as the distance of the posts from his eye increases. The length of the second post appears only equal to the distance *mn* as measured on the first post, while the length of the third post appears only equal to the distance *os* as measured on post No. 1.

11. In the same way that the lamp-posts appear to diminish in size as they recede from the eye, the parallel lines (a, a_1, a_2, etc., and c, c_1, c_2, etc.) which run along the tops and bottoms of the posts appear to converge as they recede, for the distance between these lines seems less and less as it is taken farther and farther away. At infinity the distance between the lines becomes zero, and the lines appear to meet in a single point. This point is called the **vanishing point** of the lines.

12. If any object, say a cube, Fig. 4, is studied, it will be seen that the lines which form its edges may be separated into groups according to their different directions; all lines having the same direction forming one group, and apparently converging to a common vanishing point. Each group of parallel lines is called a **system**, and each line an **element** of the system. For example, in Fig. 4, A, A_1, A_2, and A_3 belong to one group or system; B, B_1, B_2, and B_3, to another; and C, C_1, C_2, and C_3, to a third. Each system has its own vanishing point, towards which all the elements of that system appear to converge. This phenomenon is well illustrated in the parallel lines of a railroad track, or by the horizontal lines which form the courses of a stone wall.

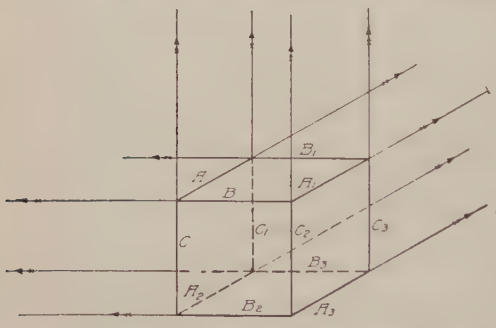


Fig. 4. Diagram Showing Construction Lines for Cube

13. As all lines which belong to the same system appear to meet at the vanishing point of their system, it follows that *if the eye is placed so as to look directly along any line of a system, that line will be seen endwise, and appear as a point exactly covering the vanishing point of the system to which it belongs.*

If, for illustration, the eye glances directly along one of the horizontal lines formed by the courses of a stone wall, this line will be seen as a point, and all the other horizontal lines in the wall will apparently converge towards the point. In other words, the line along which the eye is looking appears to cover the vanishing point of the system to which it belongs. Thus, the vanish-

Fig. 5. Sketch Showing Method of Finding Vanishing Point for Construction Lines of an Object

ing point of any system of lines must lie on that element of the system which enters the observer's eye, and must be at an infinite distance from the observer. Therefore, to find the vanishing point of any system of lines, imagine one of its elements to enter the observer's eye. This element is called the **visual element** of the system, and may often be a purely imaginary line indicating simply the direction in which the vanishing point lies. The vanishing point will always be found on this visual element and at an infinite distance from the observer.

14. To further illustrate this point, suppose an observer to be viewing the objects in space represented in Fig. 5. He desires

to find the vanishing point for the system of lines parallel to the oblique line *ab* which forms one edge of the roof plane *abcd*. There are two lines in the roof that belong to this system, namely: *ab* and *dc* If he imagines an element of the system to enter his eye, and looks directly along this element, he will be looking in a direction exactly parallel to the line *ab*, and he will be looking directly at the vanishing point of the system (§ 13). This visual element along which he is looking is a purely imaginary line parallel to *ab* and *dc*. All lines in the object belonging to this system will appear to converge towards a point situated on the line along which he is looking, and at an infinite distance from him.

This phenomenon is of great importance, and is the foundation of most of the operations in making a perspective drawing.

15. The word "vanish" as used in perspective always implies a recession. Thus, a line that vanishes upward, slopes upward as it *recedes* from the observer; a line that vanishes to the right, slopes to the right as it *recedes* from the observer.

16. It follows from paragraphs 13 and 14 that any system of lines that vanishes upward, will have its vanishing point above the observer's eye. Similarly, any system vanishing downward, will have its vanishing point below the observer's eye; any system vanishing to the right, will have its vanishing point to the right of the observer's eye; and any system vanishing to the left, will have its vanishing point to the left of the observer's eye. Any system of horizontal lines will have its vanishing point on a level with the observer's eye, and a system of vertical lines will have its vanishing point vertically in line with the observer's eye.

17. All planes that are parallel to one another are said to belong to the same system, each plane being called an element of the system.

All the planes of one system appear to approach one another as they recede from the eye, and to meet at infinity in a single straight line called the **vanishing trace** of the system. Thus, the upper and lower faces of a cube seen in space, will appear to converge toward a straight line at infinity.

18. If the eye is so placed as to look directly along one of

the planes of a system, that plane will be seen edgewise, and will appear as a single straight line exactly covering the vanishing trace of the system to which it belongs. The plane of any system that passes through the observer's eye is called the **visual plane** of that system.

19. From § 18, it follows that the vanishing trace of a system of planes that vanishes upward, will be found above the level of the eye, while the vanishing trace of a system of planes vanishing downward, will be found below the level of the eye. The vanishing trace of a system of vertical planes will be a vertical line; and of a system of horizontal planes, a horizontal line, exactly on a level with the observer's eye.

20. The vanishing trace of the system of horizontal planes is called the **horizon.**

The visual plane of the horizontal system is called the **plane of the horizon.** The plane of the horizon is a most important one in the construction of a perspective projection.

21. From the foregoing discussion the truth of the following statements will be evident. They may be called the Five Axioms of Perspective.

(*a*) *All the lines of one system appear to converge and to meet at an infinite distance from the observer's eye, in a single point called the vanishing point of the system.*

(*b*) *All the planes of one system appear to converge as they recede from the eye, and to meet at an infinite distance from the observer, in a single straight line called the vanishing trace of the system.*

(*c*) *Any line lying in a plane will have its vanishing point somewhere in the vanishing trace of the plane in which it lies.*

(*d*) *The vanishing trace of any plane must pass through the vanishing points of all lines that lie in it. Thus, since the vanishing trace of a plane is a straight line* (§ 18), *the vanishing points of any two lines lying in a plane will determine the vanishing trace of the system to which the plane belongs.*

(*e*) *As the intersection of two planes is a line lying in both, the vanishing point of this intersection must lie in the vanishing traces of both planes, and hence, at the point where the vanishing traces of the two planes cross. In other words, the vanishing point*

of the intersection of two planes must lie at the intersection of the vanishing traces of the two planes.

22. The five axioms in the last paragraph are the statements of purely imaginary conditions which appear to exist, but in reality do not. Thus, parallel lines appear to converge and to meet at a point at infinity, but in reality they are exactly the same distance apart throughout their length. Parallel planes appear to converge as they recede, but this is a purely apparent condition, and not a reality; the real distance between the planes does not change.

23. The perspective projection represents by real conditions the purely imaginary conditions that appear to exist in space.

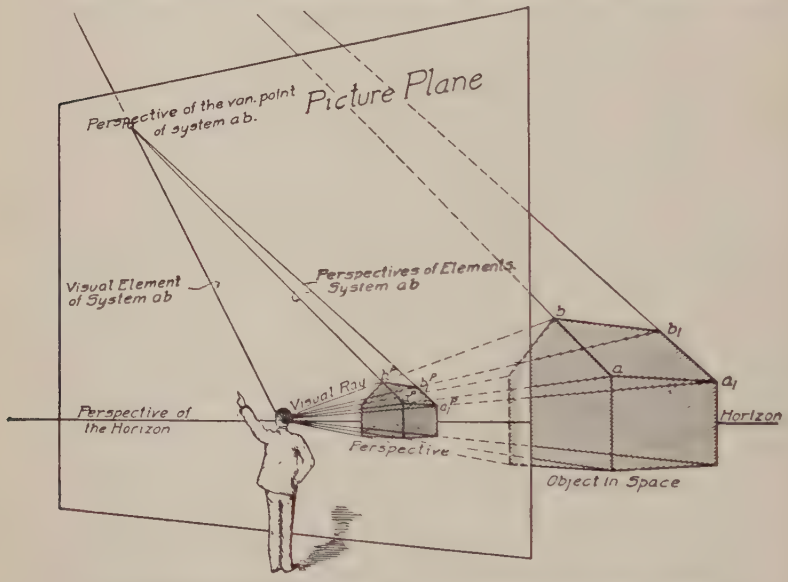


Fig. 6. Diagram Showing Method of Finding Intersection of Visual Element with Picture Plane

Thus, the apparent convergence of lines in space is represented by a real convergence in the perspective projection. Again, the vanishing point of a system of lines is a purely imaginary point which does not exist. But this imaginary point is represented in perspective projection by a real point on the picture plane.

From § 14, the vanishing point of any system of lines lies upon the visual element of that system. This visual element may be considered to be the visual ray which projects the vanish-

ing point to the observer's eye. Hence, from § 7, the intersection of this visual element with the picture plane will be the perspective of the vanishing point of the system to which it belongs. This is illustrated in Fig. 6. The object in space is shown on the right of the figure. If the observer wishes to find the vanishing point of the oblique line ab in the object in space, he imagines a line parallel to ab to enter his eye, and looks along this line (§ 13). Where this line along which he is looking pierces the picture plane, will be the perspective of the vanishing point. Furthermore, the perspective of the line ab has been found by drawing the visual rays from a and b respectively, and finding where these rays pierce the picture plane (§ 7). These points are respectively, a^P and b^P, and the straight line drawn between a^P and b^P is the perspective of the line ab. The perspective of the line a_1b_1 which is parallel to ab, has been found in a similar way, and it will be noticed that its perspective projection (a_1^P b_1^P) actually converges towards a^Pb^P in such a manner that if these two lines are produced they will actually meet at the perspective of the vanishing point of their system.

NOTE.— *It is evident that the perspective of a straight line will always be a straight line, the extreme points of which are the perspectives of the extremities of the given line.*

24. Thus, the five axioms of perspective may be applied to Perspective Projection as follows:—

(*a*) Parallel lines do converge and meet at the vanishing point of their system.

(*b*) Parallel planes do converge and meet at the vanishing trace of their system.

(*c*) The vanishing point of any line lying in a plane will be found in the vanishing trace of the plane.

Therefore, the vanishing points of all horizontal lines will be found in the horizon (§ 20).

(*d*) The vanishing trace of any plane will be determined by the vanishing points of any two lines that lie in it, and must contain the vanishing points of all lines that lie in it.

(*e*) The vanishing point of the intersection of two planes will be found at the intersections of the vanishing traces of the two planes.

To the five axioms of perspective projection already stated may be added the following three truths concerning the construction of the perspective projection: —

(*f*) The perspective of any point in space is where the visual ray through the point pierces that picture plane (§ 7).

(*g*) The perspective of the vanishing point of any system of lines is where the visual element of that system pierces the picture plane.

Rule for finding the perspective of the vanishing point of any system of lines: — Draw an element of the system through the observer's eye, and find where it pierces the picture plane.

(*h*) Any point, line, or surface which lies in the picture plane will be its own perspective, and show in its true size and shape.

25. Knowing how to find the perspective of any point, and how to find the vanishing point of any system of lines, any problem in perspective may be solved. Therefore, it may be said that the whole process of making a perspective projection reduces itself to the problem of finding where a line pierces a plane.

Before proceeding farther, the student should review the first twenty-five paragraphs by answering carefully the following questions: —

(1) What does a perspective projection represent?

(2) What is a visual ray?

(3) How is a perspective projection formed?

(4) How does a perspective projection differ from an orthographic projection?

(5) What is the plane called on which the perspective projection is made?

(6) What is meant by the term Station Point?

(7) What is the most important phenomenon of perspective?

(8) What is meant by a system of lines?

(9) What is meant by a system of planes?

(10) What is a visual element?

(11) Define vanishing point.

(12) Define vanishing trace.

(13) Describe the position of the vanishing point of any system of lines.

(14) Give the five axioms of perspective.

(15) Do parallel lines in space really converge?

(16) Do the perspective projections of parallel lines really converge?

(17) Where will the perspective projections of parallel lines meet?

(18) How is the perspective of any point found?

(19) How is the perspective of the vanishing point of any system of lines found?

(20) What will be the perspective of a straight line?

(21) What is meant by the horizon?

(22) What is meant by the plane of the horizon?

THE PLANES OF PROJECTION.

26. Two planes of projection at right angles to one another, one vertical and the other horizontal, are used in making a perspective. In Fig. 7 these two planes are shown in oblique pro-

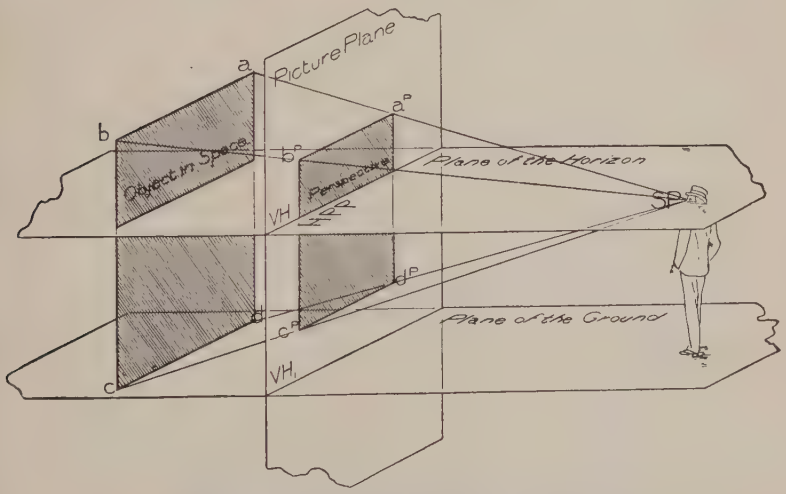


Fig. 7. Diagram Showing Planes of Projection

jection. The vertical plane is the picture plane (§ 6 and Fig. 7) on which the perspective projection is made, and corresponds exactly to the vertical plane, or vertical coordinate used in orthographic projections.

27. The horizontal plane, or plane of the horizon (§ 20 and Fig. 7), *always passes through the assumed position of the observer's eye*, and corresponds exactly to the horizontal plane or horizontal coordinate used in orthographic projections.

28. All points, lines, surfaces, or solids in space, the perspective projections of which are to be found, are represented by their orthographic projections on these two planes, and their perspectives are determined from these projections.

29. Besides these two principal planes of projection, a third plane is used to represent the plane on which the object is supposed to rest (Fig. 7). This third plane is horizontal, and is called the plane of the ground. Its relation to the plane of the horizon determines the nature of the perspective projection. To illustrate: The observer's eye must *always* be in the plane of the horizon (§ 27), while the object, the perspective of which is to be made, is usually supposed to rest upon the plane of the ground. In most cases the plane of the ground will also be the plane on which the observer is supposed to stand, but this will not always be true. The observer may be standing at a much higher level than the plane on which the object rests, or he may be standing below that plane. It is evident, therefore, that if the plane of the ground is chosen far below the plane of the horizon, the observer's eye will be far above the object, and the resulting perspective projection will be a "bird's-eye view." If, on the other hand, the plane of the ground is chosen above the plane of the horizon, the observer's eye will be below the object, and the resulting perspective projection will show the object as though being viewed from below. This has sometimes been called a "worm's-eye view," or a "toad's-eye view."

Usually the plane of the ground is chosen so that the distance between it and the plane of the horizon is about equal (at the scale of the drawing) to the height of a man. This is the position indicated in Fig. 7, and the resulting perspective will show the object as though seen by a man standing on the plane on which the object rests.

30. The intersection of the *picture plane* and the *plane of the horizon* corresponds to the *ground line* used in the study of projections in mechanical drawing. For more advanced work, how-

ever, there is some objection to this term. The intersection of the two coordinate planes has really no connection with the ground, and if the term "ground line" is used, it is apt to result in a confusion between the intersection of the two coordinate planes, and the intersection of the auxiliary plane of the ground, with the picture plane.

31. The intersection of the two coordinate planes is usually lettered VH on the picture plane, and HPP on the plane of the horizon. (See Fig. 7.) That is to say: When the vertical plane is being considered, VH represents the intersection of that plane with the plane of the horizon. It should also be considered as the *vertical projection of the plane of the horizon.*

All points, lines, or surfaces lying in the plane of the horizon will have their *vertical projection* in VH.

32. On the other hand, when the horizontal plane is being considered, HPP represents the intersection of the two planes, and also the *horizontal projection of the picture plane.* All points, lines, or surfaces in the picture plane will have their *horizontal projections* in HPP. Thus, instead of considering the intersection of the two coordinate planes a single line, it should be considered the coincidence of two lines, i.e.: First, the vertical projection of the plane of the horizon; second, the horizontal projection of the picture plane.

33. The plane of the ground is always represented by its intersection with the picture plane (see VH_1 Fig. 7). Its only use is to determine the relation between the plane of the horizon and the plane on which the object rests (§ 29). The true distance between these two planes is always shown by the distance between VH and VH_1 as drawn on the picture plane.

34. To find the perspective of a point determined by its vertical and horizontal projections.

Fig. 8 is an oblique projection showing the two coordinate planes at right angles to each other. The assumed position of the plane of the ground is indicated by its vertical trace (VH_1).

NOTE.—The *vertical trace* of any plane is the intersection of that plane with the vertical coordinate. The *horizontal trace* of any plane is the intersection of that plane with the horizontal coordinate.

The assumed position of the station point is indicated by its two projections, SP^V and SP^H. Since the station point lies in the plane of the horizon (§ 27), it is evident that its true position must coincide with SP^H, and that (§ 31) its vertical projection must be found in VH, as indicated in the figure and shown at SP^V. Let the point a represent any point in space. The perspective of the point a will be at a^P, where a visual ray from the point a to the eye at SP^H pierces the picture plane (§ 24). We may find a^P in the following manner, by using the orthographic projections of the point a^P, instead of the point itself. a^H represents the *horizontal projection*, and a^V represents the *vertical projection* of the point a. A line drawn from the *vertical projection* of the point a to the *vertical projection* of the station point, will represent the *vertical projection of the visual ray*, which passes through the point a. In Fig. 8 this vertical projection is represented by the line drawn on the picture plane from a^V to SP^V.

A line drawn from the *horizontal projection* of a to the *horizontal projection* of the station point SP^H will represent the *horizontal projection of the visual ray*, which passes through the point a. In Fig. 8, this horizontal projection is represented by the line drawn on the plane of the horizon from a^H to SP^H. Thus we have, drawn upon the planes of projection, the vertical and horizontal projections of the point a, and the vertical and horizontal projections of the visual ray passing between the point a and the station point.

35. We must now find the intersection of the visual ray with the picture plane. This intersection will be a point in the picture plane.

It is evident that its vertical projection must coincide with the intersection of the visual ray and VH, and that its horizontal projection must be in HPP (§ 32). But this intersection must also be on the visual ray through the point a, and consequently the horizontal projection of this intersection must be on the horizontal projection of the visual ray. Therefore, the horizontal projection of this intersection must be the point m^H, where the line between

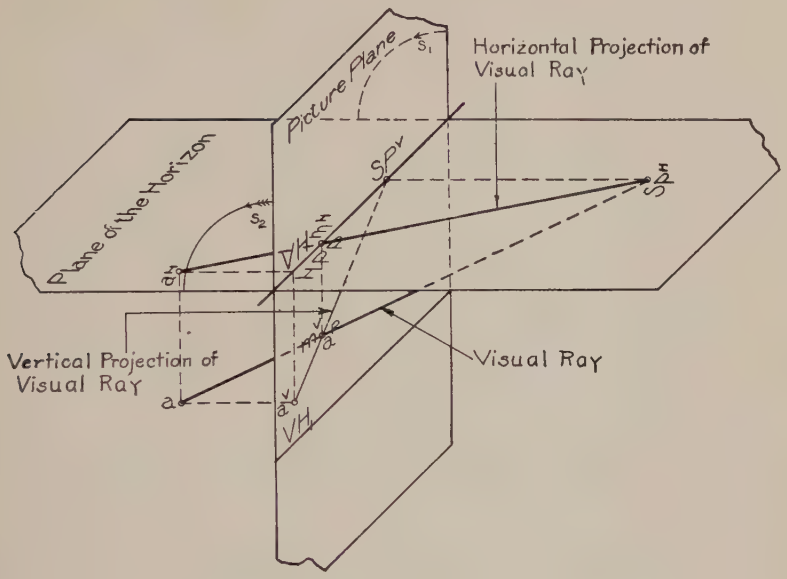


Fig. 8. Diagram Showing Difficulties of Working on Planes at Right Angles

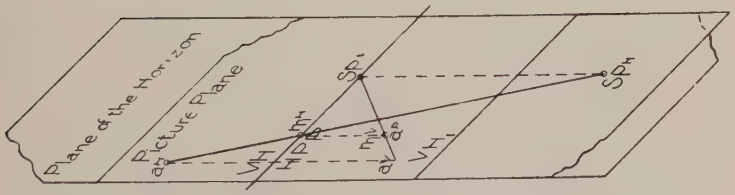


Fig. 9. Diagram Showing Plane of Horizon and Picture Plane Superposed

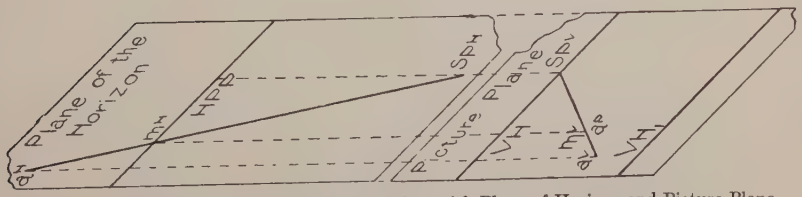


Fig. 9a. Diagram Corresponding to Fig. 9 but with Plane of Horizon and Picture Plane Pulled Apart to Facilitate Examination of Sketches

SP^H and a^H crosses HPP. The vertical projection of this intersection must be vertically in line with this point, and on the line drawn between SP^V and a^V, and hence at m^V. Since the vertical projection of the intersection coincides with the intersection itself, a^P (coincident with m^V) must be the perspective of the point a.

36. This is the method of finding the perspective of any point, having given the vertical and horizontal projections of the point and of the station point. The method may be stated briefly as follows:

Draw a line through the horizontal projection of the point to the horizontal projection of the station point. This line represents the horizontal projection of the visual ray, which passes through the point. Through the intersection of this line with HPP, draw a vertical line. Next draw a line from the vertical projection of the point to the vertical projection of the station point. The perspective of the point will be found where the vertical line crosses the vertical projection of the visual ray.

37. It would evidently be inconvenient to work upon two planes at right angles to one another, as shown in Fig. 8. To avoid this, and to make it possible to work upon a plane surface, the picture plane (or vertical coordinate) is supposed to be revolved about its intersection with the plane of the horizon, until the two coincide and form one surface. The direction of this revolution is indicated by the arrows s_1 and s_2. After revolution, the two coordinate planes will coincide, and the vertical and horizontal projections will overlap one another, as indicated in Fig. 9.

It will be noticed that the coincidence of the two planes in no way interferes with the method given in § 36 of finding the perspective (a^P) of the point a from the vertical and horizontal projections of the point. Thus, the horizontal projection of the visual ray through the point will be seen, drawn from SP^H to a^H and intersecting HPP in the point m^H. The vertical projection of the visual ray through the point will be seen passing from SP^V to a^V. And a^P is found upon the vertical projection of the visual ray directly under m^H.

It will readily be understood that in a complicated problem, the overlapping of the vertical and horizontal projections might

result in some confusion. It is, therefore, usually customary, after having revolved the two coordinate planes into the position shown in Fig. 9, to slide them apart in a direction *perpendicular to their line of intersection*, until the two planes occupy a position similar to that shown in Fig. 9a.

38. It will be remembered from the course on projections which the student is supposed to have taken, that *horizontal projections must always be compared with horizontal, and never with vertical projections*, and that in the same way, *vertical projections must always be compared with vertical, and never with horizontal projections*. It is evident that in sliding the planes apart, the relations between the projections on the vertical plane will not be disturbed, nor will the relations between the projections on the horizontal plane, and consequently it will make no difference how far apart the two coordinate planes are drawn, provided that horizontal and vertical projections of the same points are always kept in line. Thus, in Fig. 9a, it will be seen that in drawing the planes apart, a^{V} has been kept in line with a^{H}, m^{V} with m^{H}, SP^{V} with SP^{H}, etc.

39. It will be observed that in sliding the planes apart, their line of intersection has been separated into its two projections (§§ 31 and 32). HPP, being on the plane of the horizon or horizontal coordinate, is the *horizontal projection of the intersection* of the two planes, while VH, being on the picture plane or vertical coordinate, is the *vertical projection of the intersection* of the two planes. In the original position of the planes (Fig. 9) these two projections were coincident. The distance between HPP and VH always represents the distance through which the planes have been slid. This distance is immaterial, and will have *no effect* on the perspective drawing. VH_1 represents the vertical trace of the plane of the ground.

In this figure, as in the case of Fig. 9, the student should follow through the construction of the perspective of the point a, applying the method of § 36.

40. Fig. 10 shows the position of the two coordinate planes and of the plane of the ground, as they are usually represented on the drawing board in making a perspective drawing. It is essentially the same thing as Fig. 9a, except that the latter was

shown in oblique projection in order that its development from the original position of the planes (Fig. 8) might be followed more readily. The two coordinate planes are supposed to lie in the plane of the paper.

HPP represents the horizontal projection of the picture plane, and VH represents the vertical projection of the plane of the horizon.

As horizontal projections are never compared with vertical projections (§ 38), HPP may be drawn as far from, or as near, VH as desired, without in any way affecting the resulting perspective drawing. HPP and VH were coincident in Fig. 9, and the distance between them in Fig. 10 simply shows the distance that the two planes have been slid apart, as illustrated in Fig. 9a. As already stated, this distance is immaterial, and may be made whatever is most convenient, according to the nature of the problem.

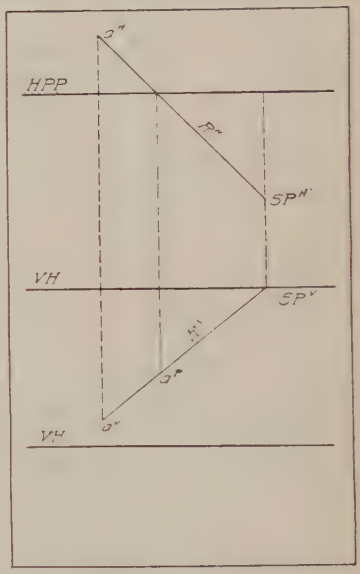


Fig. 10. Simple Sketch Showing Position of Figures in Standard Projection Drawing

If HPP should be placed nearer the top of the sheet, a^{H} and SP^{H}, both being horizontal projections, would follow it, the relation between these horizontal projections always being preserved.

On the other hand, SP^{V}, a^{V}, a^{P}, VH, and VH_1, all being projections on the vertical plane, must preserve their relation with one another, and will in no way be affected if the group of projections on the horizontal coordinate is moved nearer or farther away. It must be borne in mind, however, that, in all cases, the vertical and horizontal projections of corresponding points must be kept vertically in line. Thus, a^{H} must always be vertically in line with a^{V}. The vertical distance between these two projections does not matter, provided the distance from a^{H} to HPP, or the distance from a^{V} to VH, is not changed. This point cannot be too strongly emphasized.

41. Suppose it is desired to determine from Fig. 10 how far the station point lies in front of the picture plane. This is a horizontal distance, and therefore will be shown by the distance between the horizontal projection of the station point and the horizontal projection of the picture plane, or, in other words, by the distance between SP^H and HPP.

42. The point a is a certain distance above or below the plane of the horizon. This is a *vertical distance*, and will be shown by the distance between the *vertical projection* of the point a and the *vertical projection* of the plane of the horizon; in other words, by the distance between a^V and VH. It will be seen that in Fig. 10 the point a lies below the plane of the horizon.

43. If it be desired to find how far in front or behind the picture plane the point a lies, this is a *horizontal distance*, and will be shown by the distance between the *horizontal projection* of the picture plane and *horizontal projection* of the point a, that is, by the distance between HPP and a^H. In Fig. 10 the point a lies behind the picture plane.

44. The distance between the plane of the ground and the plane of the horizon is a vertical distance, and will be shown by the distance between the vertical projection of the plane of the horizon and the vertical projection of the plane of the ground; i.e., the distance between VH and VH_1. The distance between the observer's eye and the plane of the ground is also a vertical distance, and will be shown by the distance between SP^V and VH_1; but as SP^V must always be found in VH, the distance of the observer's eye above the plane of the ground will always be shown by the distance between VH and VH_1.

45. To find the perspective of the point a, Fig. 10, draw the visual ray through the point, and find where this visual ray pierces the picture plane (§ 24*f*). The horizontal projection of the visual ray is shown by the line R^H drawn through the horizontal projection SP^H of the observer's eye and the horizontal projection a^H of the point a. The vertical projection of the visual ray is shown by the line R^V drawn through the vertical projection SP^V of the observer's eye and the vertical projection a^V of the point a. This visual ray pierces the picture plane at the point a^P on R^V vertically in line with the point where

R^H crosses HPP (§§ 35 and 36). a^P is the perspective of the point *a*.

NOTE. — To find where any line, represented by its horizontal and vertical projections, pierces the picture plane, is one of the most used and most important problems in perspective projection. The point where any line pierces the picture plane will always be found on the vertical projection of the line, vertically above or below the point where the horizontal projection of the line crosses HPP (§§ 35 and 36).

NOTATION.

46. In order to avoid confusion between the vertical, horizontal, and perspective projections of the points and lines in the drawing, it becomes necessary to adopt some systematic method of lettering the different points and lines. The following method will be found convenient, and has been adopted in these notes.

If the student will letter each point or line as it is found, in accordance with this notation, he will be able to read his drawings at a glance, and any desired projection of a point or line may be recognized instantly.

The *picture plane* (or vertical coordinate) is indicated by the capital letters PP.

The *plane of the horizon* (or horizontal coordinate) is indicated by the capital letter H.

A *point* in space is indicated by a small letter.

The same small letter with an index ^{V}, ^{H}, or ^{P}, indicates its *vertical*, *horizontal*, or *perspective projection*, respectively.

A *line* in space is indicated by a capital letter, usually one of the first letters in the alphabet.

The same capital letter with an index ^{V}, ^{H}, or ^{P}, indicates its *vertical*, *horizontal*, or *perspective projection*, respectively.

All lines which belong to the same system may be designated by the same letter, the different lines being distinguished by the subordinate $_1$, $_2$, $_3$, etc., placed after the letter.

The *trace of a plane upon the picture plane* is indicated by a capital letter (usually one of the last letters in the alphabet) with a capital V placed before it.

The same letter preceded by a capital H *indicates the trace of the plane upon the horizontal coordinate.*

The *perspective of the vanishing trace of a system of planes* is indicated by a capital letter preceded by a capital T.

The *perspective of the vanishing point of a system of lines* is indicated by a small v with an index corresponding to the letter of the lines which belong to the system.

PP = vertical coordinate, or picture plane.

HPP = horizontal trace of the vertical coordinate, or picture plane.

H = horizontal coordinate, or plane of the horizon.

VH = vertical trace of the horizontal coordinate, or plane of the horizon.

H_1 = plane of the ground.

VH_1 = vertical trace of the plane of the ground.

a = point in space.

a^V = vertical projection of the point.

a^H = horizontal projection of the point.

a^P = perspective projection of the point.

A = line in space.

A^V = vertical projection of the line.

A^P = perspective projection of the line.

VS = trace of the plane S upon PP (vertical trace).

HS = trace of the plane S upon H (horizontal trace).

TS = perspective of the vanishing trace of the plane S. (See Note 1 below.)

v^A = perspective of the vanishing point of a system of lines, the elements of which are lettered A_1, A_2, A_3, A_4, etc. (See Note 2 below.)

NOTE 1. — A plane in space may also be designated by the letters of any two lines which lie in it. Thus, the plane AB would be a plane determined by the two lines A and B. TAB would indicate the perspective of the vanishing trace of the plane.

NOTE 2. — A straight line may be designated by the letters of any two points which lie in it. Thus, the line ab would be a straight line determined by the two points a and b. v^{ab} would indicate the perspective of the vanishing point of the line. It is sometimes convenient to use this notation in place of the general one.

ELEMENTARY PROBLEMS.

47. PROBLEM I. Fig. 11. **To find the perspective of a point. The point to be situated $\frac{1}{4}''$ behind the picture plane, and $\frac{1}{8}''$ above the plane of the horizon. The observer's eye to be $\frac{1}{2}''$ in front of the picture plane.**

First assume HPP and VH (§ 40). These lines may be drawn anywhere on the paper, HPP usually being placed some distance above VH, in order to avoid confusion between horizontal and vertical projections. The position of the point with respect to the coordinate planes must now be established by means of its vertical and horizontal projections. a^V located $\frac{1}{8}''$ above VH will represent the vertical projection of the point. Its horizontal projection must be vertically in line with a^V; and since the point

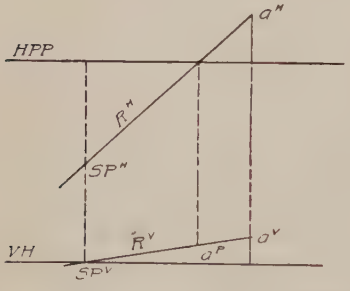


Fig. 11. Diagram for Finding Perspective of a Point

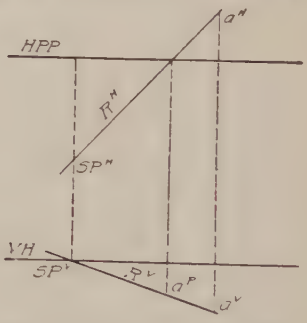


Fig. 12. Projection of a Point. Location Different from Fig. 11

is to be $\frac{1}{4}''$ behind the picture plane, its horizontal projection must be $\frac{1}{4}''$ behind the horizontal projection of the picture plane, i.e., $\frac{1}{4}''$ behind HPP. Next establish the position of the observer's eye, or station point. Its vertical projection (SP^V) may be assumed anywhere in VH. Its horizontal projection (SP^H) must be vertically in line with SP^V and $\frac{1}{2}''$ in front of HPP. The perspective of the point a will be where the visual ray through the point pierces the picture plane. A line R^H drawn through SP^H and a^H will be the horizontal projection of this visual ray. Its vertical projection will be the line R^V drawn through SP^V and a^V. The perspective a^P of the point will be found on R^V vertically in line with the intersection of R^H and HPP (§ 45, note). Compare with the construction shown in Fig. 10 and Fig 8.

48. Figs. 12, 13, and 14 illustrate this same problem.

In Fig. 12, the point *a*, as shown by its vertical and horizontal projections, is situated $\frac{1}{4}''$ *below* the plane of the horizon and $\frac{1}{4}''$ *behind* the picture plane. a^P is the perspective of the point.

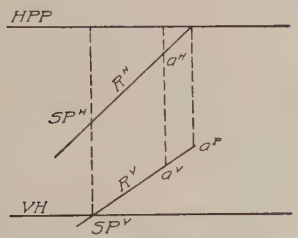


Fig. 13. Projection of a Point. Location Different from Fig. 11

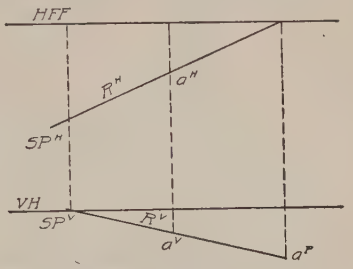


Fig. 14. Projection of a Point. Location Different from Fig. 11

In Fig. 13, the point *a* is $\frac{1}{4}''$ *above* the plane of the horizon and $\frac{1}{8}''$ *in front* of the picture plane. a^P is its perspective.

In Fig. 14, the point *a* is $\frac{1}{8}''$ *below* the plane of the horizon and $\frac{1}{4}''$ *in front* of the picture plane. a^P is its perspective.

49. PROBLEM II. Fig. 15. **To find the perspective of a line, the line being determined by its vertical and horizontal projections.**

Let HPP and VH be given, as shown in Fig. 15. Let the line A^H represent the horizontal projection of the line of which the perspective is to be found, and likewise let the line A^V be the vertical projection of this line. The points a^H, b^H, a^V, b^V will then be the horizontal and vertical projections of the ends of this line. If the perspectives of the ends of this line are found, then a line drawn between these perspective points will be the perspective of the line in the problem. The finding of the perspective of this line may be more easily followed in the four successive steps shown in Fig. 15.

Step 1 shows the horizontal and vertical projection of the line, the length of which is determined by the points a^H, b^H, a^V, b^V. The horizontal projection of the station point is at SP^H and its vertical projection is at SP^V.

Step 2 shows the horizontal projection of the visual ray passing through the points a^H b^H. Note the vertical dotted lines dropped down from the points, where the horizontal projection of the visual

ray passes through HPP, and on which the perspective points must be located.

Step 3 shows the vertical projection of the visual ray passing through the points a^V b^V. Where the vertical projection of the visual ray intersects the dotted line, dropped from the points of

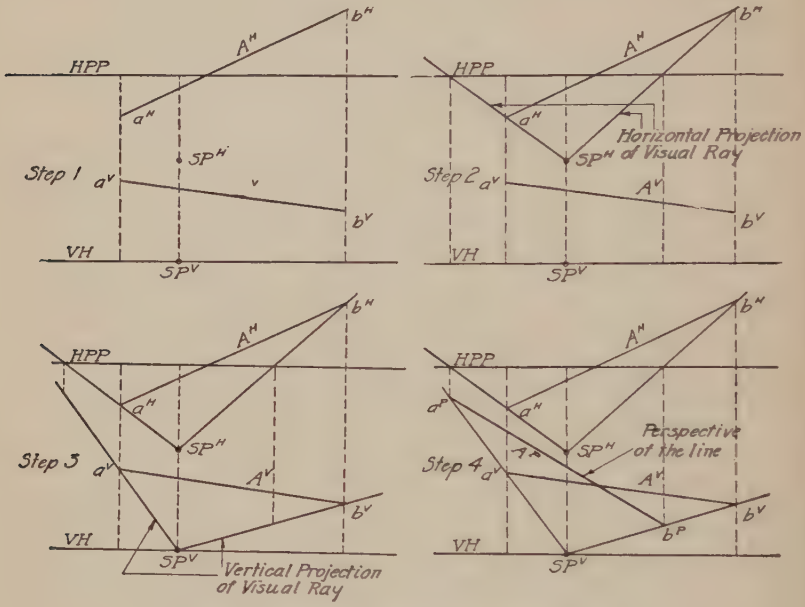


Fig. 15. Perspective Drawing for Locating a Line

intersection of the horizontal projection of the visual ray and HPP, will be located the perspective points of the ends of the line as shown at a^P b^P.

Step 4 shows the line drawn through the perspective points a^P and b^P and it is, therefore, the perspective of the line in the problem.

50. PROBLEM III. Fig. 16. **Having given the vertical and horizontal projection of any line, to find the perspective of its vanishing point.**

Let the line be given by its vertical and horizontal projections (A^V and A^H), as indicated in the figure. SP^V and SP^H represent the position of the observer's eye. To find the perspective of the

vanishing point of any line, draw through the observer's eye an element of the system to which the line belongs, and find where this element pierces the picture plane (§ 24 *g*). Through SP^H draw

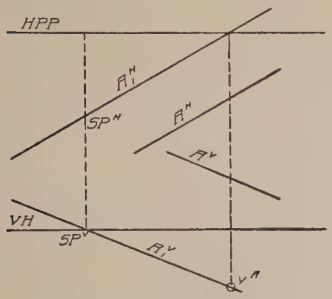


Fig. 16. Diagram for Locating Perspective of Vanishing Point of a Line

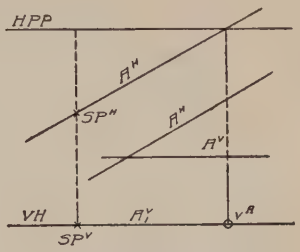


Fig. 17. Diagram Similar to Fig. 16 Showing Projections of a Horizontal Line

A_1^H parallel to AH, and through SP^V draw A_1^V parallel to A^V. A_1^H and A_1^V represent the two projections of a line passing through the observer's eye and parallel to A^HA^V. This line pierces the picture plane at v^A, giving the perspective of the required vanishing point (§ 45, note). The perspectives of all lines parallel to A^VA^H will meet at v^A.

Figs. 17 and 18 illustrate this same problem.

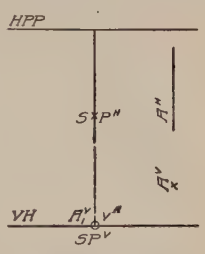


Fig. 18. Diagarm for Problem Similar to Fig. 16

51. In Fig. 17, the line, as shown by its two projections, is a horizontal one; hence, A_1^V drawn through SP^V coincides with VH, and the vanishing point for the system of the lines must be found on VH at v^A, as indicated (§ 24 *c*).

NOTE. — Systems of lines which vanish upward will have their vanishing points above VH. Systems of lines which vanish downward will have their vanishing points below VH (§ 16).

52. In Fig. 18, the given line is perpendicular to the picture plane; hence, A_1^V must be a point coincident with SP^V; and as v^A will always be found on A_1^V, the vanishing point of the line must coincide with SP^V.

NOTE. — In a perspective drawing, the vanishing point for a system of lines perpendicular to the picture plane will always coincide with the vertical projection of the observer's eye.

NOTE. — It should always be remembered that, no matter in what position $a^{V}a^{H}SP^{V}SP^{H}$ may be in relation to one another, the horizontal projection of the visual ray must always pass through $a^{H}SP^{H}$ and intersect HPP, while the vertical projection of the

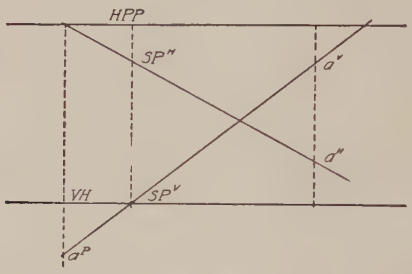


Fig. 19a

visual ray must always pass through $a^{V}SP^{V}$ and intersect the vertical line which is dropped from the point of intersection of the horizontal projection of the visual ray and HPP, as shown in Fig. 19a.

Plate I, problems I to X, should now be solved.

PERSPECTIVE DRAWING

PART II

METHOD OF THE REVOLVED PLAN

53. PROBLEM IV. Fig. 19. **To find the perspective of a rectangular block resting upon a horizontal plane 1″ below the level of the eye, and turned so that the long side of the block makes an angle of 30° with the picture plane.**

The block is shown in plan and elevation at the left of the figure. The first step will be to make an auxiliary horizontal projection of the block on the plane of the horizon, showing the exact position of the block as it is to be seen in the perspective projection. This auxiliary horizontal projection is really a revolved plan of the object, and is called a **diagram.** It is the general rule, in making a perspective projection, to place the object behind the picture plane with one of its principal vertical lines lying in the picture plane (24 *h*). HPP is usually drawn near the upper edge of the paper, leaving just room enough behind to place the auxiliary plan or diagram. In the figure the diagram is shown in the required position, i.e., with one of its long sides (*abfe*) making an angle of 30° with the picture plane. The vertical edge (*ae*) of the block is supposed to lie in the picture plane. VH may now be drawn parallel to HPP at any convenient distance from it, as indicated. VH_1, the vertical trace of the plane on which the block is supposed to rest, should be assumed in accordance with the given data, i.e., 1″ below VH (§ 44).

The position of the observer's eye should next be established. SP^H is its horizontal projection, and shows by its distance from HPP the distance in front of the picture plane at which the observer is supposed to stand. SP^V is its vertical projection, and must always be found in VH. In this problem the station point is $1\frac{1}{2}$″ in front of the picture plane.

NOTE. — As a general rule, it is well to assume the station point on a vertical line half way between two lines dropped from the extreme edges of the diagram, as indicated. This is not neces-

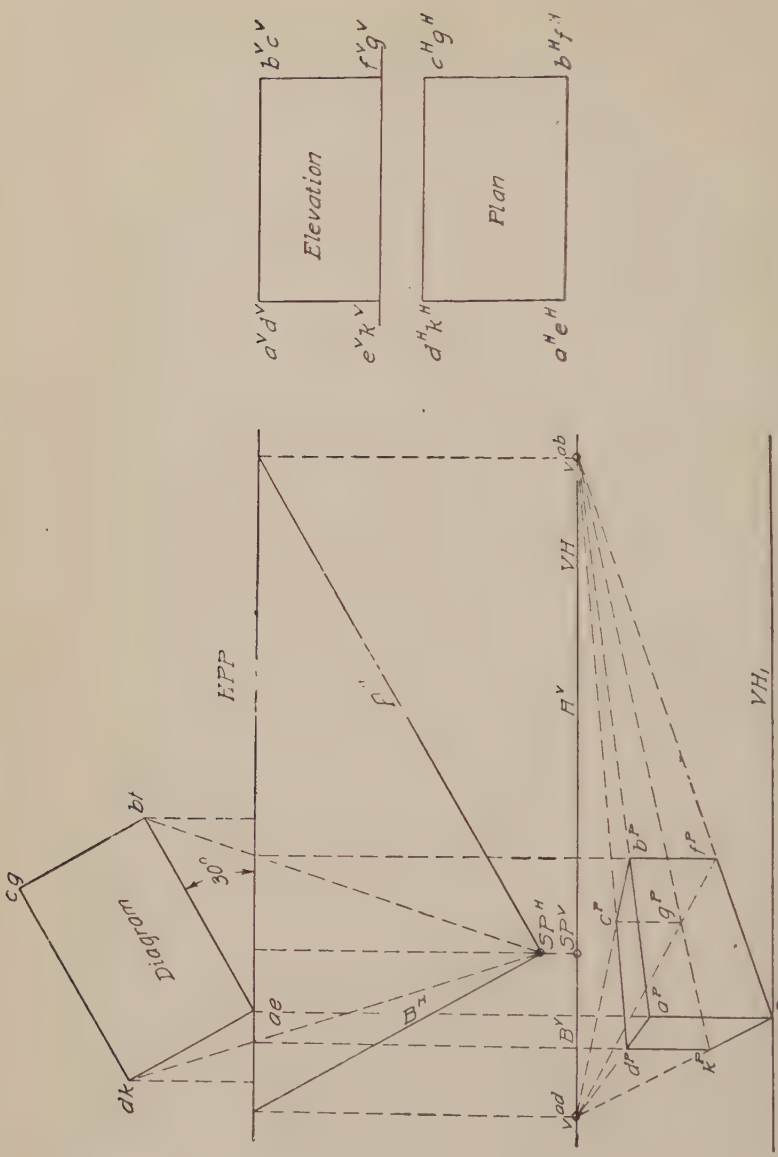


Fig. 19. Diagram for Finding Perspective of a Rectangular Block Resting on Horizontal Plane. Long Side of Block Makes Angle of 30 Degrees with a Picture Plane

sary, but, as will be explained later, it usually insures a more pleasing perspective projection.

Next find the vanishing points for the different systems of lines in the object (§ 12). There are three systems of lines in the block, formed by its three sets of parallel edges.

1st. A system formed by the four horizontal edges vanishing to the right: *ab*, *ef*, *dc* and *kg*.

2d. A system formed by the four horizontal edges vanishing towards the left: *ad*, *ek*, *bc* and *fg*.

3d. A system formed by the four vertical edges.

First find the vanishing point for the system parallel to *ab* by drawing through the station point a line parallel to *ab* and finding where it pierces the picture plane (24 *g*). A^H drawn through SP^H is the horizontal projection of such a line. Its vertical projection (A^V), drawn through SP^V, will coincide with VH, and its vanishing point will be found on VH at v^{ab} (§ 51). All lines in the perspective of the object that are parallel to *ab* will meet at v^{ab} (§ 24 *a*). In a similar manner find v^{ad}, which will be the vanishing point for all lines parallel to *ad*.

54. If the method for finding any vanishing point is applied to the system of vertical lines, it will be found that this vanishing point will lie vertically over SP^V at infinity. That is to say, since all vertical lines are parallel to the picture plane, if a vertical line is drawn through the station point, it will never pierce the picture plane. Therefore (24 *g*), the perspective of the vanishing point of a vertical line cannot be found within any finite limits, but will be *vertically over* SP^V, *and at an infinite distance from it.* In a perspective projection *all vertical lines are drawn actually vertical, and not converging towards one another.*

Note. — This is true of all lines in an object which are *parallel to the picture plane.* Thus, *the perspective of any line, which is parallel to the picture plane, will actually be parallel to the line itself;* and *the perspectives of the elements of a system of lines parallel to the picture plane, will be drawn parallel to, and not converging towards, one another.* That this must be so, is evident, since, if the perspectives of such a system of lines did converge towards one another, they would meet within finite limits. But it has just been found that the perspective of the vanishing point of such a system

is at infinity. The perspectives of the elements of any system can meet only at the perspective of their vanishing point, and must, therefore, in a system parallel to the picture plane, be drawn parallel to one another.

The directions of the perspectives of all lines in the object have now been determined, and will be as follows:

All lines parallel to *ab* will meet at v^{ab}.
All lines parallel to *ad* will meet at v^{ad}.
All vertical lines will be drawn vertical.

Since the point *e* is in the base of the object, it lies on the plane of the ground, and also, since the line *ae* lies in the picture plane, the point *e* must lie on the intersection of the plane of the ground with the picture plane. Therefore, the point *e* must lie in VH_1 and must be vertically under the point *e* in the diagram.

Since the point *e* lies in the picture plane, it will be its own perspective; and e^P will be found on VH_1 vertically under *e* in the diagram, as shown in the figure. From e^P the perspective of the lower edges of the cube will vanish at v^{ab} and v^{ad}, respectively, as indicated.

f^P is the perspective of the point *f* and will be found on the lower edge of the block vertically under the intersection of HPP with the horizontal projection of the visual ray drawn through the point *f* in the diagram.

Similarly, k^P is found on the lower edge of the block vertically under the intersection of HPP and the visual ray drawn through the point *k* in the diagram.

Vertical lines drawn through f^P, e^P, and k^P will represent the perspectives of the visible vertical edges of the block.

The edge e^Pa^P being in the picture plane will be its own perspective and show in its true size (§ 24 *h*). Therefore, a^P may be established by making the distance e^Pa^P equal to e^Va^V as taken from the given elevation.

From the point a^P two of the upper horizontal edges of the block will vanish at v^{ab} and v^{ad}, respectively, establishing the points b^P and d^P by their intersections with the vertical edges drawn through k^P and f^P, respectively. Lines drawn through d^P and b^P and vanishing respectively at v^{ab} and v^{ad} will intersect at c^P and complete the perspective of the block.

LINE OF MEASURES

55. Any line which lies in the picture plane will be its own perspective and will show the true length of the line (24 *h*). Such a line is called a **line of measures.**

In Fig. 19, the line *ae*, being in the picture plane, was a line of measures; that is to say, its length could be laid off directly from the given data, and from this length the lengths of the remaining lines in the perspective drawing could be established.

Fig. 20 shows a similar problem. The vertical line *ae* lies in the picture plane, and $a^{P}e^{P}$ is, therefore, a line of measures for the object.

56. Besides this principal line of measures, other lines of measures may easily be established by extending any vertical plane in the object until it intersects the picture plane. This intersection, since it lies in the picture plane, will show in its true size, and will be an **auxiliary line of measures.** All points in it will show at their true height above the plane of the ground.

Thus, in Fig. 20, $a^{P}e^{P}$ is the principal line of measures, and shows the true height of the block.

If the rear vertical faces of the block are extended till they intersect the picture plane, these intersections $m^{P}n^{P}$ and $o^{P}p^{P}$ will be auxiliary lines of measures, and will also show the true height of the block. These edges should be used when the front edge of the object does not lie in HPP.

It will be noticed in the figure that $m^{P}n^{P}$ and $o^{P}p^{P}$ are each equal to $a^{P}e^{P}$. Either one of these lines could have been used to determine the vertical height of the perspective of the block.

For illustration, suppose it is desired to find the height of the perspective, using the line $o^{P}p^{P}$ as the line of measures.

Assume the vanishing points v^{ad} and v^{ab} for the two systems of horizontal edges in the block to have been established. Now extend the line *bc*, Fig. 20, which represents the horizontal projection of the face *cbf*, till it intersects HPP. From this intersection drop a vertical line *op*. This line will represent the intersection of the vertical face *cbf* with the picture plane, and will be a line of measures for the face. The point p^{P}, where this line of measures intersects the plane of the ground VH_1, will be the

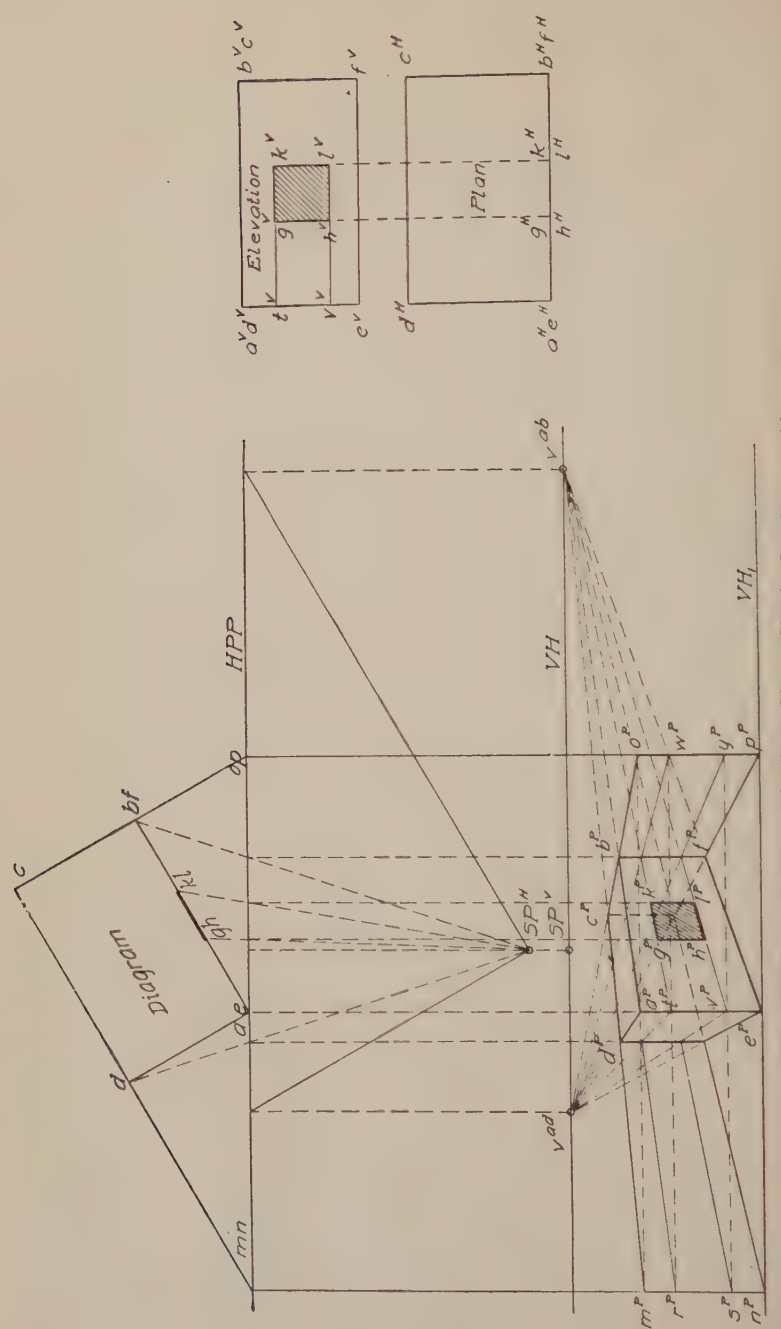


Fig. 20. Diagram of Problem Similar to Fig. 19, Showing Development of Lines of Measure

point where the lower horizontal edge (produced) of the face *cbf* intersects the picture plane. Measure off the distance $p^{P}o^{P}$ equal to the true height of the block, as given by the elevation. Two lines drawn through o^{P} and p^{P} respectively, and vanishing at v^{ad}, will represent the perspectives of the upper and lower edges of the face *cbf*, produced. The perspective (b^{P}), of the point *b*, will be found on the perspective of the upper edge of the face *cbf*, vertically below the intersection of HPP with the horizontal projection of a visual ray drawn through the point *b* in the diagram. A vertical line through b^{P} will intersect the lower horizontal edge of the face *cbf* in the point f^{P}. Lines drawn respectively through b^{P} and f^{P}, vanishing at v^{ab}, will establish the perspectives of the upper and lower horizontal edges of the face *abfe* The points a^{P} and e^{P} will be found vertically under the points *a* and *e* in the diagram. The remainder of the perspective projection may now easily be determined.

57. The perspectives of any points on the faces of the block may be found by means of the diagram and one of the lines of measures.

Let the points g^{V}, h^{V}, k^{V}, and l^{V}, in the given elevation, determine a square on the face *abfe* of the block. Let the points *g*, *h*, *k*, and *l*, represent the position of the square in the diagram. Extend the upper and lower horizontal edges of the square, as shown in elevation, until they intersect the vertical edge $a^{V}e^{V}$ in the points t^{V} and v^{V}. To determine the perspective of the square, lay off on $a^{P}e^{P}$, which is a line of measures for the face *abfe*, the divisions t^{P} and v^{P} taken directly from the elevation. Two lines drawn through t^{P} and v^{P} respectively, vanishing at v^{ab}, will represent the perspectives of the upper and lower edges (produced) of the square. g^{P} will be found on the perspective of the upper edge, vertically under the intersection of HPP with the horizontal projection of a visual ray drawn through the point *g* in the diagram. The position of k^{P} may be established in a similar manner. Vertical lines drawn through g^{P} and k^{P} respectively, will complete the perspective of the square.

58. The auxiliary line of measures $o^{P}p^{P}$ might have been used instead of $a^{P}e^{P}$. In this case, $o^{P}p^{P}$ should be divided by the points w^{P} and y^{P}, in the same way that *ae*, in elevation, is divided

by the points t and v. Through w^P and y^P, draw horizontal lines lying in the plane cbf, for which $o^P p^P$ is a line of measures. These lines will vanish at v^{ad}, and intersect the vertical edge $b^P f^P$ of the block. From these intersections draw horizontal lines lying in the plane $abef$, vanishing at v^{ab}, and representing the upper and lower edges of the square. The remainder of the square may be determined as in the previous case.

In a similar manner, the auxiliary line of measures $m^P n^P$ might have been used to determine the upper and lower edges of

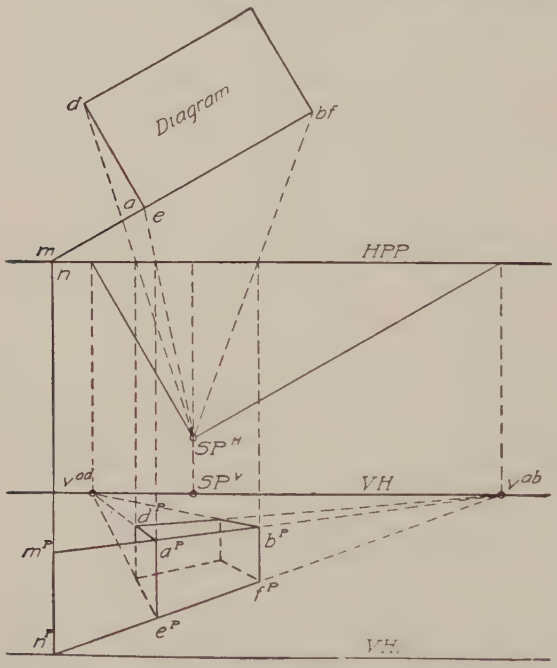


Fig. 21. Diagram Showing Extension of Certain Lines of Object to Form Auxiliary Line of Measure

the square. This construction has been indicated, and the student should follow it through.

59. It sometimes happens that no line in the object lies in the picture plane. In such a case there is no principal line of measures, and some vertical plane in the object must be extended until it intersects the picture plane, forming by this intersection an auxiliary line of measures. Fig. 21 illustrates such a case. A rectangular block, similar to those shown in Figs. 19 and 20, is

situated some distance behind the picture plane, as indicated by the relative positions of HPP and the diagram.

Its perspective projection will evidently be smaller than if the vertical edge *ae* were in the picture plane, as was the case in Figs. 19 and 20, and the perspective of *ae* will evidently be shorter than the true length of *ae*. There is, therefore, no line in the object that can be used for a line of measures. It becomes necessary to extend one of the vertical faces of the block until it intersects the picture plane, and shows by the intersection its true vertical height. Thus, the plane *abfe* has been extended, as indicated in the diagram, until it intersects the picture plane in the line *mn*. This intersection is an auxiliary line of measures for the plane *abfe*, and $m^{P}n^{P}$ shows the true vertical height of this plane.

Either of the other vertical faces of the block, as well as the face *abfe*, might have been extended until it intersected the picture plane, and formed by this intersection a line of measures for the block.

The vanishing points for the various systems of lines have been found as in the previous cases.

From m^{P} and n^{P}, the horizontal edges of the face *abfe* vanish to v^{ab}. a^{P} will be found on the upper edge of this face, vertically below the intersection of HPP with the horizontal projection of the visual ray through the point *a* in the diagram. A vertical line through a^{P} will represent the perspective of the nearest vertical edge of the block, and will establish the position of e^{P}.

In a similar manner, b^{P} will be found vertically below the intersection of HPP with the horizontal projection of the visual ray through the point *b* in the diagram. A vertical line through b^{P} will establish f^{P}, and complete the perspective of the face *abfe*. Having found the perspective of this face, the remainder of the block may be determined as in the previous problems.

NOTE. — Instead of being some distance behind the picture plane, the block might have been wholly or partly in front of the picture plane. In any case, find the intersection with the picture plane of some vertical face of the block (produced, if necessary). This intersection will show the true vertical height of the block.

At this point the student should solve Plate II.

60. PROBLEM V. Fig. 22. **To find the perspective of a house, the projections of which are given.**

The plan, front, and side elevations of the house are shown in the figure. The side elevation corresponds to the projection on the profile plan, used in the study of projections. This problem is a further illustration of the method of revolved plan and of the use of horizontal vanishing points and auxiliary lines of measures. It is very similar to the three previous problems on the rectangular blocks.

The first step in the construction of the perspective projection is to make a diagram (§ 53) which shall show the horizontal projections of all the features that are to appear in the drawing. The diagram should be placed at the top of the sheet, and turned so that the sides of the house make the desired angles with the picture plane. In Fig. 22 the diagram is shown with the long side making an angle of 30° with the picture plane. The roof lines, the chimney, and the positions of all windows, doors, etc., that are to be visible in the perspective projection, will be seen marked on the diagram.

The nearest vertical edge of the house is to lie in the picture plane. This is indicated by drawing HPP through the corner of the diagram which represents this nearest edge.

VH may be chosen at any convenient distance below HPP.

The position of the station point is shown in the figure by its two projections SP^V and SP^H. SP^V must always be in VH. The distance between SP^H and HPP shows the distance of the observer's eye in front of the picture plane (§ 43).

v^{ab} and v^{ad} may be found as in the preceding problems.

The position of the plane on which the object is to rest should next be established by drawing VH_1, the distance between VH and VH_1 showing the height of the observer's eye above the ground (§ 44).

In addition to the plane of the ground represented by VH_1, a second ground plane, represented by VH_2, has been chosen some distance below VH_1. In the figure, two perspective projections have been found, one resting on each of these two ground planes. The perspective which rests upon the plane represented by VH_1 shows the house as though seen by a man standing with his eyes

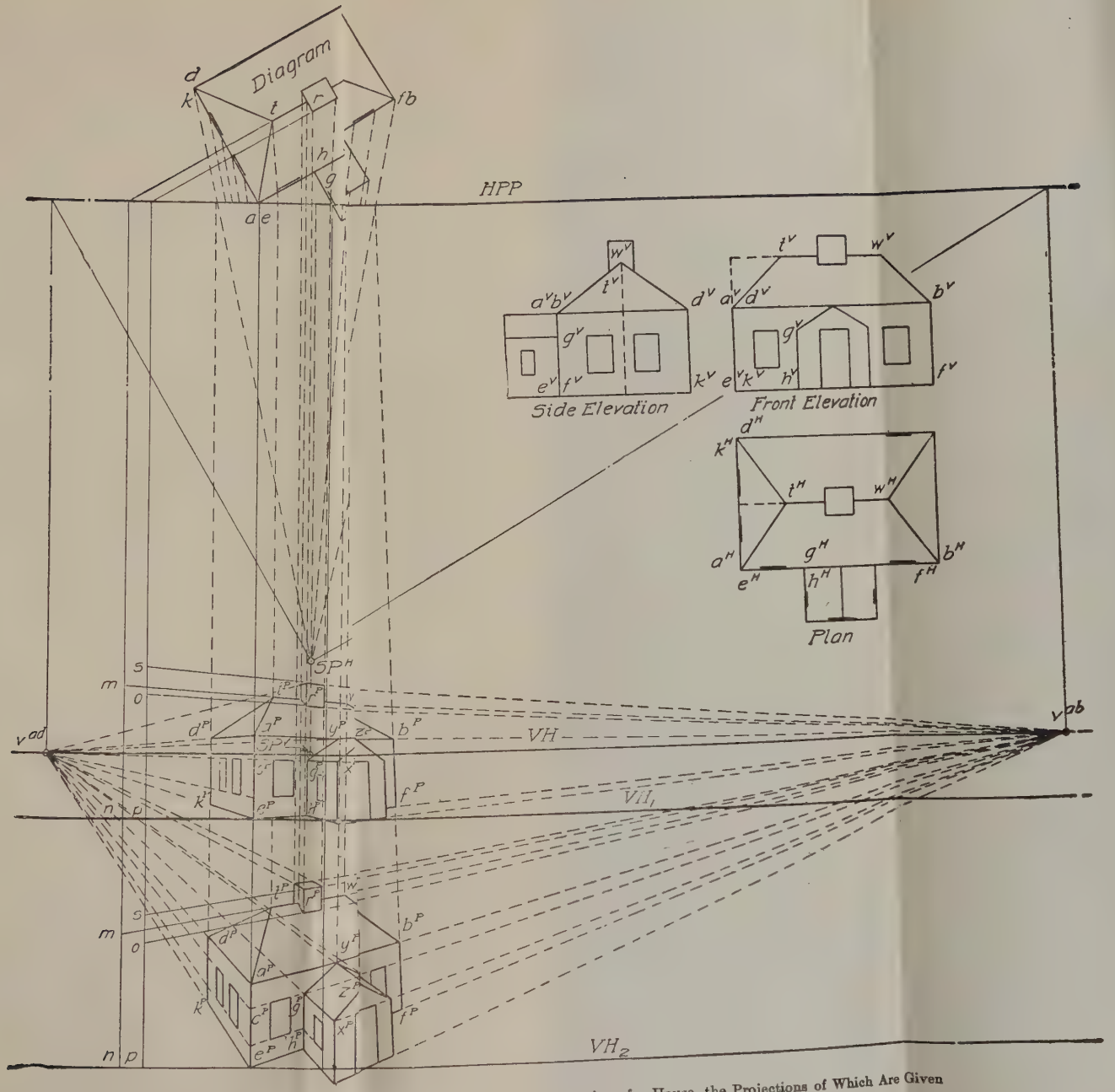


Fig. 22. Diagram Showing Method of Finding the Perspective of a House, the Projections of Which Are Given

nearly on a level with the tops of the windows (§ 29). The view which rests on the plane represented by VH_2 shows a bird's-eye view of the house, in which the eye of the observer (always in VH) is at a distance above the plane on which the view rests, equal to about two and one-half times the height of the ridge of the house above the ground.

These two perspective projections illustrate the effect of changing the distance between VH and the vertical trace (§ 34, note) of the plane on which the perspective projection is supposed to rest. The construction of both views is exactly the same. The following explanation applies to both equally well, and the student may consider either in studying the problem.

61. We will first neglect the roof of the house, and of the porch. The remaining portion of the house will be seen to consist of two rectangular blocks, one representing the main body of the house, and the other representing the porch.

The block representing the main part of the house occupies a position exactly similar to that of the block shown in Fig. 19. First consider this block irrespective of the remainder of the house. A vertical line dropped from the corner of the diagram that lies in HPP will be a measure line for the block, and will establish, by its intersection with VH_1 (or VH_2), the position of the point e^P, in exactly the same way that the point e^P in Fig. 19 was established. $e^P a^P$ shows the true height of the part of the house under consideration, and should be made equal to the corresponding height $a^V e^V$, as shown by the elevations. The rectangular block representing the main part of the house may now be drawn exactly as was the block in Fig. 19, Problem IV.

62. Having found the perspective of the main part of the house, the porch (without its roof) may be considered as a second rectangular block, no vertical edge of which lies in the picture plane. It may be treated in a manner exactly similar to that of the block shown in Fig. 21, § 59. We may consider that the *rear* vertical face of the block, which forms the porch of the house (g, q), has been extended until it intersects the picture plane in the line ae, giving a line of measures for this face, just as in Fig. 21 the *nearest* vertical face of the block was extended until it intersected the picture plane in the line of measures mn.

On $e^{P}a^{P}$, make $e^{P}c^{P}$ equal to the true height of the vertical wall of the porch, as given by the elevation. A line through c^{P}, vanishing at v^{ab}, will be the perspective of the upper horizontal edge of the rear face of the block which forms the porch. The line through e^{P}, vanishing at v^{ab}, which forms the lower edge of the front face of the main body of the house, also forms the lower edge of the rear face of the porch. Through the point h in the diagram, draw a visual ray, and through the intersection of this visual ray with HPP drop a vertical line. Where this vertical line crosses the upper and lower horizontal edges of the rear face of the porch, will establish the points g^{P} and h^{P} respectively. Having found the vertical edge $g^{P}h^{P}$, the remainder of the perspective of the porch (except the roof) can be found without difficulty, the horizontal edges of the porch vanishing at either v^{ab} or v^{ad}, according to the system to which they belong. Each vertical edge of the porch will be vertically below the point where HPP is crossed by a visual ray drawn through the point in the diagram which represents that edge. The fact that the porch projects, in part, in front of the picture plane, as indicated by the relation between the positions of the diagram and HPP, makes absolutely no difference in the construction of the perspective projection.

All of the vertical construction lines have not been shown in the figure, as this would have made the drawing too confusing. The student should be sure that he understands how every point in the perspective projection has been obtained, and, if necessary, should complete the vertical construction lines with pencil.

63. Having found the perspective of the vertical walls of the main body of the house, and of the porch, the next step will be to consider the roof of the main part of the house.

Imagine the horizontal line tw, which forms the ridge of the roof, to be extended until it intersects the picture plane. This is shown on the diagram by the extension of the line tw until it intersects HPP. From this intersection drop a vertical line, as indicated in the figure. This vertical line may be considered to be the line of measures for an imaginary vertical plane passing through the ridge of the house, as indicated by the dotted lines in the plan and elevations of the house. On this line of meas-

ures, lay off the distance nm measured from VH_1 (or VH_2), equal to the true height of the ridge above the ground as given by the elevations of the house. A line drawn from the point m, vanishing at v^{ab}, will represent the ridge of the house, indefinitely extended. From the points t and w in the diagram draw visual rays. From the intersections of these visual rays with HPP drop vertical lines which will establish the positions of t^P and w^P on the perspective of the ridge of the roof. Lines drawn from t^P and w^P to the corners of the vertical walls of the house, as indicated, will complete the perspective of the roof.

To find the perspective of the porch roof, draw a visual ray through the point y on the diagram, and from its intersection with HPP drop a vertical. Where this vertical crosses the line a^Pb^P will give y^P, one point in the perspective of the ridge of the porch. The perspective of the ridge will be represented by a line through y^P, vanishing at v^{ad}. The point z^P in the ridge will be vertically below the intersection of HPP with the visual ray drawn through the point z on the diagram. Lines drawn from y^P and z^P to the corners of the vertical walls of the porch, as indicated, will complete the perspective of the porch roof.

64. The perspective of the chimney must now be found. It will be seen that the chimney is formed by a rectangular block; and if it is supposed to extend down through the house, and rest upon the ground, it will be a block under exactly the same conditions as the one shown in Fig. 21, § 59. In order to find its perspective, extend its front vertical face, as indicated on the diagram, till it intersects HPP. A vertical line dropped from this intersection will be a line of measures for the front face of the chimney, and the distance ps, laid off on this line from VH_1 (or VH_2), will show the true height of the top of the chimney above the ground, as given on the elevation. The distance so, measured from the point s on the line of measures, will be the true vertical height of the face of the chimney that is visible above the roof. Lines through s and o, vanishing at v^{ab}, will represent the horizontal edges of the front visible face of the chimney. The vertical edges of this face will be found vertically below the points where HPP is crossed by the visual rays drawn through the horizontal projections of these edges on the diagram.

Having determined the perspective of the front face of the chimney, the perspectives of the remaining edges may be found as in the cases of the rectangular blocks already discussed. From the point r in the diagram, where the ridge of the roof intersects the left hand vertical face of the chimney, draw a visual ray intersecting HPP, and from this intersection drop a vertical line to the perspective of the ridge of the house, giving the perspective (r^P) of the point where the ridge intersects the left hand face of the chimney. A line drawn from r^P to the nearest lower corner of the front face of the chimney will be the perspective of the intersection of the plane of the roof with the left hand face of the chimney.

65. The problem of finding the perspectives of the windows and door is exactly similar to that of finding the perspective of the square *hgkl* on the surface of the block shown in Fig. 20.

It will be noticed that the intersection with the picture plane of the left hand vertical face of the porch gives a line of measures (§ 55 and § 59, note) for this face. This line may be used conveniently in establishing the height of the window in the porch.

At this point in the course the student should solve Plate III.

VANISHING POINTS OF OBLIQUE LINES.

66. The perspective of the house in the last problem was completely drawn, using only the vanishing points for the two principal systems of horizontal lines. By this method it is possible to find the perspective projection of any object. But it is often advisable, for the sake of greater accuracy, to determine the vanishing points for systems of oblique lines in the object, in addition to the vanishing points for the horizontal systems.

Take, for example, the lines g^Py^P and x^Pz^P in Fig. 22. The perspective projections of these two lines were obtained by first finding the points g^P, y^P, x^P, and z^P, and then connecting g^P with y^P, and x^P with z^P. As the distances between g and y, and x and z, are very short, a slight inaccuracy in determining the positions of their perspectives might result in a very appreciable inaccuracy in the directions of the two lines g^Py^P and x^Pz^P. These two lines

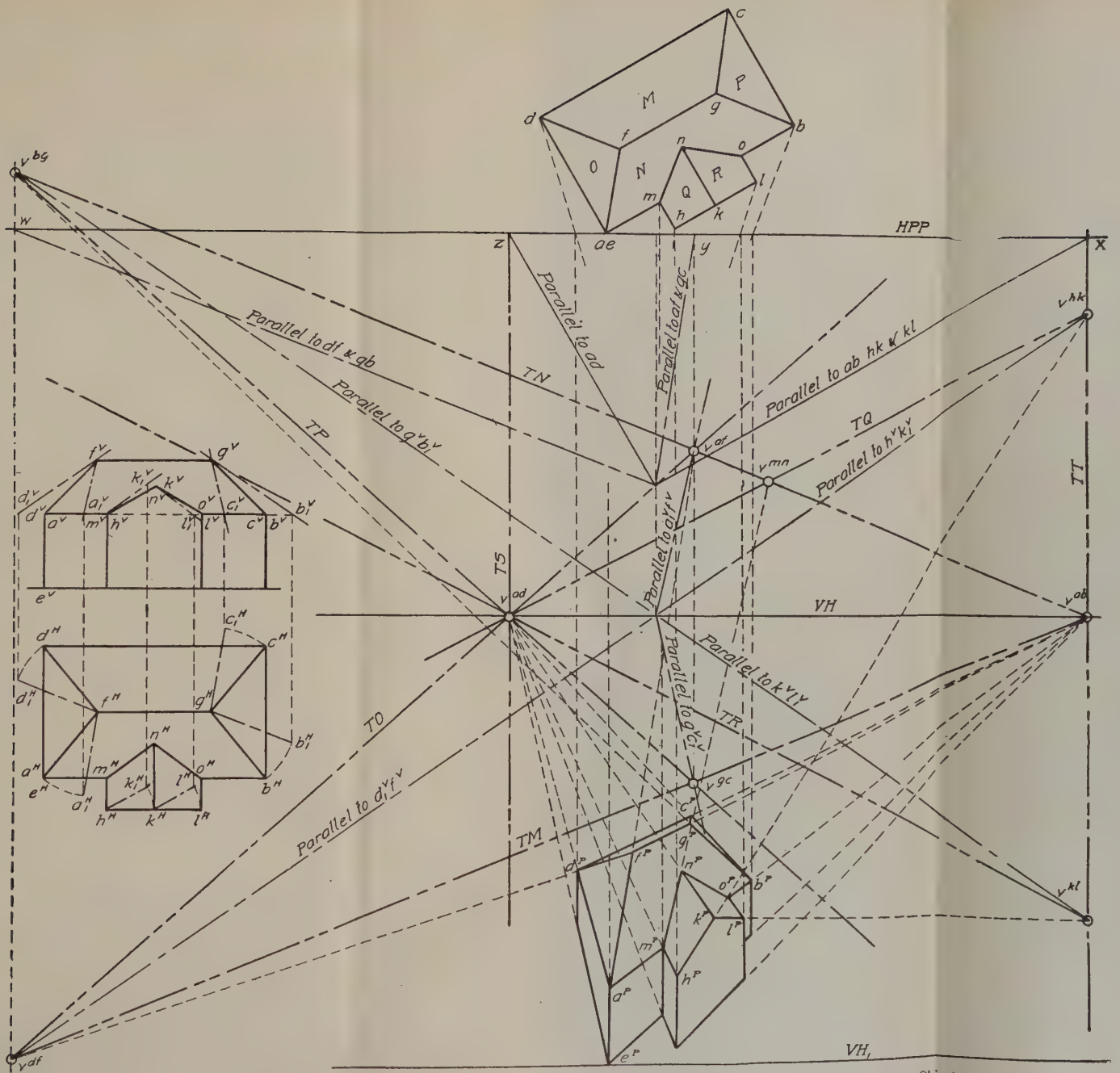


Fig. 23. Diagram Showing Method of Finding Perspective of the Vanishing Point for Each System of Lines in an Object

belong to the same system, and should approach one another as they recede. Unless the points which determine them are found with great care, the two lines may approach too rapidly, or even diverge, as they recede from the eye. In the latter case, the drawing would be absolutely wrong in principle, and the result would be very disagreeable to the trained eye. If, however, the perspective of the vanishing point of the system to which these two lines belong, can be found, and the two lines be drawn to meet at this vanishing point, the result will necessarily be accurate.

The line through r^P, which forms the intersection between the roof of the house and the left hand face of the chimney, is a still more difficult one to determine accurately. Its length is so short that it is almost impossible to establish its exact direction from the perspective projections of its extremities. If the perspective of its vanishing point can be found, however, its direction at once becomes definitely determined.

67. It is not a difficult matter to find the perspective of the vanishing point for each system of lines in an object. The method is illustrated in Fig. 23. The general method for finding the perspective of the vanishing point for any system of lines has already been stated in § 24 *g*, and illustrated in Figs. 16, 17, and 18, §§ 50, 51, and 52. It remains only to adapt the general method to a particular problem, such as that shown in Fig. 23.

The plan and elevation of a house are given at the left of the figure. The diagram has been drawn at the top of the sheet, turned at the desired angle. The assumed position of the station point is indicated by its two projections, SP^V and SP^H. VH necessarily passes through SP^V.

68. In order to find the perspective of the vanishing point of any system of lines, the vertical and horizontal projections of some element of the system must be known (see method of Problem III.). The diagram gives the horizontal projection of every line in the object which is to appear in the perspective projection. The diagram, however, has been turned through a certain horizontal angle in order to show the desired perspective view, and there is no revolved elevation to agree with the revolved position of the diagram. A revolved elevation could, of course, be con-

structed by revolving the given plan of the object until all its lines were parallel to the corresponding lines in the diagram, and then finding the revolved elevation of the object corresponding to the revolved position of the plan.

Having constructed the revolved plan and elevation of the object to agree with the position of the diagram, we should then have the vertical and horizontal projections of a line parallel to each line that is to appear in the perspective drawing, and the method of Problem III could be applied directly.

This is exactly the process that will be followed in finding the vanishing points for the oblique lines in the object, except that instead of making a complete revolved plan and elevation of the house, each system of lines will be considered by itself, and the revolved plan and elevation of each line will be found as it is needed, without regard to the remaining lines in the object.

69. All the lines in the house belong to one of eleven different systems that may be described as follows:

A vertical system, to which all the vertical lines in the house belong. The perspective of the vanishing point of this system cannot be found within finite limits (§ 54).

Two horizontal systems parallel respectively to *ab* and *ad* (see diagram). The perspectives of the vanishing points of these systems will be found in VH (§ 24 *c*, note).

Five systems of lines vanishing upward, parallel respectively to *af*, *bg*, *mn*, *on* and *hk* (see diagram). The perspectives of the vanishing points of these systems will be found to lie above VH (§ 51, note).

Three systems of lines vanishing downward, parallel respectively to *fd*, *gc* and *kl* (see diagram). The perspectives of the vanishing points of these systems will be found below VH (§ 51, note).

NOTE. — To determine whether a line vanishes upward or downward, proceed as follows:—

Examine the direction of the line as shown in the *diagram*. Determine which end of the line is the farther behind the picture

plane. If the *more distant* end of the line is *above* the nearer end in the elevation, the line vanishes *upward*, and the perspective of its vanishing point will be found above VH.

If, on the other hand, the *more distant* end of the line is *lower* than the nearer end in the elevation, the line vanishes *downward*, and the perspective of its vanishing point will be found below VH.

For illustration, consider the line *bg*. The diagram shows the point *g* to be farther behind the picture plane than the point *b*, while the given elevation shows the point *g* to be higher than the point *b*. Therefore the line *rises* as it *recedes*, or, in other words, it *vanishes upward*.

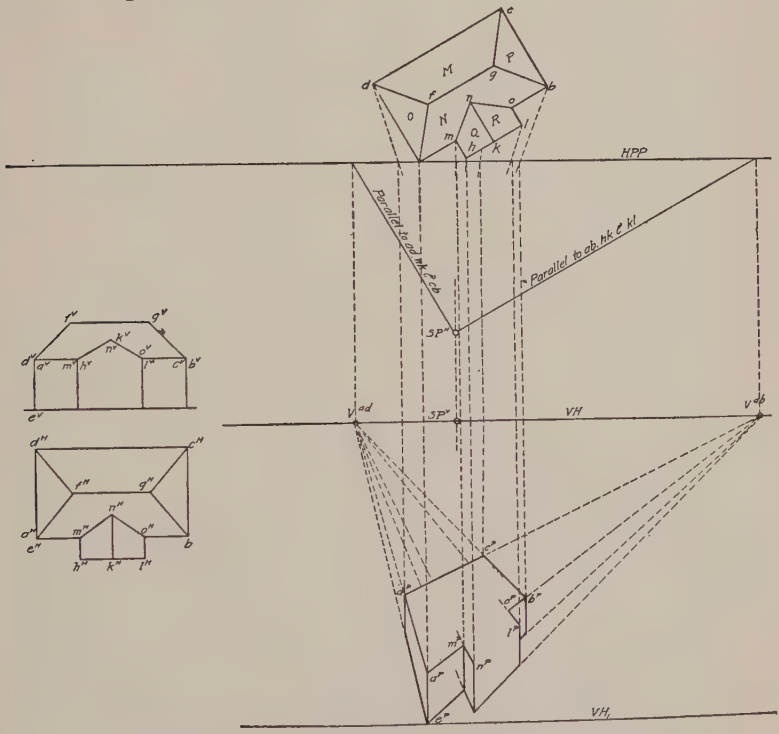


Fig. 23a. *Step 1* shows the method of finding the vanishing points for the horizontal lines of the house

In the case of the line *fd*, the diagram shows the point *d* to be farther behind the picture plane than the point *f*, while the elevation shows the point *d* to be lower than the point *f*. There-

fore the line must vanish downward, and its vanishing point be found below VH.

If the horizontal projection of a line, as shown by the diagram, is parallel to HPP, the line itself is parallel to the picture plane, and the perspective of its vanishing point cannot be found within finite limits (§ 54, note). *The perspective projections of such a system of lines will show the true angle which the elements of the system make with the horizontal coordinate.*

70. The construction for the vanishing points in Fig. 23 is shown by dot and dash lines. The solution of this problem can best be understood by following the several steps in Fig. 23a.

Step 1. This step shows the method of finding the vanishing points of the horizontal lines represented by the lines, *ab*, *bc*, *cd*, *de*, *hi*, *hm*, *kn* and *lo* in the diagram. It will be seen that all these lines belong to definite systems, some of which vanish to the right at V^{ab} while the others vanish to the left at V^{ad}.

It will be seen that the roof lines have been omitted, and as these are a definite part of the problem, each of the lines in the roof will be treated individually.

Step 2. Consider the line *af*. First, construct a revolved plan and elevation of this line to agree with the position of the diagram. Revolve the horizontal projection ($a^H f^H$) of the line in the given plan about the point f^H, or 30°, until it is parallel to the line *af* in the diagram. During this revolution, the point f^H remains stationary, while the point a^H describes a horizontal arc, until $a^H f^H$ has revolved into the position shown by the line $a_1{}^H f^H$, which is parallel to the line *af* in the diagram. The vertical projection $a^V f^V$ must, of course, revolve with the horizontal projection. The point f^V remains stationary, while the horizontal arc described by the point *a* shows in vertical projection as a horizontal line. At every point of the revolution the vertical projection of the point *a* must be vertically in line with its horizontal projection. When a^H has reached the position $a_1{}^H$, a^V will be vertically above $a_1{}^H$ at the point $a_1{}^V$, and $a_1{}^V f^V$ will be the revolved elevation of the line.

We now have the vertical and horizontal projections ($a_1{}^V f^V$ and $a_1{}^H f^H$) of an element of the system to which the roof line, represented in the diagram by *af*, belongs. The vanishing point of

this system may be determined as in Problem III. Draw through SP^H a line parallel to $a_1{}^H f^H$ (or af in the diagram), representing the horizontal projection of the visual element of the system. Draw through SP^V a line parallel to $a_1{}^V f^V$, representing the vertical projection of the visual element of the system. The visual element,

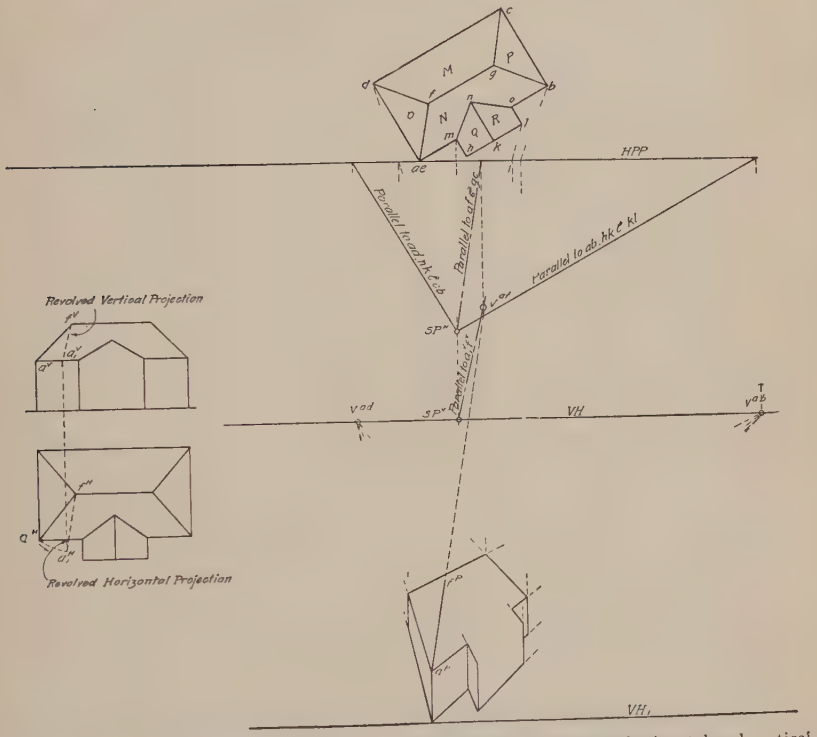


Fig. 23a. *Step 2* shows the method of revolving the roof line af to find its horizontal and vertical projection, which conforms to the position of the diagram. It will be seen that the vanishing point is shown at V^{af}

represented by the two projections just drawn, pierces the picture plane at v^{af} (§ 45, note), giving the perspective of the vanishing point for the roof line, represented by af in the diagram.

Step 3. Consider the line bg. First find, from the given plan and elevation of the object, a revolved plan and elevation of bg, to agree with the position of the line in the diagram. Revolve $b^H g^H$ in the given plan about the point g^H, until it is parallel to bg in the diagram, and occupies the position indicated by the line

$b_1{}^{H}g^{H}$. The corresponding revolved elevation is represented by the line $b_1{}^{V}g^{V}$.

$b_1{}^{H}g^{H}$ and $b_1{}^{V}g^{V}$ now represent respectively the horizontal and vertical projections of an element of the system to which the

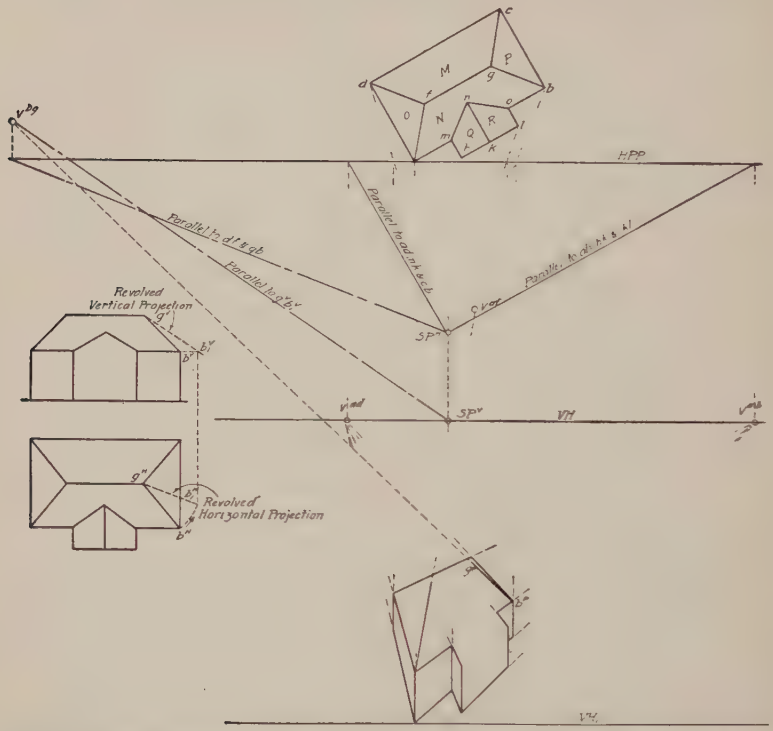


Fig. 23a. *Step 3* shows another example of revolving a line and finding its vanishing point

roof line, represented by *bg* in the diagram, belongs. The vanishing point of this system can be found by Problem III. Through SP^{H} draw a line parallel to $b_1{}^{H}g^{H}$ (or *bg* in the diagram), representing the horizontal projection of the visual element of the system; and through SP^{V} draw a line parallel to $b_1{}^{V}g^{V}$, representing the vertical projection of their visual element. The visual element, represented by these two projections, pierces the picture plane at v^{bg}, giving the perspective of the vanishing point of the roof line, represented in the diagram by the line *bg*.

By a similar process, $h^{H}k_1{}^{H}$ and $h^{V}k_1{}^{V}$ are found to represent respectively the horizontal and vertical projections of an element

of the system to which belongs the roof line represented in the diagram by the line *hk*. The perspective of the vanishing point of this line has been found at v^{hk}.

v^{af}, b^{bg} and v^{hk} have all been found to lie above VH (§ 51, note).

71. v^{fd}, v^{gc} and v^{kl} are found exactly as were v^{af}, v^{bg} and v^{hk}; but, as the systems to which they belong vanish downward, they will lie below VH (§ 51, note).

Step 4. Consider the line *fd*. $f^{H}d_1{}^{H}$ and $f^{V}d_1{}^{V}$ are respectively the horizontal and vertical projections of an element of the system

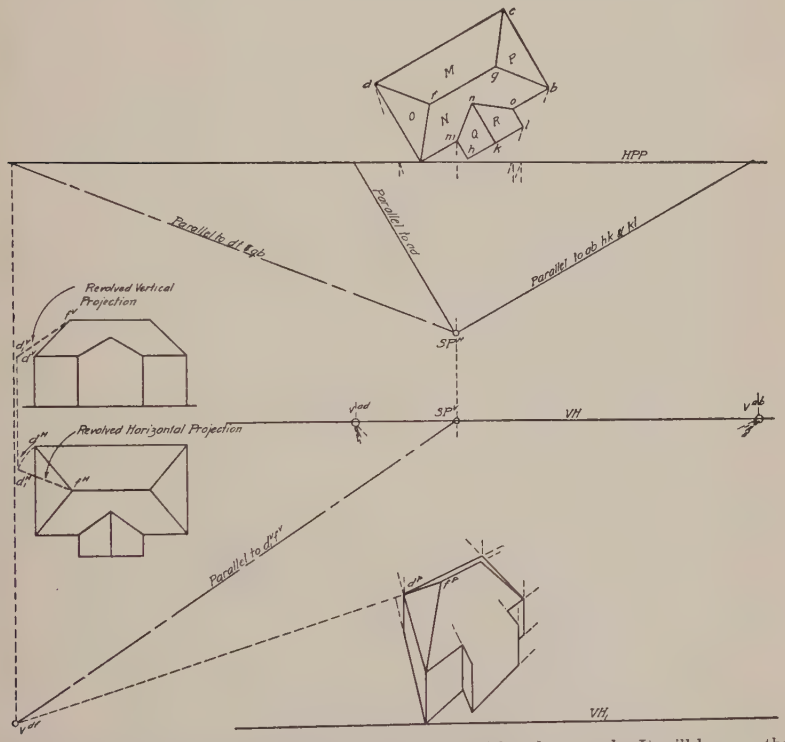


Fig. 23a. *Step 4* shows the revolution of a line which vanishes downward. It will be seen that the method of revolving the horizontal and vertical views of the line in the plan and elevation to find their horizontal and vertical projection, is the same in every case

represented by *fd* in the diagram. A line drawn through SP^{H}, parallel to $f^{H}d_1{}^{H}$ (or *fd* in the diagram), will intersect HPP in the point *w*. A vertical line through *w* will intersect a line through SP^{V} parallel to $f^{V}d_1{}^{V}$, below VH.

It will be noticed in every case that the perspectives of the vanishing points are all located on the dotted line, dropped down from the point of intersection of the horizontal projection of the visual ray and HPP and the vertical projection of the visual ray, which is the line drawn parallel to the revolved line in the elevation.

72. Having found the perspectives of these vanishing points, the perspectives of the vanishing traces of all the planes in the object should be drawn as a test of the accuracy with which the vanishing points have been constructed. The roof planes in the house are lettered with the capital letters M, N, O, P, etc., on the diagram.

The plane O contains the lines *af*, *ad*, and *fd*. Therefore, the vanishing trace (TO) of the plane must be a straight line passing through the three vanishing points, v^{af}, v^{ad}, and v^{fd} (§ 24 *d*). If all three of these vanishing points do not lie in a straight line, it shows some inaccuracy, either in draughting or in the method used in finding some of the vanishing points. The student should not be content until the accuracy of his work is proved by drawing the vanishing trace of each plane in the object through the vanishing points of all lines that lie in that plane.

The plane M contains the lines *fd*, *gc*, and *dc*. The last line belongs to the system *ab*, and hence its vanishing point is v^{ab}. The vanishing trace (TM) of the plane M must pass through v^{fd}, v^{gc}, and v^{ab}.

Similarly, the vanishing trace (TP) of the plane P must pass through v^{gc}, v^{bg}, and v^{ad}. TN must pass through v^{ab}, v^{af}, and v^{bg}. TQ must pass through v^{hk} and v^{ad}. TR must pass through v^{kl} and v^{ad}.

73. The vanishing trace of a vertical plane will always be a *vertical line* passing through the vanishing points of all lines which lie in the plane. Therefore, the vanishing trace of the vertical planes in the house that vanish towards the left will be represented by a vertical line (TS) passing through v^{ad}.

The vanishing trace of the vertical planes of the house that vanish towards the right will be represented by a vertical line (TT) passing through v^{ab}. As the vertical plane which forms the face of the porch belongs to this system, and as this plane also contains the lines *hk* and *kl*, TT will be found to pass through v^{hk} and v^{kl} as well as v^{ab}.

74. It will be noticed that the vanishing points for the lines *mn* and *on* have not been found. These vanishing points might have been found in a manner exactly similar to that in which the other vanishing points were found, or they may be determined now, directly from the vanishing traces already drawn, in the following manner:—

The line *mn* is seen to be the line of intersection of the two planes N and Q. Therefore (§ 24 *e*) v^{mn} must lie at the intersection of TN and TQ.

For a similar reason, v^{on} must lie at the intersection of TN and TR. TN and TR do not intersect within the limits of the plate, but they are seen to converge as they pass to the left, and, if produced in that direction, would meet at the vanishing point for the line *on*.

75. Having found v^{ab}, v^{ad}, v^{bg}, and v^{fd}, TN could have been drawn through v^{ab} and v^{bg}; and TO could have been drawn through v^{ad} and v^{fd}. As *af* is the intersection of the two planes N and O, v^{af} could have been found at the intersection of TN and TO without actually constructing this vanishing point.

Similarly, v^{gc} could have been determined by the intersection of TM and TP.

By an examination of the plate, the student will notice that the vanishing point for each line in the object is formed at the intersection of the vanishing traces of the two planes of which the line forms the intersection. Thus, the line *ad* forms the intersection between the plane O and the left hand vertical face of the house. v^{ad} is found at the intersection of TO and TS.

The line *fg*, which forms the ridge of the roof, is the intersection of the planes M and N. The vanishing point for *fg* is v^{ab}, and TM and TN will be found to intersect at v^{ab}. v^{hk} is found at the intersection of TQ and TT, v^{kl} is found at the intersection of TR and TT, etc.

It will be noticed also that the two lines *hk* and *kl* lie in the same vertical plane, and make the same angle with the horizontal, one vanishing upward, and one vanishing downward. Since both lines lie in the same vertical plane, both of their vanishing points will be found in the vertical line which represents the vanishing trace of that plane. Also, since both lines make equal angles with

the horizontal, the vanishing point of the line vanishing upward will be found as far above VH as the vanishing point of the line vanishing downward is below VH.

In a similar way, the line *bg* vanishes upward, and the line *fd* vanishes downward; each making the same angle with the horizontal (as shown by the given plan and elevation). These two lines do not lie in the same plane, but may be said to lie in two imaginary vertical planes which are parallel to one another. Their vanishing points will be seen to lie in the same vertical line, v^{bg} being as far above VH as v^{fd} is below it.

As a general statement, it may be said that if two lines lie in the same or parallel vertical planes, and make equal angles with the horizontal, one vanishing upward and the other vanishing downward, the vanishing points for both lines will be found vertically in line with one another, one as far above VH as the other is below it.

This principle is often of use in constructing the vanishing point diagram. Thus, having found v^{hk}, v^{kl} could have been determined immediately by making it lie in a vertical line with v^{hk}, and as far below VH as v^{hk} is above it.

VANISHING POINT DIAGRAM

76. The somewhat symmetrical figure formed by the vanishing traces of all the planes in the object, together with all vanishing points, HPP, and the vertical and horizontal projections of the station point, is called the *Vanishing Point Diagram* of the object.

77. Having found the complete vanishing point diagram of the house, the perspective projection may be drawn. VH_1 may be chosen in accordance with the kind of a perspective projection it is desired to produce (§ 29). In order that all the roof lines may be visible, VH_1 has been chosen far below VH. The resulting perspective is a somewhat exaggerated bird's-eye view.

The point e^P will be found on VH_1, vertically under the point e in the diagram. $a^P e^P$ lies in the picture plane, and shows the true height of the vertical wall of the house. From a^P and e^P, the horizontal edges of the walls of the main house vanish to i^{ab} and i^{ad}.

The points d^P, b^P, m^P, and o^P are found on the upper horizontal edges of the main walls, vertically under the points where HPP is crossed by visual rays drawn through the points d, b, m, and o in the diagram. Vertical lines from d^P and b^P complete the visible vertical edges of the main house.

In a similar manner the perspective of the vertical walls of the porch is obtained.

Each roof line vanishes to its respective vanishing point. $a^P f^P$ vanishes at v^{af}. $f^P d^P$ vanishes at v^{fd}. These two lines intersect in the point f^P. The ridge of the main house passes through f^P, vanishing at v^{ab}. $g^P c^P$ vanishes at v^{gc}, passing through the point c^P, which has already been determined by the intersection of the two upper rear horizontal edges of the main walls. $b^P g^P$ vanishes at v^{bg}, completing the perspective of the main roof.

In the porch, $h^P k^P$ vanishes at v^{hk}, passing through the point h^P, already determined by the vertical walls of the porch. $k^P l^P$ passes through l^P, and vanishes at v^{kl}. From k^P the ridge of the porch roof vanishes at v^{ad}. From m^P, a line vanishing at v^{mn} will intersect the ridge in the point n^P, and represent the intersection of the roof planes Q and N. The vanishing point for $o^P n^P$ falls outside the limits of the plate. $o^P m^P$ may be connected with a line which, if the drawing is accurate, will converge towards both TN and TR, and, if produced, would meet them at their intersection. The best method to use in a case like this is to find the vanishing point of the horizontal line in the main body of the house first, and then find the prospectus of the vanishing point to the oblique line of the roof, etc.

78. While constructing the vanishing point diagram of an object, the student should constantly keep in mind the general statements made in the note under § 69.

Plate IV should now be solved.

PERSPECTIVE DRAWING

PART III

PARALLEL OR ONE-POINT PERSPECTIVE

79. When the diagram of an object is placed with one of its principal systems of horizontal lines parallel to the picture plane, it is said to be in **Parallel Perspective.** This is illustrated in Fig. 24, by the rectangular block there shown.

One system of horizontal lines in the block being parallel to the picture plane, the other system of horizontal lines must be perpendicular to the picture plane.

The vanishing point for the latter system will be coincident with SP^v (§ 52). The horizontal system that is parallel to the

A Good Example of Perspective

picture plane will have no vanishing point within finite limits (§ 54, with note; also last paragraph of note under § 69).

The third system of lines in the object is a vertical one, and will have no vanishing point within finite limits (§ 54). Thus, of the three systems of lines that form the edges of the block, only one will have a vanishing point within finite limits. This fact has led to the term One-Point Perspective, which is often applied to an object in the position shown in Fig. 24. As will be seen, this

is only a special case of the problems already studied, and the construction of the perspective of an object in parallel perspective is usually simpler than when the diagram is turned at an angle with HPP.

80. The vertical face ($abf'e$) of the block lies in the picture plane. It will thus show in its true size and shape (§ 24 *h*). The

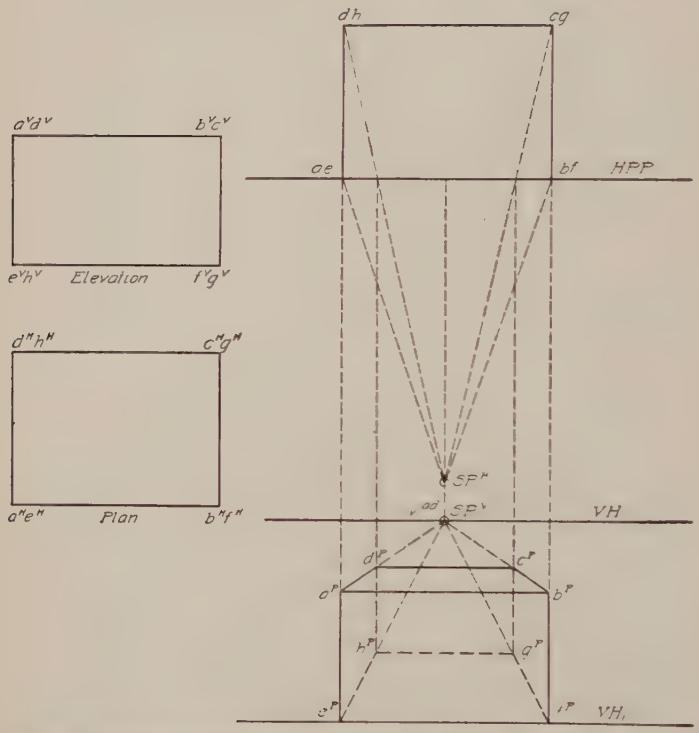


Fig. 24. Diagram Showing Method of Drawing Parallel, or One-Point, Perspective

points e^P and f^P will be found on VH_1 vertically below the points e and f in the diagram.

81. Both the edges e^Pa^P and f^Pb^P are lines of measures, and will show the true height of the block, as given by the elevation.

82. The two lines a^Pb^P and e^Pf^P, since they are formed by the intersection of the bases of the block with the picture plane, will also be lines of measures (§ 55), and will show the true length of the block, as given by the plan and elevation.

83. The perspective of the front face of the block, which is

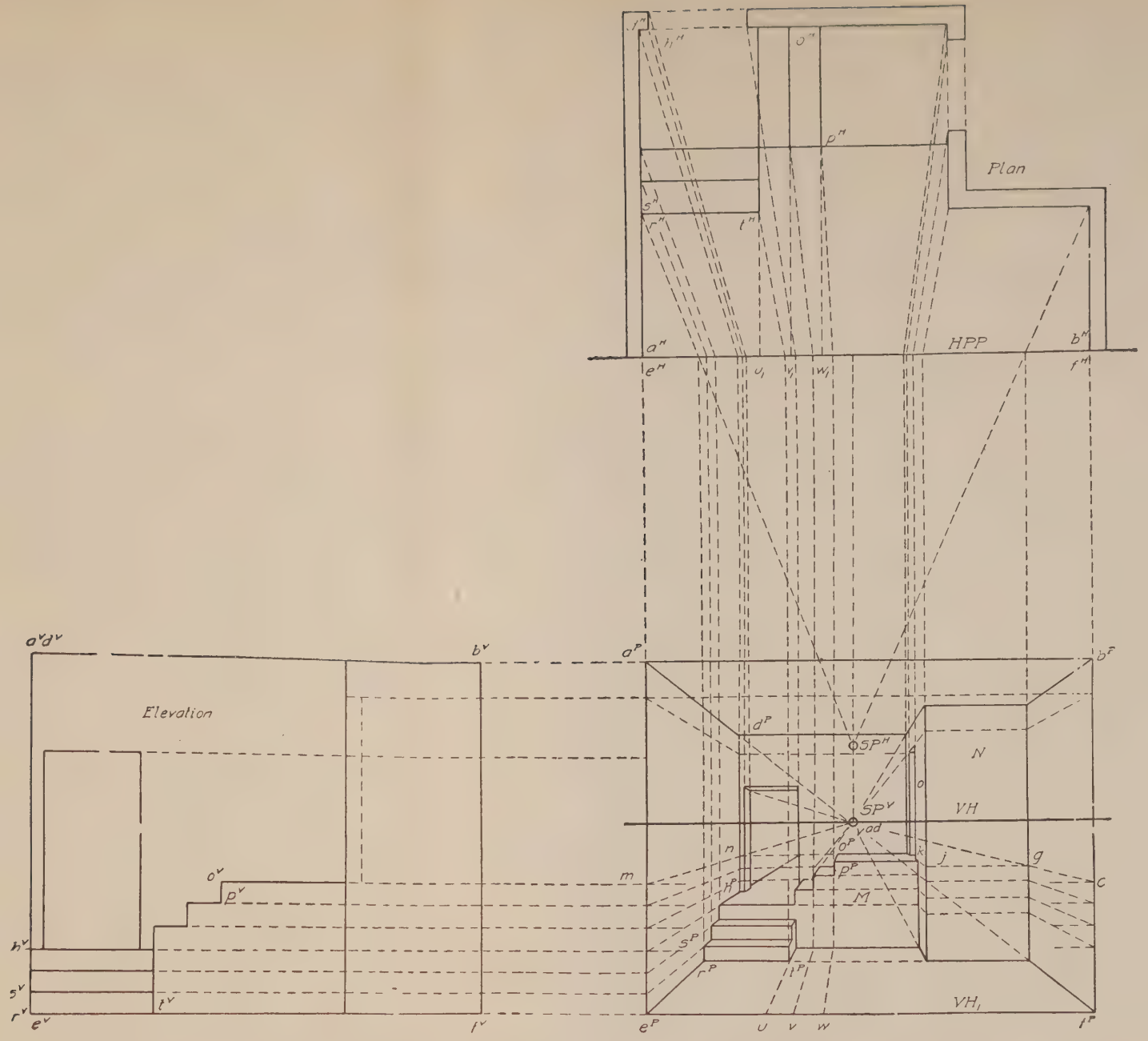


Fig. 25. Diagram Showing Method of Constructing an Interior Parallel Perspective

coincident with the picture plane, can be drawn immediately. From a^P, b^P, e^P, and f^P, the horizontal edges, which are perpendicular to the picture plane, will vanish at v^{ad} (coincident with SP^V). The rear vertical edges of the block may be found in the usual manner.

84. The lines a^Pb^P, d^Pc^P, e^Pf^P, and h^Pg^P, which form the horizontal edges parallel to the picture plane, will all be drawn parallel to one another (§ 54, note); and since the lines in space which they represent are horizontal, a^Pb^P, d^Pc^P, e^Pf^P, and h^Pg^P will all be horizontal (see last paragraph of note under § 69).

All of the principles that have been stated in connection with the other problems will apply equally well to an object in parallel perspective.

85. Interior views are often shown in parallel perspective. One wall of the interior is usually assumed coincident with the picture plane, and is not shown in the drawing. For illustration, the rectangular block in Fig. 24 may be considered to represent a hollow box, the interior of which is to be shown in perspective. Assume the face $(a^Pb^Pf^Pe^P)$ that lies in the picture plane to be removed. The resulting perspective projection would show the interior of the box. In making a parallel perspective of an interior, however, VH is usually drawn lower than is indicated in Fig. 24, in order to show the inside of the upper face, or ceiling, of the interior. With such an arrangement, three walls, the ceiling, and the floor of the interior, may all be shown in the perspective projection.

86. Fig. 25 shows an example of interior parallel perspective. The plan of the room is shown at the top of the plate. This has been placed so that it may be used for the diagram, and save the necessity of making a separate drawing. The elevation of the room is shown at the left of the plate, and for convenience it has been placed with its lower horizontal edge in line with VH_1. In this position all vertical dimensions in the object may be carried by horizontal construction lines directly from the elevation to the vertical line of measures (a^Pe^P or b^Pf^P) in the perspective projection.

87. The front face of the room $(a^Pb^Pf^Pe^P)$, which is coincident with the picture plane, may first be established. Each point

in the perspective of this front face will be found to lie vertically under the corresponding point in plan, and horizontally in line with the corresponding point in elevation. Thus, a^{P} is vertically under a^{H}, and horizontally in line with a^{V}.

All lines in the room which are perpendicular to the picture plane vanish at v^{ad} (coincident with SP^{V}).

Drawing visual rays from every point in the diagram, the corresponding points in the perspective projection will be vertically under the points where these visual rays intersect HPP. The construction of the walls of the room should give the student no difficulty.

88. In finding the perspective of the steps, the vertical heights should first be projected by horizontal construction lines from the elevation to the line of measures ($a^{P}e^{P}$), as indicated by the divisions between e^{P} and m. These divisions can then be carried along the left hand wall of the room by imaginary horizontal lines vanishing at v^{ad}. The perspective of the vertical edge where each step intersects the left hand wall may now be determined from the plan. Thus, the edge $s^{P}r^{P}$ of the first step is vertically below the intersection of HPP with a visual ray drawn through the point s^{H} in plan, and is between the two horizontal lines projected from the elevation that show the height of the lower step. The corresponding vertical edge of the second step will be projected from the plan in a similar manner, and will lie between the two horizontal lines projected from the elevation that show the height of the second step, etc.

From s^{P} the line which forms the intersection of the wall with the horizontal surface of the first step will vanish to v^{ad}, etc.

From r^{P} the intersection of the first step with the floor of the room will be a line belonging to the same system as $a^{P}b^{P}$, and will therefore show as a true horizontal line. The point t^{P} may be projected from the diagram by a visual ray, as usual. From t^{P} the vertical edge of the step may be drawn till it intersects a horizontal line through s^{P}, and so on, until the steps that rest against the side wall are determined.

89. The three upper steps in the flight rest against the rear wall. The three upper divisions on the line $e^{P}m$ may be carried along the left hand wall of the room, as indicated, till they inter-

sect the rear vertical edge of the wall, represented by the line $d^{P}h^{P}$. From these intersections the lines may be carried along the rear wall of the room, showing the heights of the three upper steps where they rest against the rear wall.

The three upper divisions on the line $e^{P}m$ have also been projected across to the line $f^{P}b^{P}$, and from this line carried by imaginary horizontal lines along the right hand wall of the room to the plane N, across the plane N to the plane O, and from the plane O to the plane M. Thus, for illustration, the upper division, representing the height of the upper step, has been carried from m to c; from c to g along the right hand face of the wall; from g to j along the plane N; from j to k on the plane O, and from k to p^{P} on the plane M.

The point p^{P} is where the line which represents the height of the upper step meets a vertical dropped from the intersection of HPP with a visual ray through the point p^{H} in the diagram. p^{P} is one corner in the perspective of the upper step, the visible edges of the step being represented by a horizontal line, $p^{P}k$, a line ($p^{P}o^{P}$) vanishing at v^{ad}, and a vertical line drawn from p^{P} between the two horizontal lines on the plane H, which represent the height of the upper step. The point o^{P} is at the intersection of the line drawn through p^{P}, vanishing through v^{ad}, with the horizontal line on the rear wall drawn through the point n, and representing the upper step where it rests against the rear wall.

The remaining steps may be found in a similar manner. The student should have no difficulty in following out the construction, which is all shown on the plate.

90. The position of the point t^{P} on the line $r^{P}t^{P}$ was determined by projecting in the usual manner from the diagram. The position of t^{P} might have been found in the following manner: In the figure the line $e^{P}f^{P}$ is a line of measures (§ 81), and divisions on this line will show in their true size. Thus, if we imagine a horizontal line to be drawn through t^{P}, parallel to the wall of the room, it will intersect $e^{P}f^{P}$ in the point u. Since $e^{P}u$ is on a line of measures, it will show in its true length. Thus, t^{P} might have been determined by laying off $e^{P}u$ equal to the distance $e^{H}u_{1}$ taken from the plan, and then drawing through the point u a line van-

ishing at v^{ad}. The intersection of this line with the horizontal line drawn through r^P will determine t^P.

In a similar manner the vertical edges of the steps, where they intersect the plane M, might have been found by laying off from u, on $e^P f^P$, the divisions uv and vw taken from the plan. These divisions could have been carried along the floor by horizontal lines parallel to the sides of the room (vanishing at v^{ad}), to the plane M, and then projected vertically upward on the plane M, as indicated in the figure. Solve Plate V.

METHOD OF PERSPECTIVE PLAN.

91. In the foregoing problems the perspective projection has been found from a diagram of the object. Another way of constructing a perspective projection is by the method of **Perspective Plan.** In this method no diagram is used, but a perspective plan of the object is first made, and from this perspective plan the perspective projection of the object is determined. The perspective plan is usually supposed to lie in an auxiliary horizontal plane below the plane of the ground. The principles upon which its construction is based will now be explained.

92. In Fig. 26, suppose the rectangle $a^H b^H c^H d^H$ to represent the horizontal projection of a rectangular card resting upon a horizontal plane. The diagram of the card is shown at the upper part of the figure. It will be used only to explain the construction of the perspective plan of the card.

First consider the line ad, which forms one side of the card. On HPP lay off from a, to the left, a distance (ae) equal to the length of the line ad. Connect the points e and d. ead is by construction an isosceles triangle lying in the plane of the card, with one of its equal sides (ae) in the picture plane. Now, if this triangle be put into perspective, the side ad, being behind the picture plane, will appear shorter than it really is; while the side ae, which lies in the picture plane, will show in its true length.

Let VH_1 be the vertical trace of the plane on which the card and triangle are supposed to rest. The position of the station point is shown by its two projections SP^H and SP^V. The vanishing point for the line ad will be found at v^{ad} in the usual manner. In a similar way, the vanishing point for the line ed, which

forms the base of the isosceles triangle, will be found at v^{ed}, as indicated. a^P will be found on VH_1 vertically under the point a, which forms the apex of the isosceles triangle ead. The line $a^P d^P$ will vanish at v^{ad}. The point e^P will be found vertically

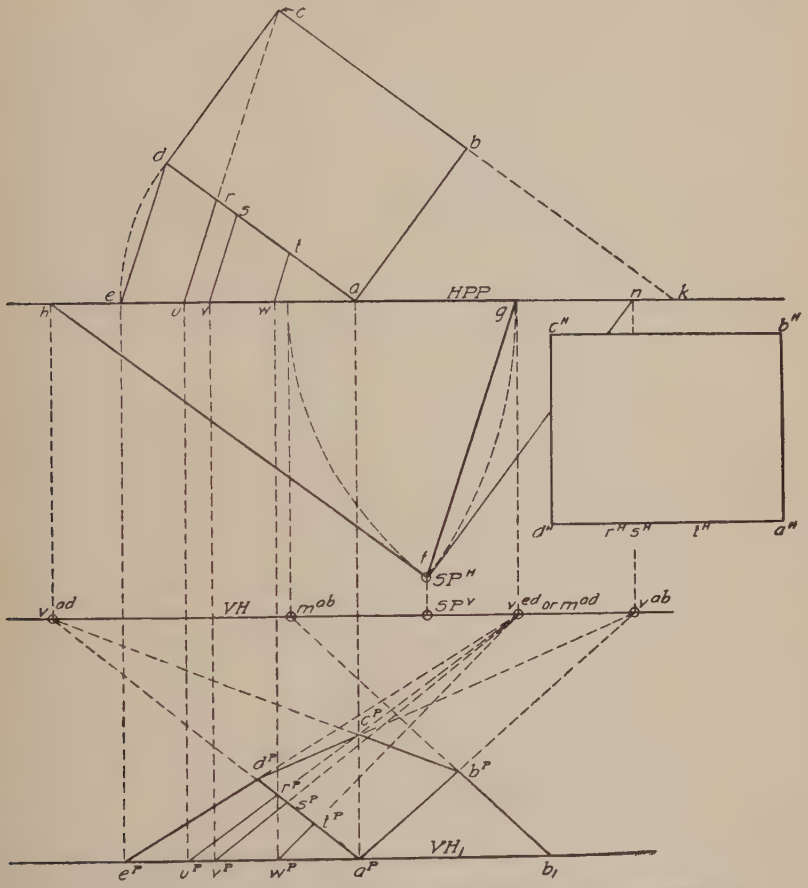


Fig. 26. Diagram Showing Perspective Constructed by Method of Perspective Plan

below the point e. $e^P d^P$ will vanish at v^{ed}, and determine by its intersection with $a^P d^P$ the length of that line. $e^P a^P d^P$ is the perspective of the isosceles triangle ead.

If the line ad in the diagram is divided in any manner by the points t, s, and r, the perspectives of these points may be found on the line $a^P d^P$ in the following way. If lines are drawn

through the points t, s, and r in the diagram parallel to the base de of the isosceles triangle (ead), these lines will divide the line ae in a manner exactly similar to that in which the line ad is divided. Thus, aw will equal at, wv will equal ts, etc. Now, in the perspective projection of the isosceles triangle, $a^{\mathrm{P}}e^{\mathrm{P}}$ lies in the picture plane. It will show in its true length, and all divisions on it will show in their true size. Thus, on $a^{\mathrm{P}}e^{\mathrm{P}}$ lay off $a^{\mathrm{P}}w^{\mathrm{P}}$, $w^{\mathrm{P}}v^{\mathrm{P}}$, and $v^{\mathrm{P}}u^{\mathrm{P}}$ equal to the corresponding distances at, ts, and sr, given in the diagram. Lines drawn through the points w^{P}, v^{P}, and u^{P}, vanishing at v^{ed}, will be the perspective of the lines wt, vs, and ur in the isosceles triangle, and will determine the positions of t^{P}, s^{P}, and r^{P}, by their intersections with $a^{\mathrm{P}}d^{\mathrm{P}}$.

93. It will be seen that after having found v^{ad} and v^{ed}, the perspective of the isosceles triangle can be found without any reference to the diagram. Assuming the position of a^{P} at any desired point on VH_1, the divisions a^{P}, w^{P}, v^{P}, u^{P}, e^{P} may be laid off from a^{P} directly, making them equal to the corresponding divisions a^{H}, t^{H}, s^{H}, r^{H}, d^{H}, given in the plan of the card. A line through a^{P} vanishing at v^{ad} will represent the perspective side of the isosceles triangle. The length of this side will be determined by a line drawn through e^{P}, vanishing at v^{ed}. The positions of t^{P}, s^{P}, and r^{P} may be determined by lines drawn through w^{P}, v^{P}, and u^{P}, vanishing at v^{ed}.

94. It will be seen that the lines drawn to v^{ed} serve to measure the perspective distances $a^{\mathrm{P}}\,t^{\mathrm{P}}$, $t^{\mathrm{P}}\,s^{\mathrm{P}}$, $s^{\mathrm{P}}\,r^{\mathrm{P}}$, and $r^{\mathrm{P}}\,d^{\mathrm{P}}$, on the line $a^{\mathrm{P}}d^{\mathrm{P}}$, from the true lengths of these distances as laid off on the line $a^{\mathrm{P}}e^{\mathrm{P}}$. Hence the lines vanishing at v^{ed} are called **Measure Lines** for the line $a^{\mathrm{P}}d^{\mathrm{P}}$, and the vanishing point v^{ed} is called a **Measure Point** for $a^{\mathrm{P}}d^{\mathrm{P}}$.

95. Every line in perspective has a measure point, which may be found by constructing an isosceles triangle on the line in a manner similar to that just explained.

NOTE. — The vanishing point for the base of the isosceles triangle always becomes the measure point for the side of the isosceles triangle which does not lie in HPP.

96. All lines belonging to the same system will have the same measure point. Thus, if the line bc, which is parallel to ad, be continued to meet HPP, and an isosceles triangle (cku) con

structed on it, as indicated by the dotted lines in the figure, the base (uc) of this isosceles triangle will be parallel to de, and its vanishing point will be coincident with v^{de}.

97. There is a constant relation between the vanishing point of a system of lines and the measure point for that system. Therefore, if the vanishing point of a system of lines is known, its measure point may be found without reference to a diagram, as will be explained.

In constructing the vanishing points v^{ad} and v^{ed}, fh was drawn parallel to ad, fg was drawn parallel to ed, and since hg is coincident with HPP, the two triangles ead and fhg must be similar. As ae was made equal to ad in the small triangle, hf must be equal to hg in the large triangle; and consequently v^{ed}, which is as far from v^{ad} as g is from h, must be as far from v^{ad} as the point f is from the point h.

If the student will refer back to Figs. 8, 9, and 9a, he will see that the point h bears a similar relation in Fig. 26 to that of the point m^{H} in Figs. 8, 9, and 9a, and that the point h in Fig. 26 is really the horizontal projection of the vanishing point v^{ad}. (See also § 32.) Therefore, as v^{ed} is as far from v^{ad} as the point h is from the point f, we may make the following statement, which will hold for all systems of horizontal lines.

98. *The measure point for any system of horizontal lines will be found on VH as far from the vanishing point of the system as the horizontal projection of that vanishing point is distant from the horizontal projection of the station point.*

NOTE. — In accordance with the construction shown in Fig. 26, SP^{V} will always lie between the vanishing point of a system and its measure point.

99. The measure point of any system of lines is usually denoted by a small letter m with an index corresponding to its related vanishing point. Thus, m^{ab} signifies the measure point for the system of lines vanishing at v^{ab}.

100. The vanishing point for ab in Fig. 26 has been found at v^{ab}. The point n, in HPP, is the horizontal projection of this vanishing point. The measure point (m^{ab}) for all lines vanishing at v^{ab} will be found on VH, at a distance from v^{ab} equal to the distance from n to SP^{H} (98). In accordance with this statement,

m^{ab} has been found by drawing an arc with n as center, and with a radius equal to the distance from n to SP^H, and dropping from the intersection of this arc with HPP a vertical line. m^{ab} is found at the intersection of this vertical line with VH.

101. The perspective of ab has been drawn from a^P, vanishing at v^{ab}. $a^P b_1$ on VH_1 is made equal to the length of $a^H b^H$ given in the plan of the card. A measure line through b_1, vanishing at m^{ab}, will determine the length of $a^P b^P$. A line from b^P vanishing at v^{ad}, and one from d^P vanishing at v^{ab}, will intersect at c^P, completing the perspective plan of the card.

102. Even the vanishing points (v^{ab} and v^{ad}) for the sides of the card may be found without drawing a diagram. Since fn is drawn parallel to ab, it makes the same angle with HPP that ab makes. Similarly, since fh is drawn parallel to ad, it makes the same angle with HPP that ad makes. The angle between fn and fh must show the true angle made by the two lines ab and ad in the diagram. Therefore, having assumed SP^H, we have simply to draw two lines through SP^H, making with HPP the respective angles that the two sides of the cards are to make with the picture plane, care being taken that the angle these two lines make with one another shall equal the angle shown between the two sides of the card in the given plan. Thus, in Fig. 27, the two projections of the station point have first been assumed. Then through SP^H, two lines (fn and fh) have been drawn, making respectively, with HPP, the angles (H° and N°) which it is desired the sides of the card shall make with the picture plane, care being taken to make the angle between fn and fh equal to a right angle, since the card shown in the given plan is rectangular.

Verticals dropped to VH from the points n and h will determine v^{ab} and v^{ad}. Having found v^{ab} and v^{ad}, m^{ab} and m^{ad} should next be determined, as explained in § 98. VH_1 should now be assumed, and a^P chosen at any point on this line. It is well not to assume a^P very far to the right or left of an imaginary vertical line through SP^V.

From a^P the sides of the card will vanish at v^{ab} and v^{ad} respectively. Measure off from a^P on VH_1, to the right, a distance ($a^P b_1$) equal to the length of the side $a^H b^H$ shown in the given plan. A measure line through b_1, vanishing at m^{ab}, will deter-

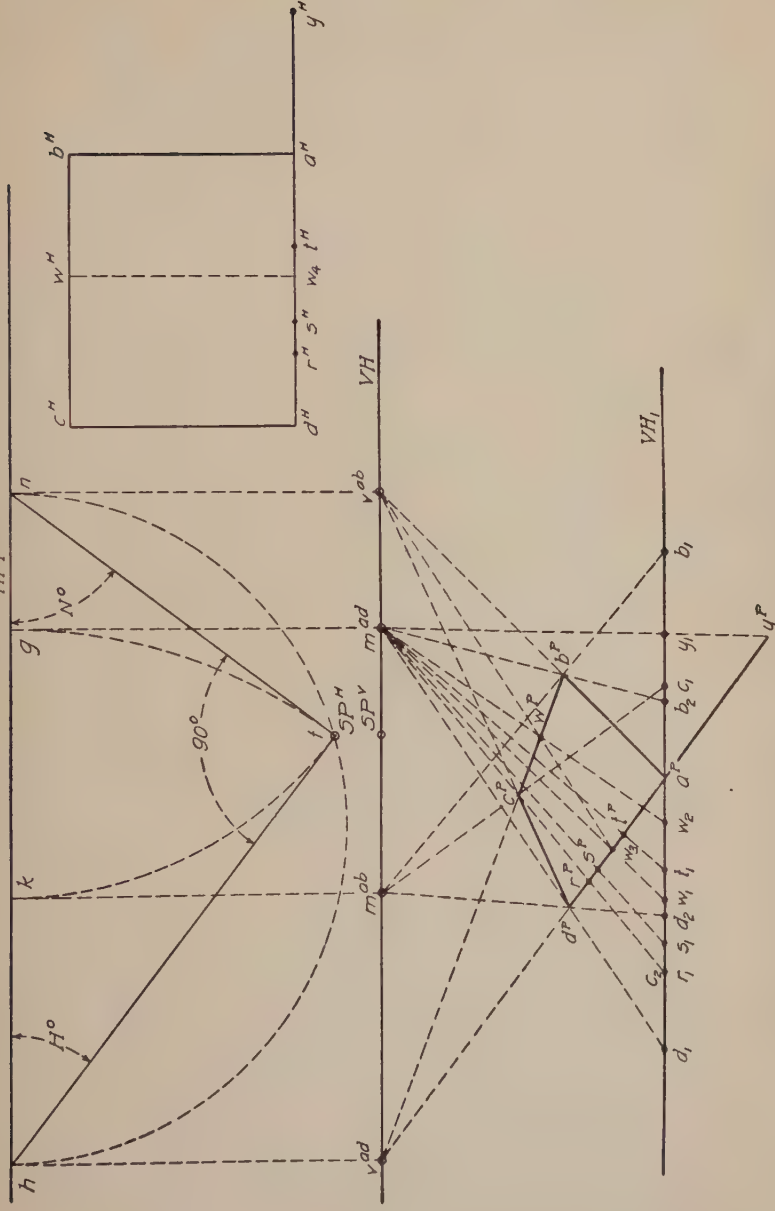


Fig. 27. Diagram Showing Method of Determining True Length of Lines by the Measure-Point System

mine the length of $a^{P}b^{P}$. Measure off from a^{P} on VH_1, to the left, a distance ($a^{P}d_1$) equal to the length of the side $a^{H}d^{H}$ shown in the given plan. A measure line through d_1 vanishing at m^{aq}, will determine the length of $a^{P}d^{P}$.

From b^{P} and d^{P}, the remaining sides of the card vanish to v^{ad} and v^{ab} respectively, determining by their intersection the point c^{P}.

The line $a^{H}d^{H}$ in plan is divided by points t^{H}, s^{H}, and r^{H}. To divide the perspective ($a^{P}d^{P}$) of this line in a similar manner, lay off on VH_1 from a^{P}, to the left, the divisions t_1, s_1, and r_1, as taken from the given plan. Measure lines through t_1, s_1, and r_1, vanishing at m^{ad}, will intersect $a^{P}d^{P}$, and determine t^{P}, s^{P}, and r^{P}.

103. As has already been stated, the true length of any line is always measured on VH_1, and from the true length, the length of the perspective projection of the line is determined by measure lines vanishing at the measure point for the line whose perspective is being determined. Care must be taken to measure off the true length of the line in the proper direction. The general rule for doing this is as follows: —

If the measure point of any line is at the *right* of SP^{V}, the true length of the line will be laid off on VH_1 in such a manner that measurements for the *more distant* points in the line will be to the *left* of the measurements for the *nearer* points.

Thus, m^{ad} is at the *right* of SP^{V}. The point d^{P} is more distant than the point t^{P}. Therefore, the measurement (d_1) for the point d^{P} will be to the *left* of the measurement for the point t^{P}. In other words, since m^{ad} is to the *right* of SP^{V}, d_1, which represents a point more distant than t_1, must be to the *left* of t_1, the distance between t_1 and d_1 being equal to the true length of $t^{P}d^{P}$, as shown by $t^{H}d^{H}$ in the given plan.

On the other hand, if the measure point for any system of lines is to the *left* of SP^{V}, the true measurements for any line of the system should be laid off on VH_1 in such a manner that measurements for more *distant points* on the line are to the *right* of measurements for the *nearer* points of the line.

Thus, m^{ab} is to the *left* of SP^V. The point b^P is *more distant* than the point a^P. Therefore, b_1, which shows the true measurement for the point b^P, must be laid off to the *right* of a^P.

104. It sometimes happens that a line extends in front of the picture plane, as has already been seen in the lines of the nearest corner of the porch in Fig. 22. It may be desired to extend the line $a^P d^P$ in Fig. 27, in front of the picture plane, to the point y^P, as indicated in the perspective projection. In this case, the point a^P being *more distant* than the point y^P, and m^{ad} being to the *right* of SP^V, the true measurement of $a^P y^P$ must be laid off on VH_1 in such a manner that the measurement for a^P will be to the *left* of the measurement for y^P. In other words, y_1 must be on VH_1 to the right of a^P, the distance $a^P y_1$ showing the true length of $a^P y^P$.

NOTE. — The true length of any line which extends in front of the picture plane will be *shorter* than the perspective of the line.

105. Having determined the perspective of any line, as $d^P c^P$, its true length may be determined by drawing measure lines through d^P and c^P. The distance intercepted on VH_1 by these measure lines will show the true length of the line. Thus, $d^P c^P$ vanishes at v^{ab}. Its measure point must therefore be m^{ab}. Two lines drawn from m^{ab}, and passing through c^P and d^P respectively, will intersect VH_1 in the points c_1 and d_2. The distance between c_1 and d_2 is the true length of $c^P d^P$. This distance will be found equal to $a^P b_1$, which is the true measure for the opposite and equal side ($a^P b^P$) of the rectangle.

In a similar manner, the true length of $b^P c^P$ may be found by drawing measure lines from m^{ad} through b^P and c^P respectively. $b_2 c_2$ will show the true length of $c^P b^P$, and should be equal to $a^P d_1$, which is the true length of the opposite and equal side ($a^P d^P$) of the rectangle.

106. The perspective (w^P) of a point on one of the rear edges of the card may be determined in either of the following ways: —

1st. From b_2, which is the intersection with VH_1 of the measure line through b^P, lay off on VH_1, to the left (§ 103), the distance $b_2 w_2$ equal to the $b^H w^H$ taken from the given plan. A

measure line through w_2, vanishing at m^{ad}, will intersect $c^P b^P$ at the point w^P.

2d. In the given plan draw a line through w^H, parallel to $a^H b^H$, intersecting $a^H d^H$ in the point w_4. On VH_1 make $a^P w_1$ equal to $a^H w_4$, as given in the plan. A measure line through w_1, vanishing at m^{ad}, will determine w_3 on $a^P d^P$. From w_3, a line vanishing at v^{ab} will determine, by its intersection with $b^P c^P$, the position of w^P.

107. In making a perspective by the method of perspective plan, it is generally customary to assume VH and HPP coincident. That is to say, the coordinate planes are supposed to be in the position shown in Fig. 9, instead of being drawn apart as indicated in Fig. 9a. This arrangement simplifies the construction somewhat.

This is illustrated in four steps, Fig. 28, showing a complete problem in the method of perspective plan. Compare this figure

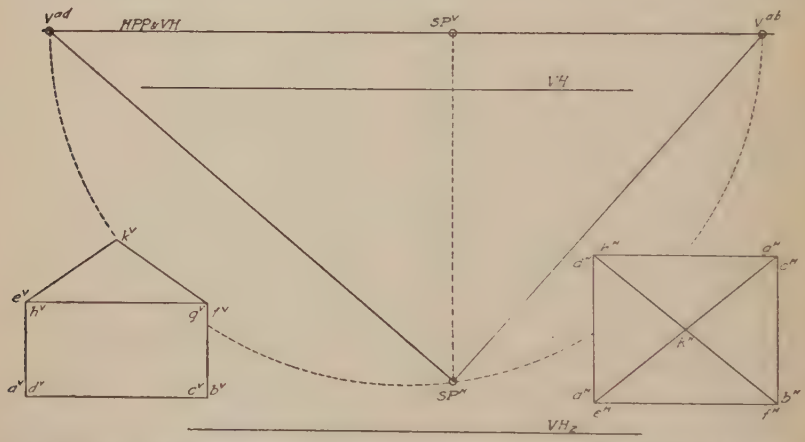


Fig. 28. *Step 1* shows the method of finding the vanishing point for the horizontal lines when no diagram is given. $SP^H V^{ab}$ and $SP^H V^{ad}$ form the angle at which the diagram is placed to HPP

with Fig. 27, supposing that, in Fig. 27, HPP with all its related horizontal projections could be moved downward, until it just coincides with VH. The point n would coincide with v^{ab}, h with v^{ad}, and the arrangement would be similar to that shown in Fig. 28. All the principles involved in the construction of the measures, points, etc., would remain unchanged.

108. The vanishing points in Fig. 28, *Step 1*, have first been assumed, as indicated at v^{ab} and v^{ad}. As the plan of the object is rectangular, SP^H may be assumed at any point on a semicircle

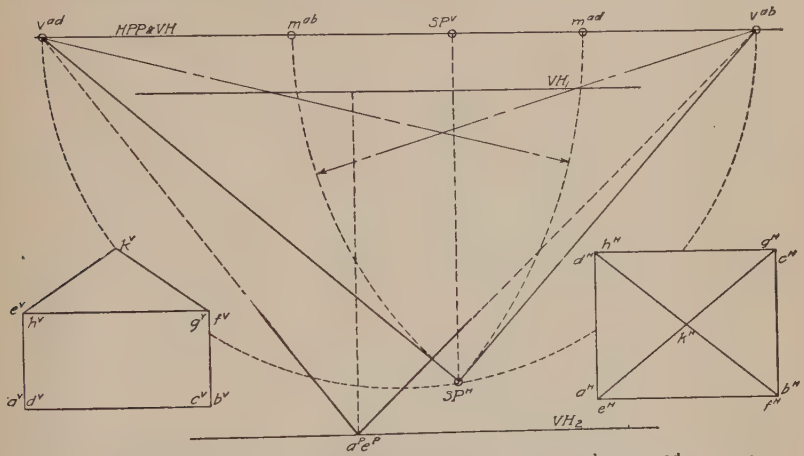


Fig. 28. *Step 2* shows the method of finding the measure points m^{ab} and m^{ad}. The distance between v^{ab} and m^{ab} on HPP and VH is equal to the radius of the arc, and is equal to $v^{ab}SP^H$

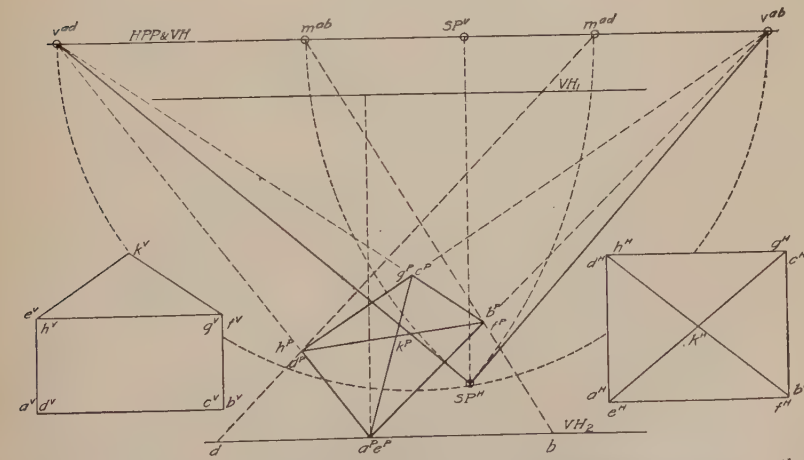


Fig. 28. *Step 3* shows the measure points on VH_2 at *db*, and the lines of measure from these points to m^{ab} and m^{ad} give the points of location for $d^P d^P$

constructed with $v^{ab}v^{ad}$ as diameter. By assuming SP^H in this manner, lines drawn from it to v^{ab} and v^{ad} respectively must be at right angles to one another, since any angle that is just contained in a semicircle must be a right angle. These lines show

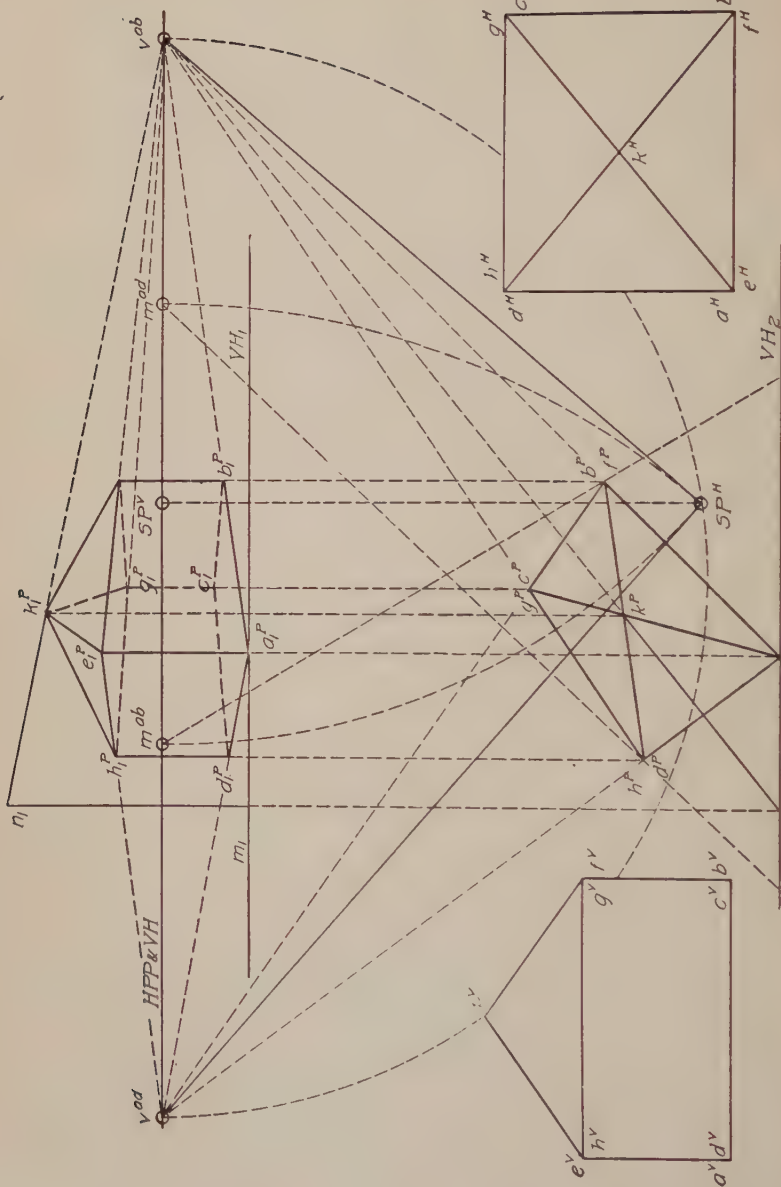


Fig. 28. *Step 4* shows the finding of the line of measure for the height of the roof in the perspective view

by the angles they make with HPP, the angles that the vertical walls of the object in perspective projection will make with the picture plane (§ 102).

109. m^{ad} and m^{ab} have been found, as shown in *Step 2* and explained in § 97, in accordance with the rule given in § 98.

VH_2 should next be assumed at some distance below VH, to represent the vertical trace of the horizontal plane on which the perspective plan is to be made (§ 91). The position of a^P (on VH_2) may now be assumed, and the perspective plan of the object constructed from the given plan, as was done in Fig. 27, *Step 3*.

110. Having constructed the complete perspective plan, every point in the perspective projection will be found *vertically* above the corresponding point in the perspective plan. VH_1 is the vertical trace of the plane on which the perspective projection is supposed to rest. a_1^P is found on VH_1 vertically over a^P in the perspective plan. $a_1^P e_1^P$ is a vertical line of measures for the object, and shows the true height given by the elevation.

To find the height of the apex (k_1^P) of the roof, imagine a horizontal line parallel to the line *ab* to pass through the apex, and to be extended till it intersects the picture plane, *Step 4*. A line of measures drawn through k^P, vanishing at v^{ab}, will represent the perspective plan of this line, and will intersect VH_2 in the point m, which is the perspective plan of the point where the horizontal line through the apex intersects the picture plane. The vertical distance $n_1 m_1$, laid off from VH_1, will show the true height of the point k above the ground. k_1^P will be found vertically above k^P, and on the line through n_1 vanishing at v^{ab}.

111. Fig. 29 is an example similar to Fig. 28. The student should follow carefully through the construction of each point and line in the perspective plan and in the perspective projection.

Plate VI should now be solved.

CURVES

112. Perspective is essentially a science of straight lines. If curved lines occur in a problem, the simplest way to find their perspective is to refer the curves to straight lines.

If the curve is of simple, regular form, such as a circle or an ellipse, it may be enclosed in a rectangle. The perspective of the

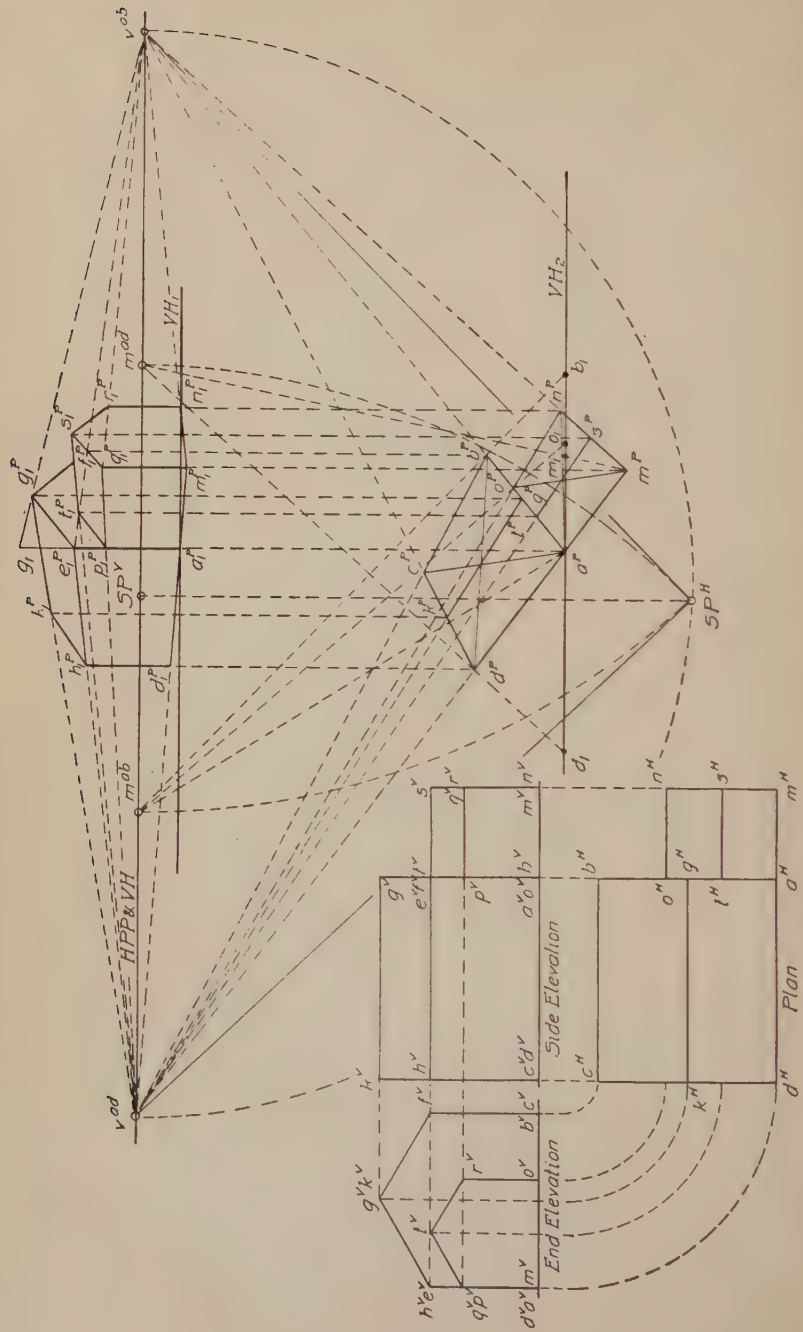


Fig. 29. Diagram Showing Perspective Plan Similar to Fig. 28

enclosing rectangle may then be found. A curve inscribed within this perspective rectangle will be the perspective of the given curve.

Fig. 30 shows a circle inscribed in a square. The points of intersection of the diameters with the sides of the square give the four points of tangency between the square and circle. The sides of the square give the directions of the circle at these points. Additional points on the circle may be established by drawing the diagonals of the square, and through the points m^{H}, k^{H}, n^{H}, and h^{H} drawing construction lines parallel to the sides of the square, as indicated in the figure.

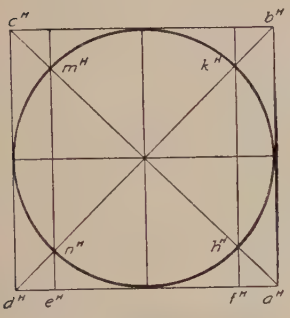


Fig. 30. Circle Inscribed in a Square

Fig. 31 shows the square, which is supposed to lie in a horizontal plane, in parallel perspective. One side of the square ($a^{P}d^{P}$) lies in the picture plane, and will show in its true size. The vanishing point for the sides perpendicular to the picture plane will coincide with SP^{V} (§ 52, note). The measure point for these sides has been found at m^{ab}, in accordance with

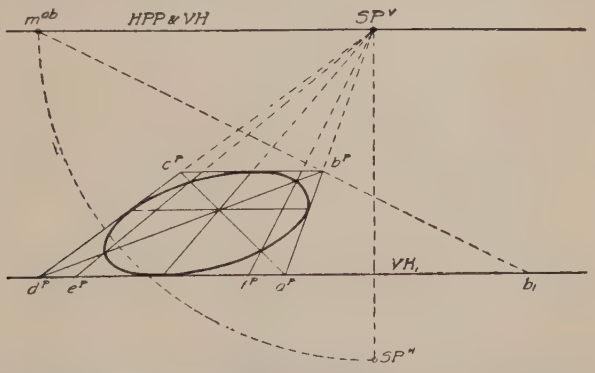


Fig. 31. Diagram Showing Square Lying in Horizontal Plane Drawn in Parallel Perspective

principles already explained. $a^{P}b_{1}$ is laid off on VH_{1} to the right of the point a^{P}, equal to the true length of the side of the square. A measure line through b_{1}, vanishing at m^{ab}, will determine the position of the point b^{P}. $b^{P}c^{P}$ will be parallel to $a^{P}d^{P}$ (§ 54, note).

The diagonals of the square may be drawn. Their intersection will determine the perspective center of the square. The diameters will pass through this perspective center, one vanishing at SP^V, and the other being parallel to $a^P d^P$ (§ 54, note).

The divisions $d^P e^P$ and $a^P f^P$ will show in their true size. Lines through e^P and f^P, vanishing at SP^V, will intersect the diagonals of the square, giving four points on the perspective of the circle. Four other points on the perspective of the circle will be determined by the intersections of the diameter with the sides of the square. The perspective of the curve can be drawn as indicated.

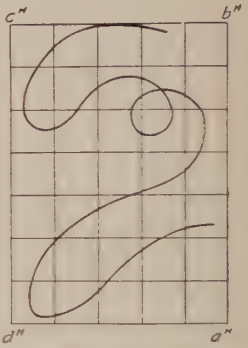


Fig. 32. Diagram of an Irregular Curve

113. If the curve is of very irregular form, such as that shown in Fig. 32, it can be enclosed in a rectangle, and the rectangle divided by lines, drawn parallel to its sides, into smaller rectangles, as indicated in the figure.

The perspective of the rectangle with its dividing lines may then be found, and the perspective of the curve drawn in free

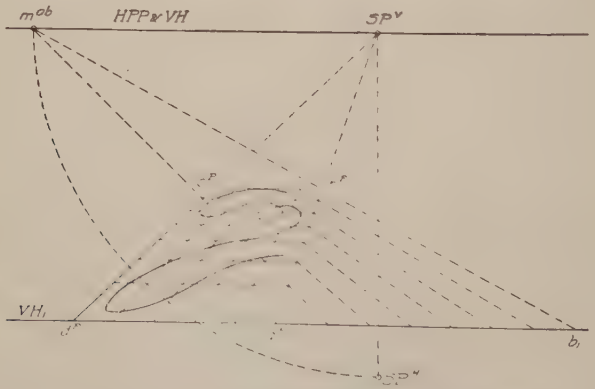


Fig. 33. Method of Making a Perspective of the Curve of Fig. 32

hand. This is shown in Fig. 33. If very great accuracy is required, the perspectives of the exact points where the curve crosses the dividing lines of the rectangle may be found.

APPARENT DISTORTION.

114. There seems to exist in the minds of some beginners in the study of perspective, the idea that the drawing of an object made in accordance with geometrical rules may differ essentially from the appearance of the object in nature. Such an idea is erroneous, however. The only difference between the appearance of a view in nature and its correctly constructed perspective projection is that the view in nature may be looked at *from any point*, while its perspective representation shows the view as seen from one particular point, and from that point only.

For every new position that the observer takes, he will see a new view of the object in space, his eye always being at the apex of the cone of visual rays that projects the view he sees (see Fig. 1). In looking at an object in space, the observer may change his position as often as he likes, and will see a new view of the object for every new position that he takes.

115. This is not true of the perspective projection of the object, however. Before making a perspective drawing, the position of the observer's eye, or station point, must be decided upon, and the resulting perspective projection will represent the object as seen from this point, and from this point only. The observer, when looking at the drawing, in order that it may correctly represent to him the object in space, must place his eye exactly at the assumed position of the station point. If the eye is not placed exactly at the station point, the drawing will not appear absolutely correct, and under some conditions will appear much distorted or exaggerated.

116. Just here lies the defect in the science of perspective. It is the assumption that the observer has but one eye. Practically, of course, this is seldom the case. A drawing is generally seen with two eyes, and the casual observer never thinks of placing his eye in the proper position. Even were he inclined to do so, it would generally be beyond his power, as the position of the station point is seldom shown on the finished drawing.

117. As an illustration of apparent distortion, consider the perspective projection shown in Fig. 23. In order that the perspectives of the vanishing points might fall within the rather

narrow limits of the plate, the station point in the figure has been assumed very close to the picture plane, the distance from HPP to SP^H showing the assumed distance from the paper at which the observer should place his eye in order to obtain a correct view of the perspective projection. This distance is so short it is most improbable that the observer will ordinarily place his eye in the proper position when viewing the drawing. Consequently the perspective projection appears more or less unnatural or distorted. But, for the sake of experiment, if the student will cut a small, round hole, one quarter of an inch in diameter, from a piece of cardboard, place it directly in front of SP^V and at a distance from the paper equal to the distance of SP^H from HPP, and if he will then look at the drawing through the hole in the cardboard, closing the eye he is not using, he will find that the unpleasant appearance of the perspective projection disappears.

It will thus be seen that unless the observer's eye is in the proper position while viewing a drawing, the perspective projection may give a very unsatisfactory representation of the object in space.

118. If the observer's eye is not very far removed from the correct position, the apparent distortion will not be great, and in the majority of cases will be unnoticeable. In assuming the position for the station point, care should be taken to choose such a position that the observer will naturally place his eyes there when viewing the drawing.

119. As a person naturally holds any object at which he is looking directly in front of his eyes, the first thought in assuming the station point should be to place it so that it will come very nearly in the center of the perspective projection.

120. Furthermore, the normal eye sees an object most distinctly when about ten inches away. As one will seldom place a drawing nearer to his eye than the distance of distinct vision, a good general rule is to make the *minimum* distance between the station point and the picture plane about ten inches. For a small drawing, ten inches will be about right; but, as the drawing increases in size, the observer naturally holds it farther and farther from him, in order to embrace the whole without having to turn his eye too far to the right or left.

121. Sometimes a general rule is given to make the distance of the station point equal to the altitude of an equilateral triangle, having the extreme dimensions of the drawing for its base, and the station point for its apex.

122. The apparent distortion is always greater when the assumed position of the observer's eye is too near than when it is too far away. In the former case, objects do not seem to diminish sufficiently in size as they recede from the eye. On the other hand, if the observer's eye is between the assumed position of station point and the picture plane, the effect is to make the objects diminish in size somewhat too rapidly as they recede from the eye. This effect is not so easily appreciated nor so disagreeable as the former. Therefore it is better to choose the position of the station point too far away, rather than too near.

123. Finally, the apparent distortion is more noticeable in curved than in straight lines, and becomes more and more disagreeable as the curve approaches the edge of the drawing. Thus, if curved lines occur, great pains should be taken in choosing the station point; and, if possible, such a view of the object should be shown that the curves will fall near the center of the perspective.

124. The student should realize that the so-called distortion in a perspective projection is only an apparent condition. If the eye of the observer is placed exactly at the position assumed for it when making the drawing, the perspective projection will exactly represent to him the corresponding view of the object in space.

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